Paper

Reflection from layered dielectric structures with combined regular and random inhomogeneities

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Abstract — Optical wave reflection from a layered half-space with regular and random inhomogeneities where the regular perturbations correspond to a linear waveguide near the half-space boundary. Random inhomogeneities are simulated in the frame of the white noise model. The problem is solved analytically in a framework of the embedding method which reduces a boundary problem to the problem with initial values considering the field as a function of the half-space boundary coordinate and obtaining then its solution as a steady-state probability density of the reflection coefficient phase. Numerical calculations revealed some features of the field behaviour under the combined influence of regular and random inhomogeneities such as the reflection coefficient phase increasing inhomogeneity from uniform distribution for small regular inhomogeneities toward a strong peak at the phase equal to $\pi/2$ for increasing ones, and some fine effects which are still greater then the calculation accuracy.

Keywords* — *optical waveguides, layered structures.

Introduction

Dielectric layered media are widely used as optical field transformers and reflectors. Particularly, artificial layered media can serve as 1D PBG structure whose transmission properties exhibit frequency bands where the propagation is forbidden, or a frequency filter in a case of periodic disturbance of the structure periodicity. Such structures were usually considered as regularly inhomogeneous media.

There are two reasons for consideration of random inhomogeneities together with regular ones in such structures. The first one is that on experimental graphical results one can usually see irregular oscillations imposing on theoretically estimated characteristics which suggests the idea of statistical behaviour in addition to the regular one. The second one follows from the well known fact that for 1D half-space with random refraction index is ideal reflector for a wave of any frequency. It allows to assume that consideration of random inhomogeneities together with regular ones can expose new possibilities of creation of structures with properties of PBG crystals.

Random inhomogeneity in high-permittivity media can take place due to interaction processes near the media layers joints as well as the media internal properties appearing in certain external conditions, particularly for nonlinear media one can select a parameters area where its behaviour is chaotic being applicable for a statistic description.

In this work, we consider a model of a stationary problem of wave propagation in a layered half-space with regular and random inhomogeneities with regular perturbations corresponding to a linear waveguide near the half-space boundary. Random inhomogeneities are simulated in the frame of the white noise model. We analyze an influence of regular and random inhomogeneities on probability distribution of the reflection coefficient phase and the wavefield average intensity at the boundary of the half-space.

Mathematical formulation of the problem

We consider a randomly inhomogeneous slab occupying the region $L_0 \leq x \leq L$. An incident field is assumed as a plane harmonic wave $E(t,x) = U_0(x)\mathrm{e}^{i\omega t}$, $U_0(x) = \mathrm{e}^{ik(l-x)}$, propagating from the region $x > L$ of a homogeneous space. Inside the inhomogeneous slab the field amplitude $U(x)$ is described by Helmholtz equation

$$\frac{\mathrm{d}^2}{\mathrm{d}x^2} U(x) + k^2(x)U(x) = 0, \tag{1}$$

where function $k(x)$ characterizes regular and statistical inhomogeneities of the medium. Boundary conditions for this equation follows from continuity of $U(x)$ and $\partial U(x)/\partial x$ on the slab boundaries:

$$\begin{aligned} \frac{i}{k}\frac{\mathrm{d}U(x)}{\mathrm{d}x} + U(x)\big|_{x=L} &= 2 \\ \frac{i}{k}\frac{\mathrm{d}U(x)}{\mathrm{d}x} - U(x)\big|_{x=L_0} &= 0\,. \end{aligned} \tag{2}$$

For the problem (1), (2) we are interested in the statistical characteristics of the wavefield when $k(x)$ contains regular and random components.

In previous works [1,2] we had shown that for a spectral parameter comparable with a wave number, a reflection coefficient phase distribution is non-uniform. It can touch the condition for average method applicability leading to false results. So, it is important to investigate the influence of the reflection coefficient phase distribution on field statistical characteristics for determination and expansion of the statistical theory frames.

The problem is solved analytically in a framework of the embedding method which reduces a boundary problem to the problem with initial values considering the field as a function of the half-space boundary coordinate and obtaining then its solution as a steady-state probability density of the reflection coefficient phase. Numerical calculation is carried out then to reveal some features of the field behaviour.

Dynamical equations solution

Wavefield effects for the considered problem are determined by scattering on inhomogeneities inside the medium and on spatial jumps of $k(L_0)$ and $k(L)$. We exclude the boundaries influence supposing that the medium occupies the half-space $x < L(L_0 \to \infty)$ and that the right boundary is adjusted: $k^2 = k^2(L) = k_L^2$.

Suppose that the wavefield $U(x) = U(x,L)$ is a function of the parameter L. Then in the framework of the embedding method the boundary problem (1), (2) is reduced [3] to the problem with initial values with respect to L, and the equation for the reflection coefficient has the form:

$$\frac{\mathrm{d}R_L}{\mathrm{d}L} = 2ik_L R_L + \frac{k_L'}{2k_L}\left(1 - R_L^2\right), \; R_{L_0} = 0\,, \qquad (3)$$

where $k_L'/k_L = \partial \ln k_L / \partial L$.

After substitution the representation for the reflection coefficient as $R_L = \rho_L e^{i\phi_L}$ into (3) and taking into account that the quantity $\rho_L = 1$ with probability equal to unit in a case when the wave is incident on a random half-space $(L_0 \to \infty)$, the following differential equation for ϕ_L is obtained:

$$\frac{\mathrm{d}\phi_L}{\mathrm{d}L} = 2k_L - \frac{k_L'}{k_L}\sin\phi_L\,. \qquad (4)$$

Combined influence of regular and random inhomogeneities on reflection coefficient is considered for the velocity profile as $c(l) = c_0\,(1 + \alpha L + \varepsilon(L))$. It is linked with $k(L)$ by the relation $k(L) = \omega/c(L)$, where ω is a cyclic frequency. Function $\varepsilon(x)$ is supposed to be homogeneous mean zero Gaussian process, that is $\langle\varepsilon(x)\rangle = 0$; correlation function B determined as $B_{\varepsilon\varepsilon}(x,x') = \langle\varepsilon(x)\varepsilon(x')\rangle = \sigma_\varepsilon^2 B\left(|x-x'|/l_0\right)$, parameter σ_ε^2 characterizes the intensity of fluctuations and l_0 denotes a correlation radius. Regular and random inhomogeneities are considered under a condition that $\sigma = |\alpha/p| < 2\,,\; p = \omega/c_0$. Assuming that the fluctuations are small $\left(\sigma_\varepsilon^2 << 1\right)$ and the regular profile is a slow function of L, Eq. (4) can be written in a shortcut form as follows:

$$\frac{\mathrm{d}\phi_L}{\mathrm{d}L} = 2p + (\alpha + \xi(L))\sin\phi_L\,,\; \xi(L) = \frac{\partial\varepsilon(L)}{\partial L}\,. \qquad (5)$$

Function $\xi(x)$ is the Gaussian process with the following parameters:

$$\langle\xi(x)\rangle = 0\,,\; B_{\xi\xi}(x,x') = \langle\xi(x)\xi(x')\rangle = -\frac{\partial^2}{\partial x^2}B_{\xi\xi}(x-x').$$

Statistical analysis of Eq. (5) is carried out in the framework of the approach of the works [1,2]. Its main ideas are the follows: we introduce the variable $z_L = \tan\phi_L/2$ instead of the phase; to simplify the further solution we assume that $\varepsilon(L)$ is the Gaussian random delta-correlated process (conditions for applicability of this approximation are considered in [3]). Then the steady-state probability density $P(z)$ of this value has the form

$$P(z) = h(z)P_+(z) + h(-z)P_-(z), \qquad (6)$$

where

$$P_+(z) = h(z_- - z)P_1(z) + h(z - z_-)h(z_+ - z)P_2(z) +$$
$$+h(z - z_+)P_3(z),\; z > 0,$$

$$P_1(z) = CA(z)\int_0^z dz_1\left(\frac{z_1 - z_-}{z_1 - z_+}\right)^{k\sqrt{d}} z_1^{k\sqrt{d+4}-1}\,,$$

$$P_2(z) = CA(z)\int_z^{z_+} dz_1\left(\frac{z_1 - z_-}{z_1 - z_+}\right)^{k\sqrt{d}} z_1^{k\sqrt{d+4}-1}\,,$$

$$P_3(z) = CA(z)\int_{z_+}^{z} dz_1\left(\frac{z_1 - z_-}{z_1 - z_+}\right)^{k\sqrt{d}} z_1^{k\sqrt{d+4}-1}\,,$$

$$P_-(z) = P_1(z),\; z < 0\,,$$

$$A(z) = \frac{1}{\sqrt{d+4}}\,\frac{(z - z_+)^{k\sqrt{d}-1}}{\left(z_1 - z_+\right)^{k\sqrt{d}+1}}\,z^{k\sqrt{d+4}}\,,$$

h is the Heaviside step-function and parameters are the follows: $d = 16/\sigma^2 - 4$. The constant value $C \equiv C(k,d)$ is defined from the condition with the parameters $k = p/8D$ and $D = p^2\sigma_\varepsilon^2\,l_0/2$. The correspondent steady-state probability distribution of the phase ϕ_L defined within the interval $(-\pi, +\pi)$ can be obtained from (6) by the formula

$$P(\phi) = \left(1 + z^2\right)\,P(z)\,/\,2\,|_{z=\tan\phi/2}\,.$$

Discussion

We had shown that for a spectral parameter comparable with a wave number, a reflection coefficient phase distribution is non-uniform. It can touch the condition for average method applicability leading to false results. So it is important to investigate the influence of the reflection coefficient phase distribution on field statistical characteristics for determination and expansion of the statistical theory frames.

Rigorous restriction for applicability of formula (6) is defined by the condition $\alpha/D << 1$. It makes possible to use the shortcut Eq. (5) to describe the wavefield behaviour in a medium with random and regular inhomogeneities. For the limit cases as 1) $k >> 1$ and $|\,\sigma\,| << 1$ and 2) $|\,\sigma\,| \to 2$ the phase has the uniform distribution or fluctuates near the deterministic solution $\phi_\infty = \pi/2$, correspondingly, which agrees with the known results.

Computer analysis of $P(z)$ behaviour was carried out for three sufficiently different cases: (A) $k = -3.13, \sigma = 0.1$; (B) $k = -3.13, \sigma = 1.0$ and (C) $k = -3.13, \sigma = 1.9$. It is shown that the case A corresponds to the phase uniform distribution. The case C describes the situation when the

phase tends to the deterministic value $\pi/2$. To extract an influence of the regular inhomogeneity on the probability distribution it was calculated in absence of the inhomogeneity for different values of k. $P(\phi)$ maximum occured to be displaced to the right when the parameter σ increases.
Calculation of the wavefield intensity on the boundary as a function of σ for different values of k showed that the results can be approximated by a linear function.

References

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