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Decision support for telecommunications and information society

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Preface

Annual International Conferences on *Decision Support for Telecommunications and Information Society*, organized by the National Institute of Telecommunications in Warsaw, Poland, assembled researchers working in several modern and important fields of telecommunications and computer science. Some papers submitted to these conferences are presented at this issue.

It is generally accepted that information and knowledge are becoming essential economic assets. Telecommunication is naturally integrated with other informational technologies, and diverse problems of telecommunications need formulation with multiple criteria. These three connected subjects: *knowledge, telecommunications* and *decision systems*, are discussed in the first paper and some others, presented here.

Routing is an important aspect of telecommunications networks because it can greatly influence overall network performance. It is also a difficult, distributed multi-objective problem, with many possible solutions. Three quite different approaches are described here: multi-objective dynamic routing model, swarm intelligence for network routing optimization and distributed asynchronous algorithms.

Routing is connected with *traffic control*. Some papers analyze optimization algorithms of traffic control and comparison between them.

The problem of telecommunications *network design*, with the objective to maximize service data flows and provide fair treatment of all services, is very up-to-date. One suggested approach is based on max-min fair solution concept and the other – on heuristic and integer programming.

Decision analysis is represented by two papers. The first one uses Decision Making Trial and Evaluation Laboratory (DEMATEL) to construct structural models of various factors that prevent safety and security of our life. The second proposes multiple criteria decision analysis of the problems with uncertainties, based on the evidential reasoning approach, and used for supplier assessment and customer satisfaction.

Knowledge representation and *data mining* are relevant to knowledge discovery – very important and promising area of science and technology. The papers in this issue discuss some aspects of structural representation and event mining based on observations.

The subjects are different but joined by one common idea – decision support.

Wiesław Traczyk
Guest Editor

Paper

Telecommunications, multiple criteria analysis and knowledge theory

Andrzej P. Wierzbicki

Abstract— Telecommunications requires multiple criteria analysis and decision support. It is shown how some basic facts from telecommunications and informational sciences can be used to formulate a rational theory of intuition, developed as a complement of multiple criteria decision support. This paper presents a method called creative space used for integrating various approaches to knowledge creation and based on SECI spiral, I^5 system and rational theory of intuition. Questions of supporting new technology creation by constructing specialized creative environments similar to decision support environments are also indicated.

Keywords— decision support, intuition, knowledge creation, knowledge integration and management, tacit knowledge, ontology and epistemology, technology creation, telecommunications.

1. Introduction

We can list diverse problems from telecommunications that need formulation with multiple criteria: in network design, in routing, in telecommunication data mining, in interconnection agreements, in strategic management of telecommunications. However, telecommunications and other informational sciences influence also our way of understanding the world in the new civilisation era of informational and knowledge-based economy. This understanding is systemic and chaotic, assumes the emergence of qualitatively new properties of complex systems which cannot be reduced to the properties of system components. On this background, it is necessary to reflect anew on the theory of knowledge.

In fact, multiple criteria decision support developed, during several decades of research, methodologies that are useful in knowledge representation and creation today. During the last decade of 20th century, several new approaches explaining knowledge creation processes were published. The first of them, Shinayakana systems approach of Nakamori and Sawaragi [21], originated in multiple criteria decision support. Much better known become another approach, originating in management science, the knowledge creating company with SECI spiral process of Nonaka and Takeuchi [24]. Several other approaches were developed and published parallel; this signified a paradigmatic change in epistemology.

2. Telecommunications and decision support versus knowledge creation

Telecommunications becomes today naturally integrated with other informational technologies. Telecommunication networks not only offer more intelligent services, but also require the use of computer intelligence and other advanced informational civilization tools, such as multiple criteria decision support, for being effectively designed, managed and developed strategically. Diverse problems from telecommunications need formulation with multiple criteria. In fact, network design is an essentially multiple criteria decision process. Routing problems have been traditionally solved by assuming *ad hoc* scalarized aggregation of multiple criteria; recently, it is becoming recognized that we must use many criteria in routing and explicitly discuss the questions how to aggregate them. Related techniques of decision support, such as data mining, are increasingly developed and used in telecommunications, simply because the amount of data available concerning the functioning of telecommunication networks is tremendous. Decision support techniques become needed when solving strategic management problems in telecommunications, such as problems related to interconnection agreements that require multiple criteria negotiation techniques.

On the other hand, during several decades of research multiple criteria decision support developed specific methods that are useful in knowledge representation and creation today. For example, *model based decision support* [39] distinguishes between *preference model* and *substantive* or *core model* of the decision situation; while the former represents information about the preferences of the decision maker, the latter represents a synthesis of knowledge about the essential aspects of the decision situation, independent of the decision maker preferences. Thus, mathematical modeling in decision support is used today in order to create *virtual laboratories*, to *represent and organize knowledge* [40].

The advances of computerised decision support, in particular related to vector optimisation and multiple criteria decision making, contributed also to the concept of user sovereignty – the sovereign role of the user of computerised decision support (e.g., [39]). This concept is also related to the assumption that human mind is capable of information processing ways as yet not duplicated by computers. This assumption and the reflection on applications of multiple criteria decision support resulted in *Shinayakana* *systems approach* of Nakamori and Sawaragi [21] as well as in formulation of a *rational theory of intuition* [37]; as we shall show later, this theory utilizes basic knowledge from telecommunications. In turn, a reflection on the needs of the beginnings of the new civilisation era of information and knowledge-based economy shows that multiple criteria decision making and the resulting rational explanation of human intuition are closely related to new developments in knowledge theory and that a new understanding of knowledge theory is necessary for the new era.

Moreover, we could use diverse methods developed for criteria aggregation in multiple criteria decision support also for synthesizing and aggregating knowledge. Thus, methods of decision support could naturally evolve into methods of knowledge integration and creativity support. These possibilities are the subject of research at the JAIST 21st Century Center of Excellence (COE) Program *Technology Creation Based on Knowledge Science*, in which scientific and technical development strategies can be formulated in cooperation with outside research organizations. At the same time, the COE contributes to the development of an education system that will demonstrate the synergetic effect of combining diverse disciplines and fields. The COE offers an advanced model of setting research priorities for three JAIST graduate schools: Information Science, Material Science and Knowledge Science. However, before commenting further on the possibility of such developments we must become aware of contemporary developments in knowledge theory.

3. New approaches to the problem of knowledge and technology creation

Historically, we could distinguish two main schools of thinking how knowledge is created.

The first school maintained that knowledge creation is essentially different activity than knowledge validation and verification, and distinguished *the context of discovery* from *the context of verification*. This school also maintained that creative abilities are irrational, intuitive, instinctive, subconscious. Such opinion was supported by many great thinkers of very diverse philosophical persuasions and disciplinary speciality. Nietzsche, Bergson, Poincare, Brouwer, Einstein, Heisenberg, Bohr, Freud, Jung, Popper, Kuhn, Polanyi were all characterizing creative abilities in such a way, although every one of them stressed different aspects of this general view.

The second stream kept to the old interpretations of science as a result of induction and refused to see creative acts as irrational. This view, represented by many hard scientists, is popular particularly in English-speaking world[1]. This view is represented also by recent works (e.g., [31]), a book rich in historical data on creativity, but refusing to analyze subconscious or unconscious aspects of creative acts. However, since the last decade of 20th century quite new approaches to knowledge creation appeared, related to these subconscious or unconscious aspects, to the concepts of tacit knowledge, of intuition and of group collaboration, most directly or indirectly related to Japanese origin.

The first of such approaches, *Shinayakana systems approach* of Nakamori and Sawaragi [21], originated in the field of multiple criteria decision support. It did not specify a process-like, algorithmic recipe for knowledge and technology creation, only a set of principles. To these principles belong: using intuition, keeping open mind, trying diverse approaches and perspectives, being adaptive and ready to learn from mistakes, being elastic like a willow but hard as a sword (*Shinayakana*).

Parallel, in management science, another approach was developed by Nonaka and Takeuchi in the book *The Knowledge Creating Company* [24]. This is the now renowned SECI spiral, with a process-like, algorithmic principle of organizational knowledge creation. This principle is revolutionary for western epistemology because it utilizes not only the collaboration of a group in knowledge creation, but also the rational use of irrational (or a-rational to a Japanese) mind capabilities, namely tacit knowledge consisting of emotions and intuition. The SECI spiral results from four consecutive transitions between four nodes on two axes. One is called *epistemological dimension* including *tacit* and *explicit knowledge*; the other is called *ontological dimension*[2] and includes *individual* and *group*. The transition from individual tacit knowledge to group tacit knowledge is called *socialization*, consists of social exchange of ideas; the transition from group tacit to group explicit – *externalization*, consists in writing down and codifying the ideas; the transition from group explicit to individual explicit – *combination*, consist in combining individual knowledge with the ideas generated by the group; the transition from individual explicit to individual tacit – *internalization*, consists in applying new knowledge in practice and thus increasing tacit (actually, intuitive) knowledge. Knowledge is increased after each such cycle, thus the name SECI spiral expresses increasing knowledge in spiral repetition.

But the problem of using irrational or a-rational mind abilities rationally was at this time perceived also by other researchers, in particular in Poland. Starting from interactive decision support, Wierzbicki published the *rational theory of intuition* [37], influenced by the formation of Shinayakana systems approach while spending a year at Kyoto University in 1990. We shall present an outline of this theory in a further part of the paper. Soon afterwards, from the mainstream of philosophy, Motycka [19] proposed another

[1] Perhaps because of the unfortunate property of English language that understands the word *science* in the sense *hard sciences*, excluding *technology*, but also excluding *soft and human sciences* – sociology, economics, law, history, etc. Other languages – such as German, Polish, Japanese – understand the word *science* more broadly and we, speakers of these languages, are prepared for the opinion that creative acts are irrational.

[2] Since also tacit and explicit knowledge are ontological elements of discourse, we shall use here rather the name *social dimension*. We also use here *transition* instead of original *knowledge conversion*, because transition indicates changing point of attention while conversion indicates transforming and using up a resource, while knowledge is a special resource that is not used up when used.

theory, this time of basic knowledge creation. She used for this purpose also irrational abilities of human mind – *instincts and myths*, not intuition, namely the concept of *collective unconscious* of Jung [12]. She postulates that, in times of a crisis of a basic science, scientist use a *regress* to myths and instincts in order to obtain stimulation of novel approaches to their field of science. These two Polish approaches were developed independently from SECI spiral, though influenced by Japanese tradition – Wierzbicki directly by Shinayakana systems approach, Motycka by Jung, and thus indirectly by Freud, Nietzsche.

A few years after international publication of the book *The Knowledge Creating Company* [24], several approaches directly stimulated by this book were also published, including several papers presented at the 37th Hawaiian International Conference on Systems Science in Hawaii 2004. For example, Gasson [9] observed that a Western company would use a process very much resembling SECI spiral but moving in opposite direction.

Further development of Shinayakana systems approach was given by Nakamori [22] in a systemic and process-like approach to knowledge creation called I^5 *system* or *Nakamori pentagram*. Five ontological elements of this system are *intelligence* (and existing scientific knowledge), *involvement* (and social motivation), *imagination* (and other aspects of creativity), *intervention* (and the will to solve problems), *integration* (using systemic knowledge). There is no algorithmic recipe (true to *Shinayakana* tradition) how to move between these nodes. Thus, I^5 *system* stresses the need to move freely between diverse dimensions of creative space.

There is no doubt that, since the beginning of the last decade of 20th century, many approaches were developed stressing and rationalizing the need of using irrational abilities of human mind in creative processes. It is, actually, a scientific revolution, because the philosophy of 20th century (explicitly in the first half of century, tacitly in the second half) was dominated by the principles of logical empiricism that refused to speak about such metaphysical aspects. We interpret this revolution as one of the indications of a new informational and knowledge civilization era.

4. Changing understanding of the world at the beginning of knowledge civilization era

The nature of the current *global information revolution* is described by various perceptions, diagnoses and concepts, but it is generally accepted that new *global information infrastructure* will gradually result in *knowledge-based economy* and in *information society* or even in *networked informational civilization*. Castells [4] rightly argues that we should use the term *informational society* rather than *information society* and that an appropriate concept is *networked society*. These ideas are augmented by the concept of *knowledge-based economy* and by disputes about the similarities and differences between the concepts of *information* and *knowledge*; thus, we might rather speak today about *informational and knowledge-based* civilization era, shortly *knowledge civilization era*.

Knowledge civilization era will be a long duration historical structure in the sense of Braudel [3], who considered such structure between the years 1440 (the rediscovery of print by Gutenberg) to 1760 (the improvement of the discovery of steam engine by Watt). Industrial civilization lasted approximately from 1760 until 1980 and informational civilization will probably last from 1980 (the combination of two earlier discoveries of computer and telecommunication networks, enabling broad social use of informational technology) until the end of 21st century (see [36, 38]). Braudel defined a long duration historical structure as a historical era in which basic ways of understanding the world are relatively stable.

In such diverse interpretations and approaches to the current information revolution, there is also a common basis. There is no doubt that *information and knowledge are becoming essential economic assets* with either private or public character and that it is necessary to develop both rules of their sharing and business models of their selling and exchange. However, not many people understand fully the informational civilisation, many see only its technological aspects or are afraid of diverse threats brought by it. To help in its understanding, the following structural model of informational civilization in the form of its three basic megatrends was proposed in [38]. These megatrends are the following.

- The technological *megatrend of digital integration* (or *convergence*). Since all signals, measurements, data, etc., might be transformed to and transmitted in a uniform digital form, we observe today a long-term process of integrating various aspects of information technology. Telecommunication and computer networks are being integrated. Diverse aspects of intelligence of networks and computers are becoming integrated. Diverse communication media are becoming integrated and there are economic and political fights who will control them. Formerly diversified information technologies – telecommunications, informatics, automatic control, electronic engineering – are becoming integrated, etc. This megatrend will define for many years yet the directions of information technology change.
- The social *megatrend of change of professions* (*of dematerialization of work*). The information technology, the automation of heavy work result together slowly in a de-materialization of work. This, however, induces a rather rapid change of existing professions; in industrial age it was sufficient to learn a profession for entire life, now we must re-learn several times in life. Some old professions disappear, others are essentially changed. The speed of this change is limited by socio-economic factors; technology would permit to build fully automated, robotic

factories even today, but what we shall do with the people that work in existing factories? Since not all people are equally adaptable, this megatrend results in the *digital divide* – on those who can speedily learn and profit from information technology and those who are excluded from this technological progress; this is accompanied by *generational divide*. The digital and generational divide affects and concerns not only people in one country, also diverse countries. These divides can threaten the existence of democratic society and market economy as we know them now. Thus, it is essential to find ways to alleviate the effects of digital and generational divide and, in particular, to devise new professions, new occupations for people in replacement of the old professions and occupations.

- The intellectual *megatrend of mental challenges, of changing the way of perceiving the world*. The perception of the world in industrial society was mechanistic, the world was perceived as a giant mechanism – *a clock turning with the inevitability of celestial spheres*. This resulted, on one hand, in the reduction principle described above, on the other hand, in the dominating belief in *inevitability*. For all specific differences, this belief motivated equally Kant (his *categorical imperative*, the transcendental moral principle *inevitably* follows logical reflection on the moral rights of fellow humans), Smith (the invisible hand of the market expresses *inevitability*) and Marx (with his *inevitability* of laws of history). Such a way of perceiving the world is still predominating (see, e.g., *The End of History* of Fukuyama [7]) and its change will be very difficult and will take time. However, it is very important to understand the change towards *systemic and chaotic way of perceiving the world, which will be typical for informational civilization*.

There are two basic concepts that were developed because of mathematical modeling that exceeded its domain and contributed essentially to the change of perceiving the world typical for the beginnings of a new civilization era. These are the concepts of *chaos* and *complexity*.

We needed to simulate random numbers in a digital computer that is an essentially deterministic device; thus, we quite early discovered the principle of a quasi-random number generator that today would be called a chaotic generator of a strange attractor type. Although this is not stressed in the typical publications on the deterministic theory of chaos (see, e.g., [10]) the quasi-random generators in digital computers were the first practical applications of the theory, preceding in fact the development of the theory. The principle of such a generator exemplifies in fact the basic principles of a strange attractor: take a dynamic system with strong nonlinearity and include in it a sufficiently strong negative feedback to bring it close to instability – or, in discrete time, apply recourse.

For specialists in mathematical modeling of nonlinear systems *there is nothing strange in strange attractors, in order emerging out of chaos, in emergence of essentially new properties* because of the complexity of the system. Order can emerge also from probabilistic chaos, as stressed by Prigogine [29]. The principle of order emerging from a probability distribution is mathematically rather simple: a strongly nonlinear transformation of a probability distribution can result in amplifying the probability of selected events, thus eventually in order.

This was only a *rational justification* of the principle of emergence, *justified also empirically* by earlier observations in nature, e.g., by biology in the concept of punctuated evolution. Additionally, technology and especially telecommunications provided a third type, *pragmatic justification*: in complex technological systems we construct today, complexity could not be mastered without assuming that higher layers of complexity require concepts irreducible to the properties of lower layers. For example, in the ISO-OSI seven layers stack of protocols of teleinformatic networks (computer networks), the functions and properties of higher layers, e.g., the highest *layer of applications*, are independent, irreducible, thus in a sense transcendental to the functions and properties of lower layers, e.g., the lowest *physical layer*.

Therefore, we can say that *biology*, but also *systems science, mathematical modeling, informational and telecommunications technology prepared a fundamental change of the way we perceive the world today*. The science of industrial civilization era perceived the world as a system explained by the behavior of its elementary parts or particles. This *reduction principle – the reduction of the behavior of a complex system to the behavior of its parts* – is valid only if the level of complexity of the system is rather low. With very complex systems today, systems science, biological but also technical and informational sciences adhere rather to *emergence principle – the emergence of new properties of a system with increased level of complexity, qualitatively different than the properties of its parts*.

We should add that the concept of complexity is used above only in its general, qualitative sense, while mathematical modeling and information sciences today developed a specific, quantitative-qualitative *theory of computational complexity*. This theory describes – qualitatively but in quantitative terms – how the computational effort needed for solving a given type of data processing or operational research problem depends on the dimension of the problem. The main conclusion of this theory is that such dependence is very seldom linear, polynomial only for rather simple problems, highly nonlinear – exponential or combinatorial – for most complicated problems.

5. The rational theory of intuition

We stress here that we are interested in intuition not as a mystic, irrational force, opposed to rationality and objectivity, which is fashionable sine at least one hundred years, even today. We are seeking a *rational explanation* of intuition as a basic creative force, necessary in times of knowledge-based economy and civilization. By *rationality* we understand here not the economic rationality of decisions, but a comprehensive rationality of a scientific theory that combines rationalism, empiricism and falsificationism, thus is close to Quine [30] and Popper [27]: a theory is rational, if it is rationalist (can be deductively derived from some abstract principles), but also empirically viable (corresponds *at its edges* to observed facts) and can be falsified with the help of an experiment or at least allows for practical conclusions that can be tested.

A rational theory of intuition can be considered as a contradiction in terms, because we tend to use the word *intuitive* with some connotation of *irrational* in everyday language. There is a long tradition of such usage of this word (see, e.g., [1]) who attached a great importance to intuition but interpreted it as a mystic force, which by definition could not be a subject of rational means of inquiry. After a century, even today *we do not want to make intuition rational, we want only to explain its functioning in rational terms*; however, we stress that such an explanation is not only possible, it is necessary.

First element of the rational theory of intuition is based on contemporary knowledge – from the field of computational complexity and telecommunications – about relative complexity of processing audio and video signals. The ratio of bandwidth necessary for transmitting audio and video signals is ca 1:100 (20 kHz to 2 MHz). Let us assume conservatively that the increase of the complexity of processing these signals (for similar purposes, such as word and picture recognition) is rather mild, say quadratic – the simplest and one of the mildest of nonlinear increases in computational complexity. Then we obtain the lower bound for the ratio of computational complexity of at least 1:10 000. Thus, the old proverb *a picture is worth one thousand words* is not quite correct: *a picture is worth at least ten thousand words*. Naturally, human mind processes signals in a different way than a digital computer, with elements of analog processing and much higher degree of parallelism, distribution, redundancy; and human vision is much better than television. However, these arguments only strengthen the estimation of such a lower bound of processing difficulty that is rather independent of the actual structure of processing device. Anyway, we need further only a qualitative conclusion that processing of visual signals (together with signals from all other human senses) is qualitatively, much more complex than processing speech.

The second element of this theory is a *dual thought experiment*. The technique of a thought experiment was suggested by T. Kuhn [14] who has shown that basic concepts applied in any scientific theory include deep, often hidden assumptions. The best way to examine consistency of such assumptions is not necessarily through empirical experiments, because more enlightening might be thought experiments. Kuhn used such technique for clarifying epistemological assumptions of historical scientific discoveries. Here we use such technique also in historical context, but in order to clarify essential aspects of modern ontology and epistemology, hence we suggest the name *dual* thought experiment.

This experiment consists in considering the question: *how people processed the signals from our environment just before the evolutionary discovery of speech*? They had to process signals from all our senses holistic, though dominant in received information was the sense of sight. Yet they were able to overcome this difficulty, developed evolutionary a brain containing $10^{11} - 10^{12}$ neurons. We still do not know how we use full potential of our brain – but it was needed evolutionary, hence probably fully used before the discovery of speech. We know that the brain processes signals with a great degree of parallelism and distribution, certainly uses neuron networks – though much more complicated than contemporary artificial neural networks – and in a holistic processing of signals uses rather fuzzy than binary logic. Biological research on real neurons shows that an appropriate model of a neuron should be dynamic and nonlinear, with extremely complex behavior. Thus, to model a neuron well we would need the computational capability of a contemporary personal computer, not a single digital switch nor a sigmoid function (the latter being used in contemporary artificial neural networks to represent a single neuron). We conclude that human brain is clearly much more complex than digital computers, though it also processes signals, only in a much more general sense[3].

Reflecting on the dual thought experiment we realize that the discovery of speech was an excellent evolutionary shortcut, we could process signals 10^4 times simpler. The use of speech for interpersonal communication enabled the intergenerational transfer of information and knowledge, we started to build up the cultural and intellectual heritage of mankind, the *third world* of Popper. The biological evolution of people slowed down and eventually stopped – including the evolution of our brains, since we discovered 10^4 times redundancy – but we accelerated intellectual and civilization evolution. Many biologists wonder *why* our biological evolution has stopped. We think that the dual though experiment described here gives a convincing explanation why it happened.

As all simplifications, this had also disadvantages. Seeking better ways of convincing other people, we devised the principle of excluded middle and thus binary logic. An argument of the type: *this must be true or false, there is no third way*, is actually an ideological or political persuasion. Binary logic contributed of course also to tremendous civilization achievements, the construction of computers and computer networks, but it still biases our way of understanding the world. The best example of this bias is *cognitivism* – the conviction that all cognitive processes – including perception, memory and learning – are based on a language-like medium, on a *language of*

[3]Searle [32] argues that human mind does not process signals, but he proves (rightly) that human mind does not process *digital* signals.

thought (see, e.g., [6, 8]) and thus functioning of mind can be modeled as the functioning of a giant computer. Note that cognitivism is a simplification to the same degree as language is a simplification of the original capabilities of our mind.

But language is only a code, simplifying the processing of information about the real world at least 10^4 *times*; therefore, each word must have many meanings, and to clarify our meaning we have to devise new words. By multiplying words, we gradually describe the world more precisely, but we faster discover new aspects of an infinitely complex world – e.g., the *microcosmic* or *macrocosmic* aspects – than we succeed in creating new words.

Our knowledge must be expressed in language, if only for interpersonal verification; since language is only an imperfect code, then *an absolutely exact, objective knowledge is not possible – not because human knowing subject is imperfect, but because he uses imperfect tools for creating knowledge, starting with language*. This fact was not seriously considered by the entire philosophy of 20th century that concentrated on language – starting with logical empiricism and ending with constructivism and postmodernism.

However, what happened to our original capabilities of holistic processing of signals – we might call them *preverbal*, since we had them before the discovery of speech? An alternative description would be *animistic*, but we had a brain greater then most animals even before discovering the speech. The discovery of speech has stopped the development of these abilities, pushed them to the subconscious or unconscious. Our conscious ego, at least its analytical and logical part, identified itself with speech, verbal articulation. Because the processing of words is 10^4 times simpler, our verbal, logical, analytical, conscious reasoning utilises only a small part of the tremendous capacity of our brain that was developed before the discovery of speech. However, the capabilities of preverbal processing remained with us – *but lacking better words, we call them intuition*, and not always know how to rationally use them.

These fundamentals of a rational theory of intuition can be now subject to first empirical validation tests. Let us we *define intuition as the ability of preverbal, holistic, subconscious* (*or unconscious, or quasi-conscious*)[4] *processing of sensory signals and memory content, left historically from the preverbal stage of human evolution*. Let us call this definition *an evolutionary rational definition of intuition*. Let us conclude that *intuitive abilities should be associated to a considerable part of the brain*. Then this should be noted in the research on the structure of brain, on neurosurgery, etc.?

And it was noted – for example, by the results of studies on the hemispherical asymmetry of the brain (see, e.g., [34]). These results suggest that a typical left hemisphere (for right-handed people; for left-handed we can observe the reverse role of brain hemispheres) is responsible for *verbal, sequential, temporal, analytical, logical, rational* thinking, while a typical right hemisphere is responsible for *non-verbal, visual, spatial, simultaneous, analog, intuitive* (!!!). In the results of such research, rational and intuitive types of thinking are typically counterpoised, following the tradition of Bergson [1]; we can accept this opposition of concepts, because *we do not maintain that intuition is equivalent to rational thinking, we only propose a rational explanation and theory of intuition*. Already in 1983, Young [42] defined intuition as the activity of the right hemisphere of the brain. However, Young's definition does not lead to a fully rational theory; we cannot conclude from it, for example, how to stimulate and better use intuition. On the other hand, we can draw such conclusions – among diverse others – from the evolutionary rational definition of intuition. To illustrate such diverse possibilities let us note the following conclusion from this definition: *memory related to intuitive thinking should have different properties than memory related to rational thinking*. And it has – modern research on the functioning of memory (see, e.g., [35]) shows that the phase of deep memorisation occurs during sleep, when our consciousness is switched off.

Each man makes everyday many intuitive decisions of quasi-conscious, operational, repetitive character. These are learned decisions: when walking, a mature man does not have to articulate (even mentally) the will to make next step. Intuitively we pass around a stone on our way, intuitively we turn the key when leaving flat, turn off the alarm-clock after waking, etc. These quasi-conscious *intuitive operational decisions* are such simple and universal that we do not attach any importance to them. But we should study them in order to better understand intuition. Note that their quality depends on the level of experience. We rely on our operational intuition, if we feel well trained. Dreyfus *et al.* [5] show experimentally that the way of decision making depends critically on the level of experience: it is analytical for beginners and deliberative or intuitive for masters.

Now there comes a critical question: *does consciousness help, or interfere with good use of master abilities? If intuition is the old way of processing information, suppressed by verbal consciousness, then the use of master abilities must be easier after switching off consciousness*. This theoretical conclusion from the evolutionary rational definition of intuition is confirmed by practice. Each sportsman knows how important is to concentrate before competition. Best concentration can be achieved, e.g., by Zen meditation practices, which was used by Korean archers before winning Olympic competition.

We suggest that *this theoretical conclusion is also applicable for creative decisions* – such as scientific knowledge creation, formulating and proving mathematical theorems, new artistic concepts. Creative decisions are in a sense similar to strategic political or business decisions. They are usually non-repetitive, one-time decisions. They are usually deliberative – based on attempt to reflect on the whole

[4] Quasi-conscious action can be defined as an action we are aware of doing, but do not concentrate on it our conscious abilities; we perform many quasi-conscious actions, such as walking, driving a car, etc.

available knowledge and information. They have often accompanied by an *enlightenment effect* (*heureka* or *aha effect*).

Let us recall that Simon [33] defined the essential phases of an analytical decision process to be *intelligence, design and choice*; later (see, e.g., [17, 38]), another essential phase of *implementation* was added. For creative or strategic, intuitive decision processes a different model of their phases was proposed in Wierzbicki [37].

- **Recognition**, which often starts with a subconscious feeling of uneasiness. This feeling is sometimes followed by a conscious identification of the type of the problem.

- **Deliberation or analysis**; for experts, a deep thought deliberation suffices, as suggested by Dreyfuses. Otherwise an analytical decision process is useful – with intelligence and design but suspending the final elements of choice.

- **Gestation**; this is an extremely important phase – we must have time for forgetting the problem in order to let our subconscious work on it.

- **Enlightenment**; the expected heureka effect might come but not be consciously noticed; for example, after a nights sleep it is simply easier to generate new ideas (which is one of the reasons why group decision and brain storming sessions are more effective if they last at least two days).

- **Rationalization**; in order to communicate our decision to others we must formulate verbally, logically, rationally our reasons. This phase can be sometimes omitted if we implement the decision ourselves[5].

- **Implementation**, which might be conscious, after rationalization, or immediate and even subconscious.

This process is not linear, recourse can occur after each of its phases. Especially important are the phases of gestation and enlightenment. Their possible mechanism relies on trying to utilize the enormous potential of our mind on the level of preverbal processing: if not bothered by conscious thought, the mind might turn to a task specified before as the most important but forgotten by the conscious ego. There exist cultural institutions supporting gestation and enlightenment. The advice of *emptying your mind, concentrating on void* or *on beauty, forgetting the prejudices of an expert* from Japanese Zen meditation or tea ceremony is precisely a useful device for allowing our subconscious mind work.

Intuition is mostly acquired by life-long learning and is preverbal, therefore, it is almost equivalent to tacit knowledge introduced by Polanyi [26]. Polanyi does not give fully rational definition of tacit knowledge (for example, he also stresses extrasensory aspects of it). On the other hand, the evolutionary rational definition of intuition has strong explanatory power, as discussed above. Because of this power, using this definition we can draw both theoretical and practical conclusions how to stimulate and better use tacit knowledge.

To illustrate this explanatory power let us discuss the issue of personal versus group tacit knowledge. From the rational theory of intuition outlined above it follows that we must formulate in words, rationalize our concepts or theories before communicating them to others. Thus, the classical discourse of Heidegger [11] about seven possible meanings of the words *nihil est sine ratione* can be supplemented by another meaning: *an intuitive judgment, by definition preverbal, must be rationalized when formulated, hence requires a ratio*. Another conclusion is as follows. If language was used as a tool of civilization evolution, individual thinkers were prompted to present their theories to the group, even to beautify and defend their theories – consistently with the Kuhnian concept of a paradigm. Such creative individuals might have been rewarded evolutionary, since eloquence might be considered as a positive aspect of mating selection. However, the evolutionary interest of the group that used the knowledge to enhance survival capabilities was opposite: too flowery personal theories and truth must have been considered suspicious, Popperian falsification was necessary. Thus, Popperian falsification and Kuhnian paradigm are two sides of the same coin.

The rational theory of intuition outlined here allows also various other practical conclusions. For example, when it comes to personal intuition, this theory implies that our best ideas for intuitive decisions might come after a long sleep, before we fill our mind with everyday life troubles. Hence a simple rule: put on your alarm clock twenty minutes before normal time of waking and try to find then solutions to your most difficult problems.

6. The concept of creative space

One of the main conclusions of the rational theory of intuition is that the old distinction between subjective and objective, rational and irrational is too coarse to describe the development of knowledge in times of informational civilisation. *There is a third, middle way: between emotions and rationality we have an important layer of intuition*.

Thus, we shall consider three layers of individual personality: *emotions, intuition, rationality*. We could also consider three layers of social human activity: *individual, group and humanity*, understood in the broadest sense because knowledge is heritage of all people. However, accepting three layers of human activity as well as three layers of individual personality would lead not to six as above, but to nine ontological elements of what we might call *creative space*. This leads to the following generalization of the SECI spiral

[5]The word *rationalization* is used here in a neutral sense, without necessarily implying self-justification or advertisement, though they are often actually included. Note the similarity of this phase to the classical phase of *choice*.

of Nonaka and Takeuchi: instead of four nodes of two-by two matrix, as represented in Fig. 1, we can consider nine *nodes of creative space*, as represented in Fig. 2, and diverse *transitions between nodes of creative space*.

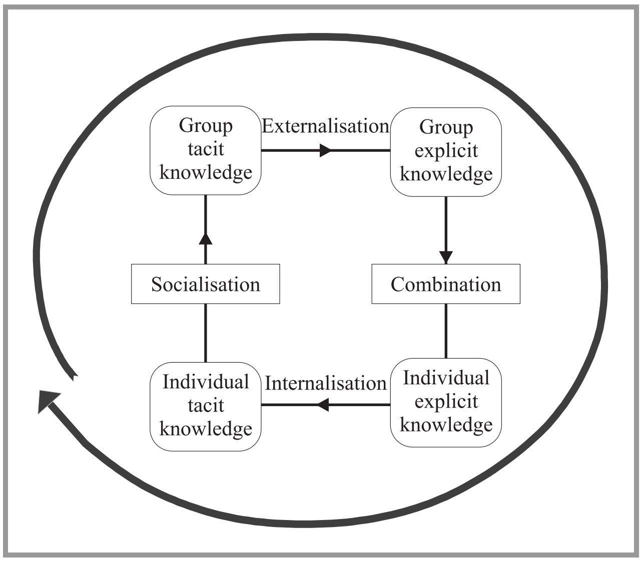

Fig. 1. A representation of the SECI spiral.

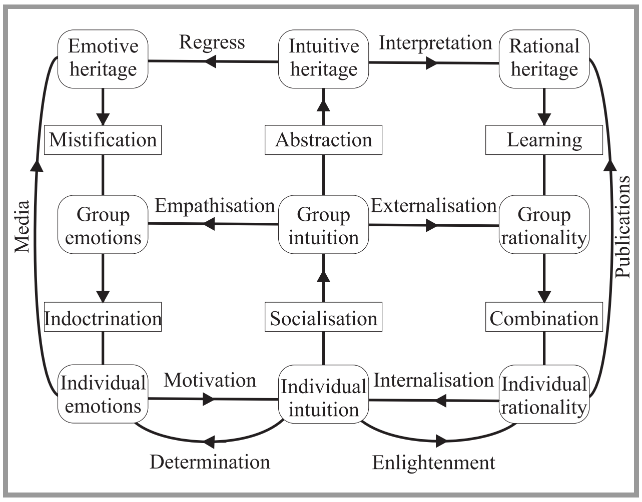

Fig. 2. Two basic dimensions of the creative space.

While the node *individual rationality* from Fig. 2 is almost equivalent to the node *individual explicit knowledge* from Fig. 1, the node individual tacit knowledge from Fig. 1 is subdivided into two nodes in Fig. 2: *individual intuition* and *individual emotions*. Similarly, the node *group explicit knowledge* from Fig. 1 is almost equivalent to the node *group rationality* in Fig. 2. However, the node *group tacit knowledge* from Fig. 1 is subdivided into two nodes, *group intuition* and *group emotions* in Fig. 2.

The nodes corresponding to emotions on all social levels include also instincts and myths; this is particularly important when we come to the third social level *humanity* in Fig. 2 that was not explicitly considered by Nonaka and Takeuchi. Yet this is a very important level, particularly in times of globalization, and playing an essential role in knowledge creation. The node *rational heritage* contains all experience and results of rational thinking – of science in its broad sense (including *hard sciences* – science and technology, *soft sciences* – humanities and history, but also *human sciences* – sociology, economy, law, medicine, etc.). It is in some sense similar to Popperian *third word*, but limited to its rational aspects. This heritage is recorded mostly in the form of books, but current informational revolution brought about here a change as important as the discovery (or re-discovery) of print by Gutenberg: change of recording medium to digital electronic records.

The emotive heritage consists of arts – music, paintings, but also literature, all fiction created by humanity, movies – that have only about a hundred year history, but recently became the main factor of trans-generational learning of emotive heritage. However, we can argue that this emotive heritage promotes also unconscious perception of *myths of humanity*. This is the concept of Jung [12] who called it *collective unconsciousness*, including in it also basic human instincts. Motycka [19] used this concept in her theory of creative behavior of scientists in time of scientific crisis or Kuhnian revolution: in order to have help in creating essentially novel concepts, scientists revert to the collective unconsciousness (Motycka called this *the process of regress*).

We do have also an intuitive heritage of humanity. Recall that Kant [13], following Platonian tradition, defined *a priori synthetic judgments* as our concepts and judgments of space and time that appear obviously true to us. Kant included in them the concept of space consistent with Euclidean axioms and the concept of time as used by Newton and other scientists before Kant. We know now that these concepts that seemed obviously true to Kant are not obvious and not necessarily true: space might be non-Euclidean, time might be relative or have several parallel scales, etc. Thus, these concepts are not a prior truth, although they seem to be true. How such preconceived ideas might be possible? A rational answer is – by intuition. We learn spatial relations when playing with blocks or Lego as children and such relations are the basis of our mathematical intuition; this intuition is strengthened by learning mathematics at school. Thus, the paradigm of teaching mathematics at school constitutes a part of the intuitive heritage of humanity. Our intuitive understanding of the world is not necessarily true, since our perception is mesocosmic, we do not often experience personally microcosmic and macrocosmic relations. But this mesocosmic perception gives us strong intuitive understanding of space and time, strengthened by the tradition of teaching mathematics. There might be other parts of the intuitive heritage of humanity – an intuitive feeling of logic related to quasi-conscious, intuitive use of language, etc. Note that this feeling is to a high degree learned, during debates in language lessons or in more advanced degree during formal training in logic. There are people that have better abilities of this intuitive feeling, there are also people that have better spatial intuition or time intuition. But there is no doubt that the intuitive heritage of humanity – including intuition for space, time, for logic – is one of the greatest achievements of our civilization.

Once we defined the ontology of nodes of creative space, we can discuss creative processes in terms of *transition between the nodes* of this space. Thus, between the nodes of *individual rationality* and *individual intuition* we might not only observe often the transition of *internalization* obtained mainly through learning by doing, as suggested by Nonaka and Takeuchi, but we can also observe sometimes the transition of *enlightenment* obtained by a creative intuitive process.

We cannot discuss here all nodes and transitions in detail that they deserve; but we shall outline shortly diverse conclusion resulting from the study of *creative space* together with its further dimensions, as suggested, e.g., by the I^5 *pentagram system* of Nakamori [22]. Beside *SECI spiral*, many other spirals of knowledge creation processes can be distinguished in *creative space*. These are:

- three spirals of organizational knowledge creation, typical for market-oriented organizations: oriental SECI spiral [24], occidental OPEC spiral [9], and brainstorming DCCV spiral [16];
- three spirals of normal academic knowledge creation, typical for normal scientific activities at universities and research institutes: hermeneutic EAIR spiral, experimental EEIS spiral, intersubjective EDIS spiral, that can be represented together in the triple helix of normal knowledge creation, all new and resulting from the concept of creative space;
- one spiral of revolutionary scientific creation processes: ARME spiral [19].

In order to shortly illustrate the tree spirals of normal academic knowledge creation processes, we present the *triple helix of normal knowledge creation* in Fig. 3, where a different graphic convention is used than in Figs. 1 and 2: small circles do not represent nodes, but transitions between nodes of *creative space*. These are the transitions: *enlightenment, analysis* (of all literature concerning the object of study), *hermeneutic immersion* (of the results of analysis), and *reflection* in the *hermeneutic EAIR spiral, enlightenment, experiment, interpretation* (of the experimental results) and *selection* (of conclusions) in the *experimental EEIS spiral, enlightenment, debate, immersion* (of the results of debate in intuition) and *selection* (of conclusions) in the *intersubjective EDIS spiral*. The triangles in Fig. 3 indicate the fact that individual researcher, having a new idea due to *enlightenment*, can switch between hermeneutic, experimental, intersubjective mode of research.

Finally, we shall shortly note importance of *computerized environments supporting creativity*. Nonaka stressed the importance of environment on creativity and introduced

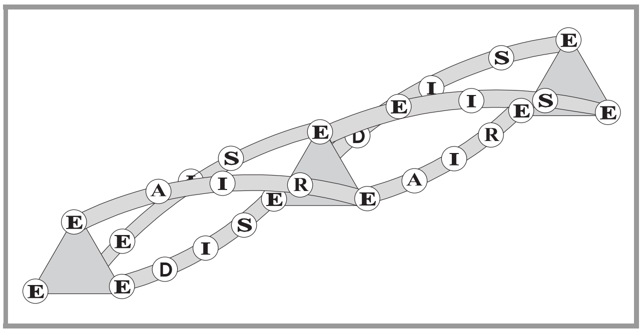

Fig. 3. Triple helix of normal knowledge creation.

the concept of *creative place Ba*. In times of informational civilization we should also use every technological possibility supporting creativity. There are many transitions in creative space and a general question might be formulated: *how to best support diverse creative transitions*? To specialists in multiple criteria decision support, there is no doubt that we can construct specialized creative environments, also for technology creation, similar to decision support environments. In particular, model-based decision support systems use the distinction between *preference model* and *core* or *substantive model*. While the former is subjective, individual, expresses the preferences of the decision maker, the latter is as objective as possible, summarizes relevant knowledge about a given decision situation. This distinction can be usefully transferred to environments supporting creative transitions between the nodes of creative space.

The discussion of this and related questions must be, however, postponed to other publications.

7. Conclusions

The conclusions of the paper are wide-ranging and we stress here only a few most important.

- Telecommunications and other informational sciences, in particular multiple criteria decision making, not only contributed the technological basis for the new era of informational and knowledge civilization, but also contributed significantly to the fundamental changes in perceiving the world characterizing the new era.
- Wittgenstein's statement [41] *"wovon man nicht sprechen kann, darüber muss man schweigen"*, though it makes a beautiful quotation, turned not to be true: we can speak today rationally about irrational metaphysical issues such as intuition and creativity.
- The science of industrial civilization era believed in the principle of reduction. We replace it today with the principle of the emergence of new properties with increased level of complexity.

- A picture is worth at least ten thousand words. This fact and an evolutionary thought experiment made it possible to formulate evolutionary rational definition of intuition and a rational theory of intuition.
- Tacit knowledge and its role in knowledge management can be analyzed in terms of the evolutionary rational definition of intuition that has a strong explanatory power.
- Language is only a code, simplifying the processing of information about the real world about 10^4 times. An absolutely exact, objective knowledge is not possible – not because human knowing subject is imperfect, but because he uses imperfect tools for creating knowledge, starting with language.
- The old distinction between subjective and objective, rational and irrational is too coarse. There is a third, middle way: between emotions and rationality we have an important layer of intuition.
- This three-valued logic and the recognition of importance of humanity emotive, intuitive and rational heritage lead to the concept of creative space, in which diverse creative processes might be considered, consisting of transitions between various nodes of this space.
- In particular, normal academic knowledge creation processes can be represented by a triple helix of normal knowledge creation.

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Paper

On the performance analysis of a heuristic approach dedicated to a multiobjective dynamic routing model

José Craveirinha, Lúcia Martins, João N. Clímaco, and Luísa Jorge

Abstract—In previous works the features and a complete formulation for circuit-switched networks of a multiple objective dynamic routing method (MODR) of periodic state dependent routing type were presented. The aim of the model is to resolve a very complex network bi-objective dynamic routing problem, by recurring to a heuristic for synchronous path selection enabling to obtain a good compromise solution in terms of two network performance measures. In this paper we present a study on the performance of variants of the MODR heuristic of synchronous path selection by using relaxations of the values previously calculated for the two network objective functions. This study permitted the development of an improved version of the initial heuristic. Also a comparison of the analytical values of the network objective functions obtained with selected variants of the initial heuristic with the corresponding results from a known reference method, the real time network routing (RTNR) method, given by a discrete-event simulator for single-service networks, is presented.

Keywords— multiple criteria analysis, routing, heuristics, telecommunications.

1. Introduction

Routing is an essential component of the functional structure of any type of telecommunication network. It has a decisive impact on the quality of service (QoS) performance of the various services provided by the network as well as on its cost and return structure. A routing method is focused on the calculation and selection of a path or set of paths between every pair of nodes, for each service request. The choice of path(s) seeks to optimise certain objective(s) and satisfy certain constraints of a technical or economical nature. The evolution of present multiservice telecommunication network functionalities leads to the necessity of dealing with multiple and heterogeneous QoS requirements. Hence the formulation of the routing problems involves the consideration (as objective functions and/or constraints) of various metrics such as delay, blocking probability, number of arcs (or "hops") or cost.

A new routing concept designated as QoS routing has emerged [22, 23] which involves the selection of a chain of network resources satisfying certain QoS requirements while seeking simultaneously to optimise the route associated metric(s). In these type of models the path calculation problem has been usually formulated as a shortest path problem where QoS requirements are often incorporated through specific constraints. Such problems are typically solved through heuristics often based on Dijkstra or Bellman-Ford shortest path algorithms. A review on QoS routing algorithms with applications can be seen in [11]. Note that these "classical" types of QoS routing models are single objective thence do not enable an explicit mathematical representation (in the form of objective functions) of potentially conflicting routing objectives.

We think there are potential advantages in considering the routing problem in integrated communication networks as a multiobjective problem, having in mind to grasp eventual conflicts and trade-offs among distinct objectives and QoS constraints. In fact multiple objective routing models enable the trade-offs among distinct QoS metrics to be treated in a mathematically consistent manner. In this context paths are normally selected in the set of non-dominated paths, i.e., paths for which (in minimisation problems) it is not possible to decrease the value of an objective function without increasing on at least the value of one of the other objective functions. Examples of multiple objective routing models in specific types of telecommunication networks can be seen in [20] and [5] (focusing on applications to asynchronous transfer mode (ATM) – networks) and [8] (dealing with a routing problem in multiprotocol label switched (MPLS) – networks). A review on multicriteria models and algorithms for telecommunication network routing problems can be seen in [4].

On the other hand the utilisation of dynamic routing methods in various types of networks is well known to have significant impact on network performance and cost, namely in overload and failure conditions [1]. This is due to the adaptive nature of dynamic routing characterised by the fact that selected routes vary dynamically as a function of varying network conditions. The routing changes are made, for example, in response to fluctuations in traffic intensities or to the state of occupation of the transmission links, corresponding to the arcs of the network representation.

In previous papers [6, 16] the essential features of a multiobjective dynamic routing method (MODR) of periodic state-dependent routing type, based on a bi-objective shortest path model, were presented. A major aspect of the MODR method (in the version for single-service traffic), beyond its specific multiobjective nature, is the explicit consideration of a "fairness" objective to be optimised together with a "classical" objective function in this type of models (network mean blocking probability). Also the consideration of a dynamic alternative routing optimisation problem (reviewed in this paper) formulated at network level is an added value with respect to classical flow-oriented QoS routing models where the paths for each node-to-node traffic flow are calculated separately, each at a time, hence giving no guarantee of obtaining "good" approximately "optimal" solutions in terms of the routes selected for all the network traffic flows. In its initial formulation, for circuit-switched networks, the model uses implied costs and blocking probabilities as metrics for the path (or route) calculation problem. Also an analytical model and a heuristic were developed [15, 17] for synchronous selection of a first choice path and an alternative path between every pair of nodes in single-service networks, seeking to obtain a set of routes which is a satisfactory compromise solution from the point of view of two global network performance objectives, namely the network mean blocking probability and the maximal end-to-end blocking probability (for all traffic flows). In [15, 17] the performance of the routing method (MODR-1) using that heuristic was compared in terms of the two network global performance metrics with the corresponding results given by a discrete event simulation model for a reference dynamic routing method, real-time network routing (RTNR) developed by AT&T, known for its efficiency and sophistication in terms of service protection mechanisms. This comparative study revealed that the method globally performed well in most situations. The extension of the MODR method to multiservice networks was outlined in [13] using a hierarchical multiobjective formulation of the dynamic alternative routing optimisation problem with $2(1+|S|)$ objectives where S represents the set of service types.

In the present work we present a study on the performance of variants of the previous MODR heuristic of synchronous path selection by using relaxations of the values previously calculated for the two network objective functions (g.o.f. in short). The consideration of these adaptations of the heuristic has in mind to enable the obtainment of approximate non-dominated solutions by travelling on the g.o.f. space in order to improve either one or the other g.o.f. with respect to the values corresponding to the solution obtained by the initial version of the heuristic. This study permitted the development of an improved version of the initial heuristic. Also a comparison of the analytical values of the g.o.f. obtained through an analytical model, with this variant of the initial heuristic and the corresponding results from the RTNR method given by a discrete-event simulator, for single-service networks, will be presented.

The major contributions of this paper with respect to previous works of the authors on the MODR model are:

- the exploration of possible variants to the heuristic described in [17] for the MODR version for single service networks;
- to analyse the relative performance of these variants in terms of the two network routing metrics for different overload factors by using an analytical model;
- to show that one of these variants, based on a simple relaxation of the value of one of global objective functions (with respect to the current minimum) may be advantageous in some practical network engineering conditions, by enabling a slight improvement in total average revenue at the cost of a small degradation of the maximal node-to-node blocking probability.

Finally note that in the context of MODR the selection of routes for every node-to-node traffic flow has to be performed in a fully automatic manner. This raises specific difficulties concerning the representation of the system of preferences, which, in a certain manner, is imbedded in key points of the considered variants of the heuristic of synchronous path selection.

The paper is organised as follows. Section 2 reviews the essential features of the MODR model and the bi-objective shortest path algorithm used as a basis for its resolution. Also the main features of the heuristic previously developed for synchronous path selection are outlined. Section 3 describes the considered new versions of the heuristic obtained by using certain relaxations of the values previously calculated for the two global objective functions. Also the behaviour of theses variants in the g.o.f. space, are analysed in this section. Section 4 presents a comparison of the network performance results obtained with a specific new variant of the heuristic with a reference dynamic routing method (RTNR) for some test networks by recurring to a discrete-event simulator. This will enable some conclusions to be drawn concerning the potential advantages and difficulties of the model and an outline of developments of this work.

2. Review of the multiobjective dynamic routing model

2.1. The MODR model

The MODR method for single-service networks, the model of which was presented in [6, 16] is a periodic state-dependent routing method, where the (loopless) paths $\{r^1(f),\ldots,r^M(f)\}$ that may be attempted by a call of each node to node traffic flow f (from node v_s to v_t) change periodically as a function of a measure of the network working conditions. The calculation of paths is based on a bi-objective shortest path model that is resolved by a very efficient algorithmic approach designated as modified multiobjective routing algorithm (MMRA). This procedure uses an extremely efficient k-shortest path algorithm [12] to search for non-dominated, including unsupported non-dominated solutions located in the interior of the convex-hull of the feasible solutions set. In the formulation of MODR for networks equivalent to circuit-switched loss networks this underlying bi-objective shortest path static routing model uses blocking probabilities and implied costs (in the sense defined by Kelly [10]) as path metrics. This model uses soft-constraints (that is constraints not directly incorporated into the bi-objective shortest path mathematical model) in the form of required and/or accepted values for each metric which define preference regions in the objective function space.

In terms of global network performance the MODR method seeks good compromise solutions to a network bi-objective alternative dynamic routing problem. In the formulation of this problem in the case of single-service networks (see [16]) the first objective is the minimisation of the network mean blocking probability B_m (this is the objective function in classical single-objective routing models). The second objective is the minimisation of the maximal marginal blocking probability B_M (maximal value of the marginal blocking probabilities $B(f)$ experienced by all traffic flows f). In the present formulation of the method a call of each traffic flow may attempt two paths (or routes) according to the alternative routing principle ($M = 2$): the first choice path $r^1(f)$ (which is the direct arc (v_s, v_t) whenever it exists) and (when $r^1(f)$ is blocked) the alternative path, $r^2(f)$. Therefore the network bi-objective alternative dynamic routing problem in the decision variables $\overline{R}_t$ is formulated as:

(Problem $\mathscr{P}_G^{(2)}$)

$$\min_{\overline{R}_t} B_m = \sum_{f\in\mathscr{F}} \frac{A_t(f)B(f)}{A_t^0} \quad (1)$$

$$\min_{\overline{R}_t} B_M = \max_{f\in\mathscr{F}} \{B(f)\} \quad (2)$$

s. t.

$$B(f) = L_{r^1(f)} L_{r^2(f)} \quad (3)$$

and equations of the teletraffic model enabling to calculate $\{B(f),\ \text{all}\ f \in F\}$ in terms of $A_t(f)$, for given route set and arc capacities C_j (for all arcs l_j,)

where $A_t^o = \sum_{f\in F} A_t(f)$ is the total traffic offered to the network, $A_t(f)$ is the traffic offered by flow f (in Erlangs) at time period t, $L_{r^i(f)}$ is the blocking probability of a call of f on route $r^i(f)$ $(i = 1,2)$ and $\overline{R}_t$ is the set of the route sets of all traffic flows $f \in F$ at time period $t = nT$ $(n = 1,2,\dots)$:

$$\overline{R}_t = \{R_t(f_1),\dots,R_t(f_{|F|})\}, \quad (4)$$

$$R_t(f) = \{r^1(f), r^2(f)\}, \quad (5)$$

The complete analytical model is described in [15, 17]. In [7] it is proved that, assuming quasi-stationary conditions in successive route updating periods (i.e., the offered traffic stochastic features remain stationary during periods which are relatively long compared to the solution time) the single-objective adaptive alternative routing problem (corresponding to the g.o.f. Eq. (1)) is NP-complete in the strong-sense, even in the "degenerated" simpler case where $M = 1$ (no alternative route provided). It should be noted that our model is a bi-objective formulation of this type of problem.

The basis of the problem resolution procedure is an algorithmic approach (designated as MMRA) which seeks good compromise non-dominated and possibly dominated solution(s) (when there is a dominated solution located in the first priority region(s) of the objective function space of the Problem $\mathscr{P}^{(2)}$ (Eq. (6)) which may be selected corresponding to some second choice route, see [6]) to the following bi-objective shortest path problem (for each flow f from node v_s to node v_t) defined in the network (V,N), where V is the node set and L the arc set:

(Problem $\mathscr{P}^{(2)}$)

$$\min\ z^n = \sum_{l_k=(v_i,v_j)\in L} \mathscr{C}_k^n x_{ij} \quad (n = 1,2) \quad (6)$$

s.t.

$$\sum_{v_j\in V} x_{sj} = 1$$

$$\sum_{v_i\in V} x_{ij} - \sum_{v_q\in V} x_{jq} = 0 \quad \forall v_j \in V, (v_j \neq s,t)$$

$$\sum_{v_i\in V} x_{it} = 1 \quad (7)$$

$$x_{ij} \in \{0,1\}, \quad \forall l_k = (v_i,v_j) \in L$$

$$(x_{ij} = 1 \text{ iff } l_k = (v_i,v_j) \in r^i(f)),$$

where

$$\mathscr{C}_k^1 = c_k (\text{implied cost on link } l_k) \quad \text{and} \quad \mathscr{C}_k^2 = -\log(1 - B_k).$$

The call blocking probability B_k (or call congestion) on arc l_k and the application of log is necessary for obtaining an additive metric. The implied cost c_k associated with link l_k is an important concept in teletraffic routing theory due to Kelly [10]. It represents the expected value of the increase in lost calls (on all routes of all traffic flows which use l_k) which results from accepting a call of a given traffic flow on link l_k. Note that each c_k depends on $\{c_j\}$, $\{C_j\}$, $\{B_j\}$, $\{A_t(f)\}$ and $\overline{R}_t$. The equations of the teletraffic model (in [15]) also imply that each B_k depends on $\{B_j\}$, $\{C_j\}$, $\{A_t(f)\}$ and $\{R_k\}$ (set of routes which at a given period may use link l_k). The arcs are supposed undirected and the paths for each flow f are node disjoint, loopless, and have a predefined maximal number of arcs.

From the analytical model (see [17]) it can be easily shown that there are interdependencies between the objective functions coefficients $\{c_k\}$ and $\{B_k\}$ in $\mathscr{P}^{(2)}$ and between these two sets and the current total route set $\overline{R}_t$, via the set of routes R_k, which, at a given time t, may use link l_k. MMRA enables solutions to $\mathscr{P}^{(2)}$ to be computed assuming fixed values of $\{c_k\}$ and $\{B_k\}$ and given $\{A_t(f), \text{all } f \in F\}$ and the capacities C_k of all links l_k.

Taking into account the NP completeness (in the strong sense) nature of the network problem $\mathscr{P}_G^{(2)}$ and the aforementioned interdependencies between the mathematical entities $\{c_k\}$, $\{B_k\}$ and $\overline{R}_t$, it can be concluded of the extreme intractability of the network problem $\mathscr{P}_G^{(2)}$.

Concerning the possible conflict between the objective functions in $\mathscr{P}_G^{(2)}$ it can be said that in many situations (in networks using alternative routing) the minimisation of B_m is associated with a deterioration on $B(f)$ for "small" traffic flows $A_t(f)$, leading to an increase in B_M. In conventional single-objective routing models this effect is usually limited by imposing upper bounds on $B(f)$.

The use of MMRA as a basis for seeking approximate solutions to $\mathscr{P}_G^{(2)}$ relies on the property that minimising z^1 in $\mathscr{P}^{(2)}$ corresponds to minimising B_m, when searching for a path for flow f assuming all the remaining conditions in the network (namely the routes assigned to all other flows and all the link implied costs) were maintained constant while the minimisation of z^2 in $\mathscr{P}^{(2)}$ tends to achieve the minimisation of B_M, under similar assumptions. Of course from the analysis on the problem overall complexity it is clear that these assumptions (all remaining conditions in the network are maintained constant) do not hold, which leads to an unstable behaviour of MMRA solutions as reviewed in the next section.

Concerning the traffic modelling aspects, underlying the calculation of B_k and c_k, we must clarify that we used a one-parameter simplification, based, for the multiservice networks case, on the Kaufman [9] or Roberts [21] algorithms [14]. It is well known in teletraffic theory that these models represent an oversimplification (from a stochastic point of view) which leads to significant errors, specially for low blocking probabilities. The reason for this choice was purely instrumental taking into account the great numerical efficiency of the used procedures which is absolutely critical in a model of this nature. In fact the traffic calculation subroutines used for resolving the system of equations (involving implied costs and blocking probabilities for each traffic type in every link) enabling to estimate $(\overline{c}, \overline{B})$ have to be executed a very large number of times in each run of the heuristic for final route selection. Note that the importance of the accuracy of the results given by the traffic calculation model, in the context of MODR, is in terms of relative values of the associated route metrics (since the aim is just to compare routing solutions with respect to those metrics) rather than in terms of absolute errors. Also note that these simplified models were used/recommended in single-objective global routing optimisation models, for off-line application, such as in Mitra *et al.* [18]. In a dynamic routing environment, specially when a very complex and lengthy calculation procedure is at stake, the need for a very efficient approximation (albeit simplistic) is unavoidable for tractability reasons.

2.2. First version of a heuristic of path selection

The interdependencies between key mathematical entities of the model $\mathscr{P}^{(2)}$ and the great complexity of the global problem $\mathscr{P}_G^{(2)}$ make the direct application of the bi-objective algorithm MMRA (to every pair of nodes) to generate unstable solutions, possibly leading to poor network performance (under the bi-objective model (B_m, B_M)) as shown in [16]. In fact direct application of MMRA to obtain the "best" compromise alternative paths for every node to node traffic flow as a function of the network state leads typically to situations where the chosen path sets $\overline{R}_t$ may oscillate between a few sets of solutions. This is associated with the fact that in a certain iteration certain links will be very loaded (as a result of contributing to many paths) while others are lightly loaded; in the following iteration the more loaded and the less loaded links will tend to reverse their condition. This is a new and specific "bi-objective" case of a known instability problem in single objective adaptive shortest path routing models which was extensively studied in packet switched networks (see for example [3, Chapter 5]) and also analysed in some single-objective dynamic alternative routing models.

This path instability phenomena in the context of MODR was extensively analysed in [16].

A heuristic was developed in [17] for selecting path sets $\overline{R}_t$ $(t = nT; \quad n = 1, 2, \dots)$ capable of guaranteeing a good compromise solution in terms of the two network performance criteria (B_m, B_M), at every updating period. The basis of that procedure is to search for the subset of the alternative path set

$$\overline{R}^a_{t-T} = \{r^2(f), \quad f \in \mathscr{F}\} \tag{8}$$

the elements of which should possibly be changed in the next updating period, seeking to minimise B_m while not letting an excessive increase in max $\{B(f)\}$. The authors proposed in [16] the following criterion for choosing candidate paths for possible improvement which depends explicitly both on the first choice path $r^1(f)$ and on the alternative path $r^2(f)$:

$$\xi(f) = F_1 F_2 = \left(2C^1_{r^1(f)} - C^1_{r^2(f)}\right)\left(1 - L_{r^1(f)} L_{r^2(f)}\right), \tag{9}$$

$$C^1_{r^i(f)} = \sum_{l_k \in r^i(f)} c_k \,. \tag{10}$$

The objective of the factor F_1 is to favour (concerning the need to change the 2nd route) the flows for which the 2nd route has a high implied cost and the 1st route a low implied cost. The factor 2 of $C^1_{r^1(f)}$ was introduced for normalising reasons taking into account that $r^1(f)$ has one arc and $r^2(f)$ two arcs, in fully meshed networks. In a more general case, where $r^1(f)$ has n_1 arcs and $r^2(f)$ n_2 arcs $(n_1 \le n_2)$:

$$F_1 = (n_2 - n_1) c'_1 + C^1_{r^1(f)} - C^1_{r^2(f)}, \tag{11}$$

c'_1 being the average implied cost of the arcs in $r^1(f)$. The second factor F_2 intends to favour the flows with worse end-to-end blocking probability. An important issue tackled in the procedure is the specification of how many and which of the second choice routes $r^2(f)$ with smaller value of $\xi(f)$ should possibly be changed by applying MMRA once again. Among the recalculated routes only those which lead to solutions which dominate previous ones (in terms of B_m and B_M) are finally selected as routes to be changed in each path updating period. This implies that the effect of each candidate route (in terms of B_m and B_M) is previously anticipated by solving the corresponding analytical model. This heuristic procedure uses two variables that control the current number of candidate paths for improvement in the two main cycles of the heuristic. The first variable is initialised to the total number of node pairs and controls an external cycle where the second variable is initialised; the second variable is used in an internal cycle that seeks to obtain new alternative paths $r^2(f)$ able of improving B_m and/or B_M.

The MODR heuristic uses a specific "service protection scheme", aimed at preventing excessive network blocking degradation in overload situations, associated with the utilisation of alternative routes for all node-to-node traffic flows. This mechanism designated as alternative path removal (APR) is based on the elimination of the alternative paths of all traffic flows for which the value of the scalar function (convex combination of the two objective functions) of the bi-objective shortest path model $\mathscr{P}^{(2)}$ is greater than or equal to a certain parameter z_{APR} that is adapted dynamically to overload conditions. Details and a formal description of this heuristic are in [17].

In [15] the performance of the global routing method using that heuristic (MODR-1) was compared in terms of the two global performance network metrics with the corresponding results given by a discrete event simulation model for a reference dynamic routing method, real-time network routing developed by ATT&T, known for its efficiency and sophistication in terms of service protection mechanisms. This comparative study revealed that the method globally performed well in most situations.

3. New versions of the heuristic

Having in mind the very complex nature of the network bi-objective dynamic alternative routing problem $\mathscr{P}_G^{(2)}$ we have considered the analysis of variants of the previously described version of the heuristic, namely by using relaxations of the values calculated for the two network objective functions in $\mathscr{P}_G^{(2)}$, B_m and B_M. This had in mind to enable the calculation of approximate non-dominated solutions by travelling in the network objective function's space in order to improve one of the objective functions, relaxing the other with respect to the values corresponding to the solution obtained by the initial version of the heuristic (designated hereafter as MODR-1). This also enabled the analysis of the behaviour of the variants of the heuristic with respect to the objective function values and test possible improvements of MODR-1. The test networks are the same which were used in previous studies: the network in [19] (fully meshed, with six nodes) widely used in studies on dynamic routing methods (network M in short) and two networks with the same topology designated as networks B and A. Network M has strong asymmetries in many arc capacities, with respect to the direct traffic offered to them. Network B was engineered by recalculating the link capacities of network M for the same values of traffic offered $A_{t_0}(f)$ with a standard dimensioning method for dynamic routing circuit-switched networks [2]. Network A has a different matrix of nominal traffic offered with a smaller variation in traffic intensities than in network B and M; its link capacities were obtained by the same method as network B. The specification of each of these networks, including the initial route set $\overline{R}_{t_0}$ computed by the mentioned method [2], is given in Table 1.

3.1. Versions of the heuristic

Firstly the path selection procedure (heuristic) was changed so that the routes are chosen by seeking to minimise separately one of the network metrics: B_m and B_M. The solutions obtained are denoted by (B_m^*, B_M^+) and (B_m^+, B_M^*) and correspond to the approximations to the minimum of B_m (B_m^*) and B_M (B_M^*) which the heuristic was capable of obtaining. These solutions are designated as **extremes-H**.

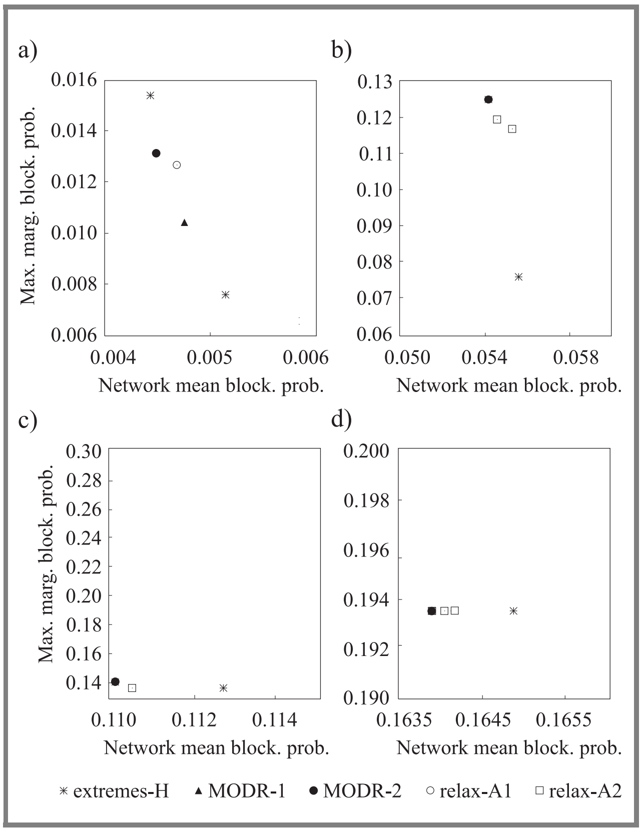

Fig. 1. Network B: overload factor (a) 0%; (b) 10% (c) 20%; (d) 30% .

Table 1

Test networks A, B and M

O-D pair	Network A link capac.	Network A offer. traf.	Network A intermed. node	Network B link capac.	Network B offer. traf.	Network B intermed. node	Network M link capac.	Network M offer. traf.	Network M intermed. node
1-2	36	27	3	41	27.47	3	36	27.47	3
1-3	13	6	4	13	6.97	4	24	6.97	5
1-4	33	25	5	276	257.81	5	324	257.81	–
1-5	27	20	6	33	20.47	6	48	20.47	3
1-6	31	20	2	45	29.11	2	48	29.11	5
2-3	29	25	4	29	25.11	4	96	25.11	–
2-4	17	10	5	112	101.61	5	96	101.61	3
2-5	37	30	6	88	76.78	6	108	76.78	3
2-6	25	20	1	94	82.56	1	96	82.56	3
3-4	17	11	5	18	11.92	5	12	11.92	1
3-5	14	8	6	11	6.86	6	48	6.86	6
3-6	19	13	1	21	13.25	1	24	13.25	2
4-5	13	9	6	87	79.42	6	192	79.42	1
4-6	27	20	1	94	83.0	1	84	83.0	5
6-6	18	12	1	137	127.11	1	336	127.11	–

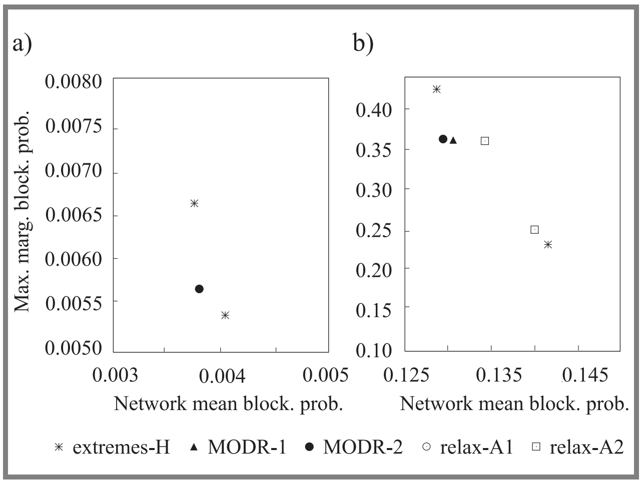

Fig. 2. (a) Network A: overload factor 0%; (b) network M: overload factor 60%.

Next two new versions of the heuristic were implemented which seek solutions $\overline{R}_t$ satisfying:

$$\text{(A1)}\ \ \min B_m : B_M < B_M^+ - n\Delta^1,\ \ n = 9, \ldots, 1 \qquad (12)$$

$$\text{(A2)}\ \ \min B_M : B_m < B_m^+ - n\Delta,\ \ n = 9, \ldots, 1 \qquad (13)$$

where

$$\Delta = \frac{B_m^+ - B_{m_{MODR}}}{10},$$

$$\Delta^1 = \frac{B_M^+ - B_{M_{MODR}}}{10}$$

and $(B_{m_{MODR}}, B_{M_{MODR}})$ are the objective function values corresponding to the solution obtained by the initial version of the MODR heuristic. A1 (A2) corresponds to the relaxation of B_M (B_m) by successive increments equal to Δ^1 (Δ) in the interval $]B_{M_{MODR}}, B_M^+[$ ($]B_{m_{MODR}}, B_m^+[$). In Figs. 1 and 2 the solutions obtained with A1 and A2 correspond to the points signalled as **relax-A1** and **relax-A2**.

3.2. Insight on the heuristic

Finally to give some insight on the behaviour of the solutions generated by the major cycles of this type of heuristic a fourth version of the heuristic was implemented.

This is a variant of the heuristic where, in the search for solutions which minimise B_M and B_m, the currently selected solutions have to satisfy the condition:

$$\text{(B)} \qquad B_m^* \le B_m \le B_m^+ \quad \text{and} \quad B_M^* \le B_M \le B_M^+. \qquad (14)$$

In Figs. 3 and 4 the solutions from this version correspond to the points signalled as **val. interv**. The consideration of this version has to do with the fact that in the initial version of the heuristic from one iteration to the next it is not accepted a generated solution which worsens any of the two objective functions values. It was observed that this condition was too strict regarding the prosecution of the main search for solutions. In the new version the controlled relaxation of this condition with respect to the two

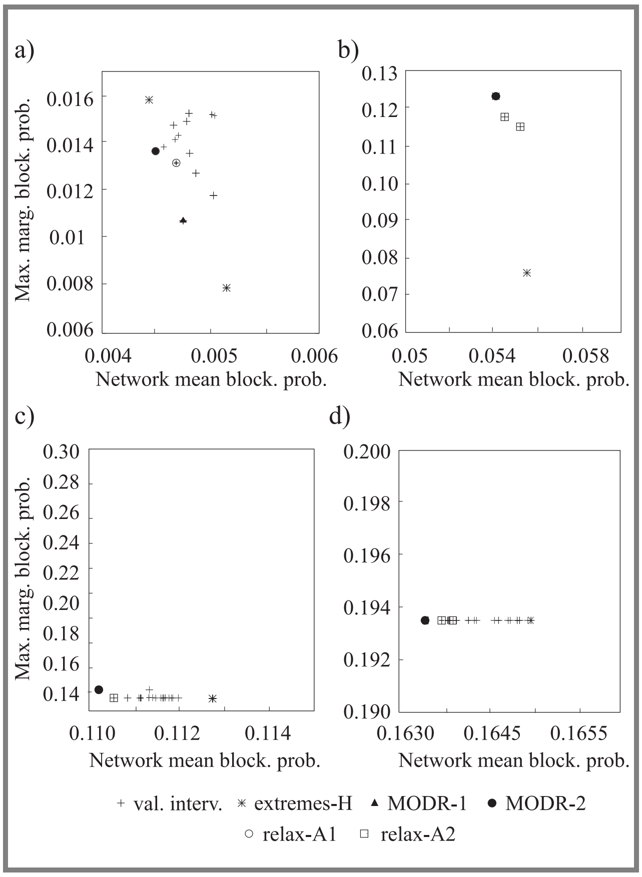

Fig. 3. Network B: overload factor (a) 0%; (b) 10%; (c) 20%; (d) 30%.

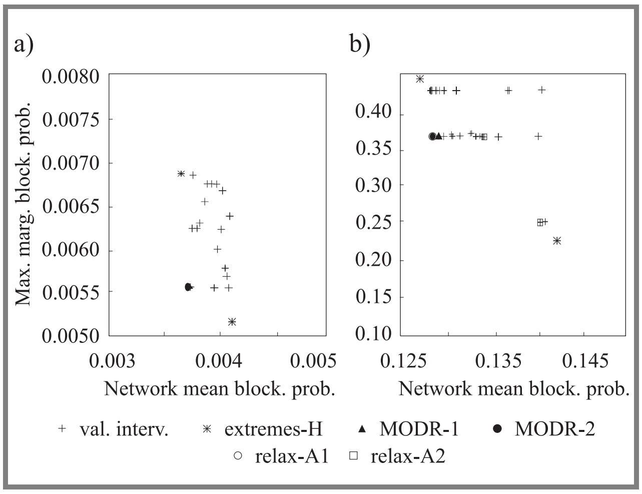

Fig. 4. (a) Network A: overload factor 0%; (b) network M: overload factor 60%.

metrics might enable that the solutions calculated in this manner could lead at a later iteration to solution(s) worthwhile considering.

3.3. Analysis of results

The most significant results obtained with the described versions of the heuristic are depicted in Figs. 1 and 2, for the three test networks.

The first conclusion is that the solutions obtained by MODR-1 are in almost all the cases non-dominated with respect to the solutions from all the other versions of the heuristic and are good compromise solutions in terms of B_m and B_M. The only exception was in case Fig. 2b, where the MODR-1 solution was slightly dominated by the solution from **relax-A1** with respect to the metric B_m. This situation can be explained by the very complex nature of the problem $\mathscr{P}_G^{(2)}$, previously reviewed, namely the strong interdependencies between the objective functions and between the parameters of the functions and the calculated path sets. Related to the situation in Fig. 2b, we can say that by considering some relaxation of B_M (version **relax-A1** of the heuristic) we might also obtain solutions which are non-dominated with respect to those of MODR-1 but for which B_m is better than for MODR-1 while B_M is just slightly worse. It may also happen that some of these slight differences in the values of B_M or B_m result from numerical imprecision associated with the lengthy and complicated numerical procedures involved in the resolution of the network teletraffic model. Having all this in mind (as well as other experiments) and to enable that such solutions may be selected, a new version of the MODR heuristic, designated as MODR-2 was implemented which seeks solutions which tend to minimise B_m and B_M while accepting those for which B_M is 3% worse than the current minimum. This new version of the heuristic enabled slight improvements in the network performance results in terms of B_m in some situations (as illustrated in the next section). Note that from an engineering point of view it is correct to accept solutions with somehow better B_m at the cost of a slight worsening in B_M, since the former metric is directly related to the average revenue associated with the total traffic carried in the network.

Other interesting aspect to be assessed in these results concerns the sets of solutions with the same value of B_M, which occur for higher overloads (Figs. 1c, 1d, 2b, 3c, 3d, 4b). This phenomenon can be explained as a result of the elimination of the alternative route for some traffic flows which are then the only flows in the corresponding direct arcs. One of the flows in these conditions (the one which suffers the highest congestion) determines the value of B_M. Hence the value B_M does not change while (in the solutions obtained from the different versions of the heuristic) there is no alternative route of other flow(s) which uses the direct arc associated with that flow or while that flow does not have an alternative route.

4. Network performance

In order to evaluate the network performance in terms of the two metrics B_m, B_M obtained with the initial version of the heuristic (MODR-1) and the new version, MODR-2, described in the previous section, Tables 2 to 4 show the corresponding analytical results, for the three test networks

Table 2
Global network performance for network M

Overl. factor [%]	MODR–1 Analytical model		MODR–2 Analytical model		RTNR Simulation model	
	B_m	B_M	B_m	B_M	$B_m \pm \Delta$	$B_M \pm \Delta$
0	$< 10^{-3}$	0.001	$< 10^{-3}$	0.001	$\mathbf{< 10^{-3}}$	$\mathbf{< 10^{-3}}$
10	0.001	0.009	0.001	0.009	$\mathbf{0.001 \pm 1.1 \cdot 10^{-4}}$	$\mathbf{0.005 \pm 1.1 \cdot 10^{-3}}$
20	0.005	0.035	0.005	0.035	$\mathbf{0.004 \pm 3.0 \cdot 10^{-4}}$	$\mathbf{0.025 \pm 2.4 \cdot 10^{-3}}$
30	0.019	0.076	**0.019**	**0.076**	$0.027 \pm 1.5 \cdot 10^{-3}$	$0.144 \pm 1.3 \cdot 10^{-2}$
40	0.063	0.141	**0.063**	**0.141**	$0.063 \pm 1.6 \cdot 10^{-3}$	$0.257 \pm 5.5 \cdot 10^{-3}$
50	0.103	0.192	0.103	**0.192**	$\mathbf{0.101 \pm 1.8 \cdot 10^{-3}}$	$0.335 \pm 3.3 \cdot 10^{-3}$
60	0.130	0.361	**0.130**	**0.362**	$0.138 \pm 1.5 \cdot 10^{-3}$	$0.397 \pm 3.7 \cdot 10^{-3}$
70	0.169	0.397	**0.166***	**0.398**	$0.173 \pm 1.7 \cdot 10^{-3}$	$0.446 \pm 2.9 \cdot 10^{-3}$
80	0.203	0.429	**0.196***	0.484	$0.204 \pm 1.6 \cdot 10^{-3}$	$\mathbf{0.479 \pm 1.4 \cdot 10^{-3}}$

Table 3
Global network performance for network B

Overl. factor [%]	MODR–1 Analytical model		MODR–2 Analytical model		RTNR Simulation model	
	B_m	B_M	B_m	B_M	$B_m \pm \Delta$	$B_M \pm \Delta$
0	0.005	0.011	**0.005**	**0.011**	$0.007 \pm 6.7 \cdot 10^{-4}$	$0.029 \pm 6.4 \cdot 10^{-3}$
10	0.054	0.124	**0.054**	**0.124**	$0.058 \pm 1.1 \cdot 10^{-3}$	$0.180 \pm 9.7 \cdot 10^{-3}$
20	0.110	0.140	**0.110**	**0.140**	$0.111 \pm 1.3 \cdot 10^{-3}$	$0.257 \pm 1.2 \cdot 10^{-2}$
30	0.164	0.194	**0.164**	**0.194**	$0.193 \pm 2.1 \cdot 10^{-3}$	$0.296 \pm 3.8 \cdot 10^{-3}$
40	0.214	0.246	**0.214**	**0.246**	$0.216 \pm 1.2 \cdot 10^{-3}$	$0.315 \pm 7.7 \cdot 10^{-3}$

Table 4
Global network performance for network A

Overl. factor [%]	MODR–1 Analytical model		MODR–2 Analytical model		RTNR Simulation model	
	B_m	B_M	B_m	B_M	$B_m \pm \Delta$	$B_M \pm \Delta$
0	0.004	0.006	0.004	0.006	$\mathbf{0.003 \pm 5.3 \cdot 10^{-4}}$	$\mathbf{0.006 \pm 1.5 \cdot 10^{-3}}$
10	0.031	0.038	**0.031**	**0.038**	$0.041 \pm 2.9 \cdot 10^{-3}$	$0.061 \pm 4.4 \cdot 10^{-3}$
20	0.078	0.153	**0.077**	0.153	$0.090 \pm 2.7 \cdot 10^{-3}$	$\mathbf{0.133 \pm 8.9 \cdot 10^{-3}}$
30	0.119	0.198	**0.118**	0.198	$0.129 \pm 2.2 \cdot 10^{-3}$	$\mathbf{0.186 \pm 8.7 \cdot 10^{-3}}$
40	0.157	0.242	**0.156**	0.242	$0.167 \pm 1.8 \cdot 10^{-3}$	$\mathbf{0.226 \pm 1.1 \cdot 10^{-2}}$

and different overload factors. Since the major objective of this study was to perform a comparison between the relevant variants of the heuristic only analytical results are given in these tables. A simulation study using a discrete-event platform (in report [15]) confirmed the relations between the results obtained from the two variants of MODR for the test networks and the different overload factors. Also the results from a reference dynamic routing method (RTNR), obtained from a discrete event simulator, are displayed with 95% confidence intervals. The results presented for RTNR are intended as reference values for each case.

The major conclusion is that MODR-2 enables slight improvements in B_m at the cost of slight increases in B_M, specially in high overload conditions. These results also confirm that both versions of the heuristic globally perform well when compared to RTNR, specially in overload conditions, as already concluded in [15] and in [17] for the MODR-1 case. In fact, excepting for the case of the poorly engineered network M for low and moderate overload (where the values B_m and B_M obtained by the heuristic were even so very low and always below standardised requested values) and for very low blocking in network A and B the solutions of the heuristics either dominate the RTNR solutions or are non-dominated with respect to the latter. Only for low or very low overload where even so MODR-1 and MODR-2 values for B_m are normally below typical required values (e.g., $\leq 0.5\%$ at 0% overload), RTNR tends to give better results than MODR-1 in terms of B_m. A detailed comparison of the network performance with the solutions from the MODR-1 heuristic, with the corresponding results for the RTNR solutions, using discrete event simulation models for both dynamic routing methods is described in [15, 17]. Those simulation studies have shown that MODR-1 globally tends to have better performance than RTNR, specially in overload conditions. Note that MODR-2 performs at least as well as MODR-1 with respect to the total average network revenue (or network mean blocking probability).

5. Conclusions and further work

A study was presented on the performance of variants of a heuristic for synchronous path selection in a bi-objective dynamic alternative routing model, by using relaxations of the values previously calculated for the two network objective functions.

This work permitted the specification of a new version of the heuristic which enables slight improvements in the network mean blocking probability possibly at the cost of a slight increase in the maximal node to node blocking probability which is advantageous in some practical network engineering situations. Also a comparison of the network performance (as measured by the two metrics) obtained with two versions of the heuristic and the dynamic routing method RTNR enabled the confirmation of the globally good performance of the MODR method, namely in overload conditions.

Further work concerns the extension and complete formulation of the MODR model for multiclass traffic loss networks as outlined in [13]. This includes the development of a heuristic capable of finding good compromise solutions for a bi-hierarchical multiple objective dynamic alternative routing problem where the first priority global objective functions concern the global network level metrics and the second priority network objective functions are concerned with the quality of service metrics associated with the different services. Also extensive simulational comparative studies have to be carried out in this context, in order to evaluate with more precision the results of the heuristics for various test networks.

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Paper

Swarm intelligence for network routing optimization

Peter Dempsey and Alfons Schuster

Abstract— This paper presents the results of a comparative study of network routing approaches. Recent advances in the field suggest that swarm intelligence may offer a robust, high quality solution. The overall aim of the study was to develop a framework to facilitate the empirical evaluation of a swarm intelligence routing approach compared to a conventional static and dynamic routing approach. This paper presents a framework for the simulation of computer networks, collection of performance statistics, generation and reuse of network topologies and traffic patterns.

Keywords— network routing, swarm intelligence, ant algorithms.

1. Introduction

Computer networks are handling larger amounts of traffic and experience tells us that they will continue to grow in size and demand for efficiency. Improvements in network hardware will offer performance improvements but need to rely upon intelligently designed policies and protocols to achieve optimality. Many of the problems that are encountered in the design of network communication policies have no easy or completely satisfactory solution. This is because they are often compromises between conflicting objectives. The problem that this paper investigates is known as network routing.

2. Routing and ant metaheuristic

Consider a network of devices, either hosts or routers, connected by point-to-point communication links. Any device may communicate with another by sending an addressed data packet to a neighbour who will then forward the packet on, eventually to the intended recipient. The decision of which outward link to send an incoming packet along is made by the network layer's routing algorithm. Routing is an important aspect of computer networks because it can greatly influence overall network performance; good routing can cause greater throughput or lower average delays, all other conditions being the same [6]. Routing is a difficult problem because it is a distributed multi-objective optimization problem. This has two important implications:

- Because the problem is distributed it is impossible for any one device to have an accurate picture of the overall problem state at the time when it must make decisions affecting the performance of the network.
- A good solution to the problem will be a compromise between conflicting requirements. For example, throughput is desirable but not to the extent that it will unfairly penalise some hosts.

Traditional approaches to network routing include static [13] routing algorithms and various dynamic routing approaches [4, 8]. Static routing algorithms compute the least costly paths through the network when the network is first booted using known information about the communication links used. The algorithm used in our simulation framework used Djikstra's shortest path algorithm calculating the distance using cost weightings for the communication links [13]. The obvious disadvantage of such an approach is that it is unable to adjust its policy to minimise the build-up of localised congestion in one area of the network. Also the network must implement another protocol to handle the failure of a communication link. The dynamic routing algorithm treated in this paper is a distance-vector algorithm similar to *Routing Information Protocol* (RIP) [2]. In this approach nodes (hosts or routers) periodically send a packet to each neighbour notifying them of the minimum number of packets that are queued along their best route to every other node in the network. When a node learns of a better route to a node it rewrites its routing table to begin using the new route. This gives dynamic routing algorithms the ability to direct traffic around congested areas to relieve congestion. This flexibility can backfire resulting in the situation where traffic is diverted between routes in constant oscillations that increase congestion in the local area. Invulnerability to this effect is an advantage of static routing algorithms over dynamic routing algorithms. Swarm intelligence routing approaches seem to offer the flexibility of distance-vector algorithms without the drawback of undamped traffic oscillations [6].

3. Ant metaheuristic

Swarm intelligence is a soft computing technique that has gained considerable attention in the research community over the last couple of years [1, 5, 6]. It was proposed for various tasks including the control of robot swarms, power saving in mobile networks and network routing, for example [6, 11].

The swarm intelligence approach we use is an ant routing algorithm. An important characteristic of swarm intelligence is its use of stigmergy. Stigmergy refers to a communication method that encodes information about the problem (and its solutions) on the environment of the problem. A good example of stigmergy in swarm intelligence is the ant metaheuristic. This refers to the method by which ant swarms find best paths. Information is encoded on the environment in the form of pheromone (scent chemicals) deposited on the ground as the ants walk over it. In the case illustrated in Fig. 1 ants continuously pour out of the nest toward the food. When a fork in the path at A is encountered ants choose either path with a probability based

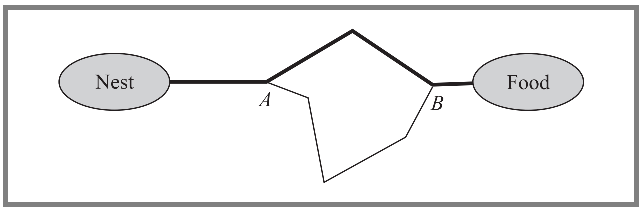

Fig. 1. An example of the shortest path-finding behaviour.

on the concentration of pheromone on that path. Initially unscented the swarm will divide evenly between the paths. The ants that took the shortest path will reach the food first and turn back toward the nest. When they reach the fork at B the shortest path will be more strongly scented because more ants will have emerged from it. The scent of the shortest path will be reinforced until the whole swarm converges on the shortest path.

3.1. Ant routing

The ant metaheuristic has lead to the creation of ant algorithms such as the AntNet system [6]. Ant routing algorithms generate ants (packets with random addresses) that traverse the network and collect timestamps as they pass through each node. The ants are routed by a stochastic process that is weighted by the goodness of a particular route [6]. When an ant reaches its destination it generates a backward ant which follows the same route as the original ant back to its source. As the backward ant travels through each node it updates the stigmergy table, which holds the goodness values for the different routes. An example of a stigmergy table typically produced in our study is illustrated in Table 1.

The table is consulted when sending an ant to the node whose address is the column name. The values represent the probabilities of using the node whose address is the row name as the next node on the path. The table is taken from node 0. Note how the sum of probabilities in each column is 1.

Each node also maintains a data structure that contains, for each other node in the network, the mean delay to that node (μ), the variance of observed delays (σ^2) and the number of observations (n) that contributed to μ and σ^2. This data structure is used when calculating the reinforcement to the goodness of a route.

First a raw measure (r) of the goodness of the reported time (t) is calculated as illustrated by Eq. (1):

$$r = \frac{t}{2\mu}, \tag{1}$$

with out-of-scale (> 1) values of r being saturated to 1.

If the mean is considered unstable ($\sigma^2/\mu \geq 0.25$) then the value for r is corrected as follows: for good values of r ($r < 0.5$) a value U is added to r, and for bad values of r ($r \geq 0.5$) U is subtracted from r, where:

$$U = 0.1^{\sigma^2/\mu}. \tag{2}$$

This rule mitigates the costly oscillatory behaviour that arises in dynamic routing algorithms by reducing the reinforcement effect if the mean delay to a node fluctuates.

If the mean is considered stable ($\sigma^2/\mu < 0.25$) then the value for r is corrected as follows: for good values of r ($r < 0.5$) a value S is subtracted from r, and for bad values of r ($r \geq 0.5$) S is added to r, where:

$$S = 0.1^{2\sigma^2/\mu}. \tag{3}$$

This rule amplifies the reinforcement effect for routes with stable mean delays, effectively increasing the learning rate when we trust observations and believe that it is safe to do so.

The corrected r is used to update the stigmergy table using the following rules.

For the probability (P_0) for the neighbour which the time relates to:

$$P_0' = P_0 + (1-r)(1-P_0). \tag{4}$$

For all other neighbour's probabilities yields:

$$P_n' = P_n + (1-r)P_n. \tag{5}$$

The rule for updating the values in the stigmergy table simply ensures that the value being reinforced is increased in proportion to the goodness (r). The other values are decreased in proportion to r and their own relative magnitudes while keeping the sum of all the probabilities equal to 1.

Data packets are not routed stochastically, they are always sent to the neighbour with the greatest goodness value for the intended destination. When forward ants revisit a node the circuit that they have travelled in is cleared from their memory to avoid reinforcing circular routes. To attempt to provide a faster feedback mechanism backward ants have priority over all other packets. A common criticism of this system is that a faster yet feedback mechanism would be to design forward ants to update the routing tables of nodes with regard to the section of the trip that they already completed. To this we respond that an essential feature of the ant metaheuristic is that the reinforcement from poor routes must be delayed proportionally.

Table 1
A stigmergy table from node 0 (taken from simulation output)

Address	0	1	2	3	4	5	6	7	8	9
0	0	0	0	0	0	0	0	0	0	0
1	0	0.99993	$3.45 \cdot 10^{-5}$	0	0.00475	0.113151	0.22	0	0	0.03413
2	0	$7.48 \cdot 10^{-5}$	0.61347	0	0.00475	0.113151	0.195	$2.74 \cdot 10^{-5}$	0.436324	0.03413
3	0	0	0.23308	0.185908	0.00475	0.127658	0.195	0	0	0.8635
4	0	0	0	0	0	0	0	0	0	0
5	0	0	0	0	0	0	0	0	0	0
6	0	0	0	0	0	0	0	0	0	0
7	0	0	0.13051	0	0.02975	0.113151	0.195	0.99997	0	0.03413
8	0	0	0.02291	0.814092	0.95599	0.532888	0.195	0	0.563676	0.03413
9	0	0	0	0	0	0	0	0	0	0

3.2. Traffic patterns and network topology

The traffic generator generates a random number of packets less than ten with random sizes below 1000 bytes. Self-addressed packets will not be generated. Traffic patterns are saved for reuse during simulations with different routing approaches. At the beginning of each simulation every node is preloaded with its traffic pattern after which no more traffic is added to the nodes.

The network generator included in the framework is a random network generator (as opposed to scale-free). In our study the network generator was typically used to generate a network of 10 nodes with a probability of any pair of nodes being connected at 0.35. A graph traversal function guarantees that only connected graphs proceed to simulation. The generator will not create self-to-self arcs. Topologies are saved for reuse during simulations with different routing approaches.

The choice to not use a scale-free network generator was an important one. For more than 40 years the study of networks was based on work by Paul Erdös and Alfréd Rényi. They suggested, in 1959, that networks could be described by nodes connected by randomly placed links. While their work revolutionised graph theory it has since been shown that scale-free networks are much more common. The physical structure of the Internet and the link structure of the world wide Web (WWW) have both been shown exhibit scale-free organisation [7]. The choice of generation method and implementation of a suitable algorithm is a considerable undertaking as can be seen in the recent work of Spencer and Sacks, for example [12]. For this study we have decided to simulate random networks only.

3.3. Characteristics of random networks

The random networks are those that are formed by the creation of links between randomly chosen pairs of nodes. Random networks are also known as exponential networks because the probability that a node is connected to k other nodes decreases exponentially as k increases. If the frequency of nodes is plotted against the number of links a normal distribution is evident as in Fig. 2.

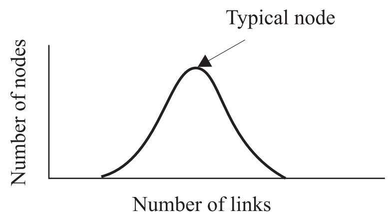

Fig. 2. Poisson distribution of node linkages in random networks.

One interesting point of random networks is that they are vulnerable to fragmentation by the removal of a number of randomly chosen nodes leaving the remaining nodes hopelessly separated from most of the rest of the network.

3.4. Characteristics of scale-free networks

Scale-free networks consist of clusters of nodes connected to a central hub that is connected to other hubs like it. They are characterised by power-law distribution of node linkages. This means that the probability of a node being connected to k other nodes is $1/k^n$. Scale-free networks all seem to have values of n between two and three. So for example, a node is four times as likely to have only half the number of links another node has. Figure 3 illustrates this behaviour.

In scale-free topologies the vast majority of nodes have roughly the average number of links, but a few "hubs" have thousands times the average number of links. When plotted on a double logarithmic scale the node linkage distribution is a straight line. This behaviour is illustrated in Fig. 4.

In contrast to random networks, scale-free networks are resilient to the removal of randomly chosen nodes to a high degree. As many as 80 percent of randomly selected nodes can fail and the remainder will still form a compact con-

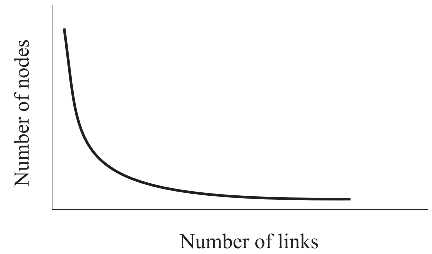

Fig. 3. Power-law distribution of node linkages in scale-free networks.

nected cluster [3]. However they are very sensitive to the removal of selected hubs. In fact, scale-free networks are only more robust to node removal than random networks if more than 95 percent of the removed nodes are chosen at random [10].

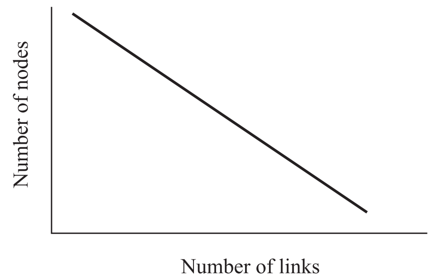

Fig. 4. Power-law distribution plotted on double logarithmic scale (scale-free networks).

The peculiar characteristics of scale-free networks are due to the way they are created. The Internet did not come into being as a set of randomly connected routers and hosts, nor do new nodes attach themselves to the network at random points. The Internet grew. When new users become connected there are reasons why they connect at specific points. The mechanisms which create scale-free networks are *growth* and *preferential attachment*. Growth implies that the organisations (nodes) that are oldest will have accumulated more links. Preferential attachment refers to the fact that the more links a node has the more attractive it is to be connected to. Other factors accentuate this effect; for example strategic positioning of service providers and users clustering around a preferred service.

4. Results

The results in this study are based on five simulation runs. Each run uses the same traffic patterns and topology for each of the three routing approaches discussed. In the case of critical events, the framework outputs event tags to delimited text files. These files were then analysed by importing into a spreadsheet. The results of the final analysis are illustrated in Table 2.

The values provided are mean values across the five simulations. Static routing offers the best throughput of the three approaches but dynamic routing offers shorter mean packet delays. This would suggest that the dynamic routing

Table 2
Summary of results from analysis of simulation output

Metric	Routing approach		
	static	dynamic	ant
Mean packet delay [ms]	1537.78	1457.00	3878.30
No. packets delivered	36	36	35.8
Time taken [ms]	6006	7362	17 388
Total data transfer [B]	17 537	17 537	17 537
Throughput [B/s]	2.92	2.39	1.03
Total busy time [ms]	31 455.60	34 446.60	61 104.40
Percentage utilisation	52.44	46.94	35.76

algorithm sacrifices raw efficiency for fairness (keeping the average delay of packets lower). In this case ant routing shows inferior performance when compared to the other two. We contend that this is due to a coarse system of adjusting the learning rate which would be improved with fine-tuning.

5. Summary and future work

We developed a framework for simulation of a computer network and implemented static, dynamic and ant routing algorithms. We collected 15 results sets in total from five simulations. Upon analysis we find static and distance-vector routing perform similarly. Our ant routing algorithm performs sub optimally but demonstrates the principle of stigmergetic communication successfully.

The poor performance of the ant algorithm will be investigated further with special consideration given to the learning rate adjustments. Swarm intelligence may yet not prove more efficient than traditional network routing algorithms but its ability to self-organise operating purely on local information may prove useful in *ad hoc* networks like the Bluetooth world, for example [9]. An investigation of other network topologies, using our simulation framework, such as scale-free networks would be useful. Also the work can be taken into new fields. For example, consider a network controlled by ants using stigmergy to communicate information on billing, virus infection, hardware failure, usage patterns, etc. One can also envision a genetic algorithm evolving the different ant variants and producing super-ants tailored to a cable company's own network [14]. We believe however, that great caution must be exercised in the application of fitness criteria to ants. Ants in a technical sense are computer viruses, it is not hard to imagine what harm could be done if they spread across networks or evolved to escape detection.

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Paper

Distributed asynchronous algorithms in the Internet – new routing and traffic control methods

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Abstract— The paper presents several new algorithms concerning the third (network) and the fourth (transport) layer of ISO/OSI network model. For the third layer two classes of the shortest paths algorithms – label correcting and auction algorithms – are proposed. For the fourth layer an application of price decomposition to network optimization and Internet congestion control is suggested.

Keywords— computer networks, optimization, shortest path, traffic control, decomposition, distributed computations, asynchronous algorithms.

1. Introduction

In the last decade two new approaches were intensively studied to solve shortest path/routing problems in parallel and distributed environment, namely, label correcting [4, 8], and auction [18] algorithms.

The label correcting algorithms may be treated as a generalization of the Bellman-Ford algorithm. The name "label" refers to the distance from the origin node "1" to a node $i \neq 1$, the correction is its change in subsequent iterations of the algorithm leading towards the optimum (these algorithms may be similarly applied to problems with multiple origins and a single destination, by reversing the roles of origins and destinations and the direction of each arc). A notion of the candidate list of nodes is introduced. Different shortest path algorithms are distinguished by the method of selecting the node to exit the candidate list at each iteration. The simplest method from this family is the Bellman-Ford method – where the candidate list is simply a FIFO queue. More sophisticated label correcting methods maintain two queues and use more complex removal and insertion strategies. The objective is to reduce the number of node reentries to the candidate list. Some of these methods – namely SLF (small label first) and SLF-LLL (large label last) will be described in the paper.

An interesting alternative to these algorithms seem to be auction algorithms. In the basic version [2, 8] an auction algorithm maintains a path starting at the origin and a price for each node. The terminal node of the path "bids" for neighboring nodes basing on their prices and the lengths of the connecting arcs. At each iteration the path is either extended by adding a new node or contracted by deleting its terminal node. When the destination becomes the terminal node of the path, the algorithm terminates.

In recent years a considerable progress was also done in the area of distributed traffic control in the Internet. A special interest was paid to the asynchronous price method. In particular, the theory presented in [14] was thoroughly examined by the author of this paper. Unfortunately, an error in the proof of convergence of the asynchronous version of the algorithm was found. Later on this proof was corrected [12], so this interesting and important theory has been saved. At the same time Steven Low and his collaborates have shown how important for the Internet this theory is. In their works they used it not only as a tool to analyze the stability of different implementations of TCP congestion control protocols [16, 17], but also as a basis for the development of new, more efficient protocols [10]. At the end of the article some of these results will be presented.

2. Classical Bellman-Ford routing algorithm

We consider a directed graph consisting of n nodes (routers). Let us denote by N the set of all these nodes and by N_i the set of neighbours of the ith node (that is, the set of all nodes from the set N, to which arcs starting from i go). Let us assume, that every arc $(i, j) : j \in N_i$ is characterized by a positive scalar value a_{ij}, which we will treat as the cost of passage from i to j, that is the distance measure (metric).

Let us choose a node $m \in N$ and assume, that it may be reached from all other nodes. It can be easily proved (e.g., by the contradiction), that the paths of the minimum costs $\hat{d}_{im}$ (so-called shortest paths) can be obtained through the solution of the following set of equations:

$$\hat{d}_{im} = \min_{j \in N_i} \left(a_{ij} + \hat{d}_{jm}\right) \quad i \neq m . \tag{1}$$

Let us take now

$$h_{im}(d) = \begin{cases} \min_{j \in N_i} (a_{ij} + d_{jm}) & i \neq m \\ d_{ii} & i = m \end{cases} \tag{2}$$

and

$$d = [d_{11} d_{12}, \ldots, d_{1n}, \ldots, d_{n1}, d_{n2}, \ldots, d_{nn}] , \tag{3}$$

$$h_i(d) = [h_{i1}(d), \ldots, \ldots, h_{in}(d)] , \tag{4}$$

$$h = [h_1, h_2, \ldots, h_n] . \tag{5}$$

To find the solution we may apply the Bellman-Ford algorithm

$$d := h(d) \tag{6}$$

starting from $d_{ii} = 0$; $d_{ij} = \infty, i \neq j, \forall i, j \in N$. This algorithm is based on the order preserving (monotone) mapping h, which may be implemented in a distributed, totally asynchronous version [6].

The optimization algorithm (1)–(6) may be applied respectively – to adapt routing to the current situation in a network. In that case the cost a_{ij} should be a measure of the quality of transmission, dependent on the current flow (transmission rate) f_{ij} between nodes i and j. A very popular flow cost function $a_{ij}(.)$ is:

$$a_{ij}(f_{ij}) = \frac{f_{ij}}{(c_{ij} - f_{ij}) + \varepsilon_{ij}} + d_{ij} \cdot f_{ij}, \tag{7}$$

where c_{ij} is the transmission capacity of arc (i, j) and d_{ij} is the processing and propagation delay (of course we assume $0 \leq f_{ij} \leq c_{ij}$; $\varepsilon_{ij} > 0$ is a small constant to avoid zero in the denominator). However, in this – adaptive – case one should remember, that the cost functions (7), dependent monotonically on flow, should be augmented with constant components δ_{ij} (so-called bias factors, interpreted as link costs/lengths at zero load), because otherwise oscillations (in subsequent optimal routings) may occur [5].

The presented adaptive routing approach is used in the Internet [7] in demons *routed*, *gated* and protocols RIP and Hello.

In the first protocol so-called "hop count metrics" is used, what means, that simply all elementary arcs are counted for; in the second "network delay metrics", that is the time of transmission, is taken into account. In the active state, all messages used to the optimization of the routing tables (i.e., the tables of the shortest path neighbours for different destinations) are sent by every computer to all direct neighbours every 30 seconds.

3. Generic shortest path algorithm

The algorithm presented in the previous section may be treated as a special case from a more general class of algorithms. We will present these algorithms to solve problems as formulated in the cited works.

Let us take now that for each node $i \in N$ we want to find a path of minimum length (cost) that starts at node 1 and ends at i (these algorithms may be similarly applied to problems with multiple origins and a single destination, by reversing the roles of origins and destinations and the direction of each arc). We assume, that all arc lengths are positive and that there exists at least one path from node 1 to each other node.

A general class of algorithms to which belongs, among others, Bellman-Ford algorithm are label correcting algorithms. The name "label" refers to the distance $d_i \triangleq d_{1i}$ from the origin node "1" to a node $i \neq 1$, the correction is its change in subsequent iterations of the algorithm leading towards the optimum. A notion of the candidate list of nodes is introduced. Let us denote it by V. In addition to this, let us denote by A the set of all arcs in the directed graph (that is, the set of all links in the network). Assuming that V is nonempty, a typical iteration of a shortest path algorithm (not necessarily of label correcting type) is as follows [3]:

Initialization:

$$d_1 = 0, \; d_i = \infty \quad \text{for} \quad i \neq 1,$$

$$V = \{1\}.$$

Typical iteration of the generic shortest path algorithm:

Remove a node i from the candidate list V.
For each outgoing arc $(i, j) \in A$, with $j \neq 1$,
if $d_j > d_i + a_{ij}$, set:

$$d_j := d_i + a_{ij} \tag{8}$$

and add j to V if it does not already belong to V.

Different shortest path algorithms are distinguished by the method of selecting the node to exit the candidate list V at each iteration. For example in Dijkstra method the node exiting V is the node whose label is minimum over all other nodes in V. This guarantees, that every node enters and exits V exactly once and its label is not changed in later iterations. Because of that, these methods are called label setting methods. Label correcting methods avoid the overhead associated with finding the minimum label node at the expense of multiple entrances of nodes into V. In these methods a queue Q is used to maintain the candidate list V. Bellman-Ford method is the simplest method from this family. In the terms of the above generic shortest path algorithm it maintains V in a FIFO queue Q; nodes are removed from the top of the queue and are added to the bottom of Q.

4. SLF and LLL strategies

More sophisticated label correcting methods maintain V in one or in two queues (eg., in so-called threshold algorithms the candidate list is partitioned into two separate queues on the basis of some threshold parameter) and use more complex removal and insertion strategies. The objective is to reduce the number of node reentries in V. Some of these methods are significantly faster than Bellman-Ford method. The most effective proved to be SLF and SLF-LLL methods [4].

In the SLF method the candidate list V is maintained as a double ended queue Q. At each iteration, the node removed is the top node of Q. The rule for inserting new nodes is as follows:

Let i be the top node of Q, and j be a node that enters Q,

if $d_j \leq d_i$, then enter j at the top of Q;
else, enter j at the bottom of Q.

The LLL method defines a more complicated strategy of removal a node from Q, which aims to remove from Q nodes with small labels. At each iteration, when the node at the top of Q has a larger label than the average node label in Q (defined as the sum of the labels of the nodes in Q divided by the cardinality $\overline{\overline{Q}}$ of Q), this node is not removed from Q, but rather it is repositioned to the bottom of Q. It may be described as follows:

Let i be the top node of Q, and let $s = \sum_{j \in Q} d_j / \overline{\overline{Q}}$,
if $d_i > s$, then move i to the bottom of Q.
Repeat until a node i such that $d_i \leq s$ is found and is removed from Q.

It is possible to combine the SLF queue insertion and the LLL node selection strategies. The resulting method is denoted by SLF-LLL. The proof of convergence of these two algorithms closely resembles the proof of the convergence of Bellman-Ford algorithm and is based on the monotonicity property of the label modification mapping [4, 8].

In the parallel implementation, each processor removes the top node from Q (perhaps after some shifts of the queue in the case of LLL strategy), updates the labels of its adjacent nodes and adds these nodes (if necessary) into Q according to SLF insertion strategy. This means, that several nodes can be simultaneously removed from the candidate list and the labels of the adjacent nodes can be updated in parallel. In the distributed version an additional processor responsible for maintaining the candidate queue Q is useful. When the algorithm is implemented in an asynchronous version, a new node may be removed from the candidate list by some processor while other processors are still updating the labels of other nodes. Of course, only one processor at a time can modify a given label. Hence, it is very easy to implement this method on a parallel shared-memory or ccNUMA machines using locks to assure the consistency of data.

A multiple queues version of this algorithm showed also very good features [4]. In this version each processor uses a separate queue (a node can reside in at most one queue). It extracts nodes from the top of its queue, updates the labels for adjacent nodes and uses a heuristic procedure for choosing the queue to insert a node that enters V. For example, it may be the one with the minimum current value of the sum of the outgoing arcs in that list. This heuristic is easy to implement and ensures good load balancing among the processors. For checking whether a node is present in the candidate list (that is, in some queue) it is suggested [4, 8] to associate with every node a Boolean variable, which is updated each time a node enters or exits a candidate list. This algorithm is easily generalized to the problem of finding several distinct shortest paths [8] showing in many problems very high efficiency.

Other label correcting methods (however, not so efficient in tests), e.g., with threshold dividing queues, are presented in [4].

5. Auction algorithm

A more efficient alternative to the presented algorithms seem to be auction algorithms. In the basic version [2, 18] such an algorithm maintains a path starting at the origin s and a price for each node. The terminal node of the path "bids" for neighboring nodes basing on their prices and the lengths of the connecting arcs. At each iteration the path is either extended by adding a new node or contracted by deleting its terminal node. When the destination becomes the terminal node of the path, the algorithm terminates.

To present the algorithm in a formal way, let us denote by P a path starting at the origin, that is: $P = (s, i_1, i_2, \ldots, i_k)$, where $(i_m, i_{m+1}) \in A$, $m = 1, \ldots, k-1$. We assume that $i_{j_1} \neq i_{j_2}$, $j_1 \neq j_2$, that is a path does not contain any cycle. The node i_k is called the terminal node of P. The degenerate path $P = (s)$ may be also obtained in the course of the algorithm.

If i_{k+1} is a node that does not belong to a path $P = (s, i_1, i_2, \ldots, i_k)$ and (i_k, i_{k+1}) is an arc, extending P by i_{k+1} means replacing P by the path $(s, i_1, i_2, \ldots, i_k, i_{k+1})$. If P does not consist of just the origin node s, contracting P means replacing P with the path $(s, i_1, i_2, \ldots, i_{k-1})$.

In addition to the path, the algorithm maintains a price p_i for each node $i \in N$ in the network. Let us denote by p the vector of all prices p_i. We say, that a path-price pair (P, p) satisfies complementary slackness (CS) if the following relations hold:

$$p_i \leq a_{ij} + p_j, \ \forall (i, j) \in A, \tag{9}$$

$$p_i = a_{ij} + p_j \tag{10}$$

for all pairs of successive nodes i and j of P.

An important property is that if a path-price pair (P, p) satisfies CS, then portion of P between node s and any node $i \in P$ is the shortest path from s to i and $d_{si} = p_s - p_i$ is the corresponding shortest distance.

The algorithm proceeds in iterations, transforming a pair (P, p) satisfying CS into another pair satisfying CS. At each iteration the path P is either extended by a new node or contracted by deleting its terminal node. In the latter case the price of the terminal node is increased strictly. In may be described in the following way:

Typical iteration:

Let i be a terminal node of P.

- *Step 0:* (Scanning of successor nodes)

  If $p_i < \min_{\{j | (i,j) \in A\}} \{a_{ij} + p_j\}$ (11)

  go to Step 1; else go to Step 2.

- *Step 1:* (Contract path)

  Set $p_i := \min_{\{j|(i,j)\in A\}} \{a_{ij} + p_j\}$ (12)

  and if $i \neq s$ contract P.

- *Step 2:* (Extend path)

  Extend P by node j_i, where

  $$j_i = \arg \min_{\{j|(i,j)\in A\}} \{a_{ij} + p_j\} . \tag{13}$$

  If j_i is the destination d, stop; P is the desired shortest path.

The algorithm starts with the default pair:

$$P = (s), \quad p_i = 0, \, \forall i$$

and stops when the destination node d becomes the terminal node of the path. This iteration is called the "forward algorithm". It is possible to apply also the "reverse algorithm", where not the source s, but the destination node d is fixed (and forms the initial path) and the path extends by inserting and contracts by deleting the starting node, and to use a combined algorithm, where there are two paths: P_f – starting at the origin s and P_r – ending at the destination d. In this algorithm alternately several forward and reverse iterations are performed at least one of which leads to an increase of, respectively, origin price p_s or the destination price p_d. This two-sided algorithm terminates when the two paths have a common node. In many tests [2] this hybrid approach proved to be the most effective, much faster than the sided Dijkstra algorithm.
When there are several origin nodes, the shortest path auction algorithm may be implemented in a parallel way in a distributed environment [2]. In the basic (i.e., forward) version for each origin i there is a separate processor that executes the forward algorithm and keeps in local memory a price vector p^i (build of the snapshots of prices p_l^j sent by other nodes) and a corresponding path P^i satisfying CS together with p^i. The price vectors are communicated at various times to other processors, perhaps irregularly. A processor operating on P^i upon reception of a price vector p^j from another processor j, adopts as the price of each node l the maximum of the prices of l according to the existing and the received (i.e., the snapshots) price vectors, that is:

$$p_l^i := \max(p_l^i, p_l^j), \ \forall l \in N . \tag{14}$$

This guarantees keeping the CS property, monotonicity of the mapping:

$$p_i := \min_{\{j|(i,j)\in A\}} \{a_{ij} + p_j\} \tag{15}$$

and the asynchronous convergence of the algorithm.

The parallel, synchronous and asynchronous implementations of the two-sided different shortest path algorithms for different problems (including many origin – many destinations routing problem) are more complicated due to the possibility of losing CS and, in the consequence, the oscillation of prices. To avoid it, all nodes are equipped with counters for forward and reverse extensions without an intervening contractions. However, in most tests (on a shared-memory machine) much simpler, one sided, forward scheme showed superiority. The details are described in [18].

6. The application of the price method to network flow optimization

In this section we will return to a problem presented in [11], because since that time there were some progress both in a better justification of this approach and in the understanding of its importance for the Internet. There were also successful implementations of this method to improve Internet congestion control protocols at the TCP level.
We will consider the situation, where the capacities of links are too small to carry all traffic and it is necessary to reduce the users' transmission rates. So, we will deal with the decision variables – flexible transmission rates x_w, where $w \in W$ is the connection and W is the set of all active connections (i.e., source-destination pairs).
The transmission rates should belong to some intervals:

$$\underline{x}_w \le x_w \le \overline{x}_w . \tag{16}$$

Every customer, that is the user of the network, assesses the satisfaction from the use of the network through his utility function $U_w(x_w)$, defined on the interval $[\underline{x}_w, \overline{x}_w]$. In this problem, instead of minimization of the total cost of transmission, which is not so important for the operator of the network (because most links are fully used), we will strive to maximize the satisfaction of the customers, that is the sum of their utility functions.
Hence, our problem will be as follows:

$$\max_x \sum_{w\in W} U_w(x_w) , \tag{17}$$

$$\underline{x}_w \le x_w \le \overline{x}_w, \ \ w \in W , \tag{18}$$

$$f_{ij} = \sum_{w\in W_{ij}} x_w \le c_{ij}, \ \ \forall (i,j) \in A . \tag{19}$$

The last inequality expresses capacity constraints of the links; W_{ij} denotes the set of connections (virtual paths) traversing arc (i,j), that is:

$$W_{ij} = \{w|(i,j) \in A_w\} , \tag{20}$$

where A_w is the set of arcs (links) used by connection w.
The Lagrange function for problem (17)–(19) will be as follows:

$$\mathrm{L}(x,p) = \sum_{w\in W} U_w(x_w) - \sum_{(i,j)\in A} p_{ij} \left(\sum_{w\in W_{ij}} x_w - c_{ij} \right), \tag{21}$$

where

$$x = \Big[x_w, w \in W \Big] . \tag{22}$$

Let us notice now that:

$$\sum_{(i,j)\in A} p_{ij}\left(\sum_{w\in W_{ij}} x_w - c_{ij}\right)$$
$$= \sum_{(i,j)\in A} p_{ij} \sum_{w\in W_{ij}} x_w - \sum_{(i,j)\in A} p_{ij} c_{ij}$$
$$= \sum_{w\in W} \sum_{(i,j)\in A_w} p_{ij} x_w - \sum_{(i,j)\in A} p_{ij} c_{ij}$$
$$= \sum_{w\in W} x_w \sum_{(i,j)\in A_w} p_{ij} - \sum_{(i,j)\in A} p_{ij} c_{ij}$$
$$= \sum_{w\in W} x_w p_w - \sum_{(i,j)\in A} p_{ij} c_{ij}\,, \tag{23}$$

where

$$p_w = \sum_{(i,j)\in A_w} p_{ij} \tag{24}$$

is the price for the connection w along its path formed of arcs $(i,j)\in A_w$.

Applying (23) to (21) we get:

$$\mathrm{L}(x,p) = \sum_{w\in W}\Big(U_w(x_w) - x_w\,p_w\Big) + \sum_{(i,j)\in A} p_{ij}\,c_{ij}\,. \tag{25}$$

According to the duality theory, the optimal solutions, both the optimal distribution of flows $(\hat{x}_w, w\in W)$ and the vector of optimal prices $[\hat{p}_{ij}, (i,j)\in A]$ may be obtained via the two-phase procedure:

$$\min_p \left[L_D(p) = \sum_{w\in W} \max_{\underline{x}_w\le x_w\le \overline{x}_w}\Big(U_w(x_w) - x_w\,p_w\Big) + \sum_{(i,j)\in A} p_{ij}\,c_{ij}\right]. \tag{26}$$

In this way we obtained $\overline{\overline{W}}$ problems of optimization of connection transmission rates and an $\overline{\overline{A}} = n$-dimensional problem of optimization of prices of unit bandwidth ($\overline{\overline{V}}$ denotes the number of elements of the set V). It can be proved [12, 14], that they may be solved in a distributed, partially asynchronous way[1].

More precisely, the wth user solves the local optimization problem:

$$\max_{\underline{x}_w\le x_w\le \overline{x}_w}\Big(U_w(x_w) - x_w\,\tilde{p}_w(t)\Big)\,, \tag{27}$$

where $\tilde{p}_w(t)$ is the current estimate of the price of transmission w, that is:

$$\tilde{p}_w(t) = \sum_{(i,j)\in A_w}\sum_{\tau=t-B}^{t} \eta_{ij}^{w}(t,\tau)\,p_{ij}(\tau)\,. \tag{28}$$

In the last equation $\eta_{ij}^{w}(t,\tau)$ are (usually unknown) nonnegative coefficients such that:

$$\sum_{\tau=t-B}^{t} \eta_{ij}^{w}(t,\tau) = 1 \tag{29}$$

[1]The proof of the asynchronous convergence theorem in [14] had a serious error pointed out and corrected in [12].

and B is the length of the time window (i.e., the measure of asynchronizm). The optimal solution of the local problem (27) may be determined analytically [14] from the expression:

$$\hat{x}_w = \Big[U_w^{'-1}(\tilde{p}_w)\Big]_{\underline{x}_w}^{\overline{x}_w}\,, \tag{30}$$

where $[z]_a^b = \min\{\max\{z,a\},b\}$ and $U_w^{'-1}$ is the inverse of $= U_w^{'}$.

The optimal link prices $\hat{p}_{ij}$ in problem (26) may be calculated in different ways. In the simplest case the steepest descent method is applied. According to Eq. (21) this is realized by the iteration:

$$p_{ij}(t+1) = \left[p_{ij}(t) + \gamma\Big(\tilde{f}_{ij}(t) - c_{ij}\Big)\right]^{+}, \tag{31}$$

where

$$\tilde{f}_{ij}(t) = \sum_{w\in W_{ij}}\sum_{\tau=t-B}^{t} \eta_{ij}(t,\tau)\,x_w(\tau) \tag{32}$$

is the estimate of the total flow through the link (i,j). In the last equation $\eta_{ij}(t,\tau)$ are (usually unknown) nonnegative coefficients such that:

$$\sum_{\tau=t-B}^{t} \eta_{ij}(t,\tau) = 1 \tag{33}$$

and B is the length of the time window. Due to the theory presented in [12, 14], for sufficiently small values of the stepsize γ and bounded time intervals between consecutive updates of the optimal link prices p_{ij} and transmission rates x_w, the algorithm converges partially asynchronously. Since the algorithm was devoted to flow control in the Internet, where sending information on current internal prices of links (from the operator to users) and the calculation of current average transmission rates in all virtual paths would mean some additional equipment and the communication overheads, the following estimation mechanisms were proposed [1, 15]:

1. Instead of calculation of the aggregated flow rate $\tilde{f}_{ij}(t)$ in the link (i,j), the link operator measures the link buffer occupancy $v_{ij}(t)$. This occupancy evolves according to the equation:

$$v_{ij}(t+1) = \left[v_{ij}(t) + \sum_{w\in W_{ij}} x_w(t) - c_{ij}\right]^{+}, \tag{34}$$

   where $x_w(t)$ is the current flow rate of the transmission w. Then, the new link price $p_{ij}(t+1)$ is set as:

$$p_{ij}(t+1) = \gamma v_{ij}(t)\,. \tag{35}$$

2. Instead of passing the users directly the information on the current price of the unit of the bandwidth, the link operator applies random exponential marking (REM) algorithm. It allows for encoding this information in only one bit[2] of the stream of packets.

[2]We mean explicit congestion notification (ECN) bit in the IP header.

Namely, it is assumed, that the link (i,j) marks each packet with a probability $m_{ij}(t)$ which is exponentially increasing in the price $p_{ij}(t)$ (or in the buffer occupancy $v_{ij}(t)$ – see (35)):

$$m_{ij}(t) = 1 - \phi^{-p_{ij}(t)}, \tag{36}$$

where $\phi > 0$ is a constant. Once a packet is marked, its mark is carried to the destination and then conveyed back to the source via acknowledgement (due to the TCP/IP protocol). The end-to-end probability that a packet of the connection w is marked after traversing the whole its way made of arcs form the set A_w is then:

$$m_w(t) = 1 - \prod_{(i,j)\in A_w} (1 - m_{ij}(t)) = 1 - \phi^{-p_w(t)}, \tag{37}$$

where $p_w(t) = \sum_{(i,j)\in A_w} p_{ij}(t)$ is the price for the transmission of the unit of bandwidth along the virtual path w. Then, the customer using this connection estimates the price of it $\tilde{p}_w(t)$ by the fraction $\tilde{m}^p(t)$ of his packets marked in some window before time t and inverting (37), that is:

$$\tilde{p}_w(t) = -\log_\phi (1 - \tilde{m}_w(t)). \tag{38}$$

Owing to this two improvements, there is no need for administrative communication between the operators of links and the end users of the network.
It should be noted, that the prices in this model are regarded rather as a control signal to guide sources' decisions, than a part of the charge a user pays [14]. In particular, these prices (i.e., Lagrange multipliers) equal zero when the traffic is below the capacity of the network. In other words: the user pays nothing for the highest desirable quality! On the contrary, he pays more and more for connections of lower quality, when there is a congestion in the network. Such pricing mechanism would be hardly acceptable for a human. So it is rather a tool for software agents.
In the latest articles Low and collaborates [16, 17] present few such agents and interpret their utility functions. It is shown, that in the Internet these agents are simply different implementations of TCP congestion control protocols. The basic idea is to regard the process of congestion control as carrying out a distributed computation by sources and links over a network in real time to solve an optimization problem. The objective is to maximize aggregate source utility subject to capacity constraints. The source rates are interpreted as primal variables, congestion measures as dual variables, and TCP/AQM (active queue management) protocols as distributed primal-dual algorithms to solve this optimization problem and its associated dual problem. Different protocols, such as Reno, Vegas, RED, and REM, all solve the same prototypical problem with different utility functions, Moreover, all these protocols generate congestion measures (Lagrange multipliers) that solve the dual problem in equilibrium. It is described in the next section.

7. TCP window flow control through the price method and the consequences

The classical congestion control method, currently used in the Internet at the TCP level, is based on Jacobson algorithm [9] called TCP-Reno. If we denote by: $z(t)$ – the length of the source window at time t, that is the maximum number of unacknowledged packets that the source can inject into the network at the time t; RTT – round trip time, that is the time between sending the packet and receiving its acknowledgement; ACK – the acknowledgement packet; TOUT – timeout for waiting for ACK, this algorithm may be described in the following way:

1. Slow-Start phase
   - z(0) = 1,
   - after every ACK received $z(t+1) = z(t) + 1$ until attaining SSTRESH (slow-start threshold).
2. Congestion avoidance phase:

$$z(t+1) = \begin{cases} z(t) + \frac{1}{z(t)} & \text{OK} \equiv \text{ACK received before TOUT} \\ \frac{1}{2} z(t) & \text{the loss of packet} \equiv \text{ACK has not arrived before TOUT or 3 previous have been received} \end{cases} \tag{39}$$

In the recent works [16, 17]:

- z are treated as primal variables (actually it is taken $x = c \cdot z$, $c = const.$),
- the link congestion measures are treated as dual variables p,
- the dynamic (state) equations describing the modification of z (or rather x) and p are treated as a distributed primal-dual algorithm.

One may transform the most important congestion avoidance phase of TCP-Reno to the problem (17)–(19) taking:

$$\dot{z}_w(t) = \kappa_w(t) \cdot \left(1 - \frac{p_w(t)}{v_w(t)}\right), \tag{40}$$

where

$$\kappa_w(t) = \frac{1}{T_w(t)};\ v_w(t) = \frac{3}{2z_w^2(t)} \tag{41}$$

and $T_w(t)$ is an estimate of RTT at time t for the wth connection.

Let us denote the transmission rate by $x_w(t)$:

$$x_w(t) = z_w(t)/T_w(t) \tag{42}$$

and define an utility function:

$$U(x_w) = \frac{\sqrt{3/2}}{T_w} \tan^{-1}\left(\sqrt{\frac{2}{3}} x_w \cdot T_w\right). \tag{43}$$

The dual variable p_w may be interpreted as the probability of the loss of a packet for the wth connection.

The TCP-Reno has some drawbacks:

- at the beginning the window grows too slowly,
- when the packets are lost the window is shortened too abruptly,
- often oscillations,
- the big packets are privileged (small packets are punished although the congestion is almost always caused by big packets).

Because of these drawbacks, some proposals appeared how to improve the effectiveness of using the link capacities and to make the network more just. The most successful proved to be the following model:

$$\kappa_w(t) = \gamma \cdot \alpha_w;\ \nu_w(t) = \alpha_w / x_w(t), \tag{44}$$

$$U(x_w) = \alpha_w \log x_w, \tag{45}$$

where κ_w, ν_w are from the state equation (40) and $U(x_w)$ is from the source optimization problem (17)–(19). The utility function is derived from the notion of weighted proportional fairness introduced by Kelly [13].

A vector of rates $\hat{x} = (\hat{x}_w, w \in W)$ is *weighted proportionally fair* if it is feasible and if for any other feasible vector x, the aggregate of proportional changes is zero or negative:

$$\sum_{w \in W} \alpha_w \frac{x_w - \hat{x}_w}{\hat{x}_w} \leq 0. \tag{46}$$

It easy to check, that the performance index (45) as a concave function satisfies weighted proportional fairness condition (46) and, if we treat all connections as a game between users, it is a Nash-equilibrium point [19].

The dual variable p_w in this model is interpreted as a delay caused by queues in routers for the wth connection.

This very approach became a basis for the development in California Institute of Technology (Caltech) by prof. S. Low group of the new TCP control protocol called FAST (Fast Active queue management Scalable Transmission control protocol). It was designed for high speed data tranfers over large distances, e.g., tens of gigabyte files across the Atlantic [10]. At the time of writing this paper the world record of the Internet transmission (8.6 Gbps from Los Angeles to Geneva (CERN) via Chicago) belonged to this very group (as the previous one).

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Paper

Optimization approach with ρ-proximal convexification for Internet traffic control

Adam Kozakiewicz

Abstract—The optimization flow control algorithm for traffic control in computer networks, introduced by Steven H. Low, works only for concave utility functions. This assumption is rather optimistic and leads to several problems, especially with streaming applications. In an earlier paper we introduced a modification of the algorithm based on the idea of proximal convexification. In this paper we extend this approach, replacing the proximal method with the ρ-proximal method. The new method mixes the quadratic proximal term with higher-order terms, achieving better results. The algorithms are compared in a simple numerical experiment.

***Keywords*— nonlinear programming, price method, convexification, network control.**

1. Introduction

It is possible to formulate the problem of traffic control in the Internet as an optimization problem. Many proposed control methods are based on the idea of solving such a problem using a distributed algorithm based on the dual approach. Recently this approach was even used to model TCP Vegas [4]. However, most of such proposals assume convexity of the problem, eliminating the possibility of a duality gap. The model is then simplified, but not necessarily realistic. Nonconvexity, while definitely not desireable, may be important. There are a lot of methods for convexification of a nonconvex optimization problem, but most of them do not correspond to the structure of the network. In [2] we proposed a way of introducing convexification in the algorithm proposed by Steven H. Low ([1] with a correction in [5]). The method we proposed is not the only one possible and in this paper some modifications are suggested.

In Section 2 we describe the problem, introduce the concept of proximal convexification and the partial convexification algorithm for traffic control from [2]. Section 3 presents the ρ-proximal convexification method, and Section 4 explains how it can be adapted to our problem. Finally, in Section 5 we present some simulation results and conclude in Section 6 with a summary.

Note: We use the term *convex problems* for a class of problems including both minimization of convex functions and maximization of concave functions, both over convex sets. Converting a problem to fit this class is therefore called *convexification*. In the case of a maximization problem, convexification means making the goal function concave (hence the names *convexification parameter* and *center of convexification*).

2. Optimization in traffic control

An often mentioned (e.g., [6–10]) way of dealing with the problem of sharing network resources in the best possible way is maximizing average (or, equivalently, total) utility, as defined by the sources' individual utility functions. The optimal solution of such a problem is *proportionally fair*. This approach is the essence of Steven H. Low's optimization flow control algorithm, presented in [1] (with a correction to the proof in [5]).

The network is modelled as a set of unidirectional links, L with capacities $c_l, l \in L$, used by a set of sources, S. The links may also be bidirectional with the capacity limit on the sum of traffic in both directions. Each source uses a predefined set of links L_s (static routing), although the model can be modified to choose from multiple paths. The relation between sources and links defined by sets L_s can be represented as the binary routing matrix $\mathbf{A}$. Each source's rate x_s is bounded by limits m_s and M_s and derived from the state of the network and its own utility, defined as a function $U_s(x_s)$, preferably zero for $x_s = m_s$ and increasing. Utility may be normalized (to be 1 at $x_s = M_s$) or not. Capacities and rates can be expressed as vectors $\mathbf{c} = [c_l]_{l \in L}$, $\mathbf{x} = [x_s]_{s \in S}$. For feasibility it is necessary that for each link the sum of lower limits of sources using this link be lower than its capacity. To eliminate the possibility of a duality gap, strong concavity of the utility function for each source is also necessary. Combining both assumptions leads to certain difficulties, explained later.

The problem in this case can be formulated as follows:

$$\begin{aligned} &\max_{\mathbf{x} \in I} \sum_{s \in S} U_s(x_s) \\ &\text{subject to } \mathbf{A}\mathbf{x} \leq \mathbf{c} \\ &I = \bigtimes_{s \in S} I_s \\ &I_s = [m_s, M_s] \end{aligned} \tag{1}$$

and it can be solved by application of the dual approach.

Using prices (Lagrange multipliers) to communicate the state of the network to sources it is possible to distribute effectively the computations. A simple, synchronous algorithm follows:

Algorithm 1 (synchrononous gradient projection)
At times $t = 1, 2, \ldots$:

- Link (router) l:
  1. Collects the rates x_s of all sources using the link ($s \in S_l$) and computes $x^l = \sum_{s \in S_l} x_s$.
  2. Computes a new price $\xi_l(t+1) = [\xi_l(t) + \gamma(x^l(t) - c_l)]^+$, where $\gamma > 0$ is a stepsize, and $[z]^+ = \max\{z, 0\}$.
  3. Communicates new price $\xi_l(t+1)$ to all sources that use link l.
- Source s:
  1. Receives from the network the total price along its path $\xi^s(t) = \sum_{l \in L_s} \xi_l(t)$.
  2. Computes its new rate $x_s(t+1) = \arg\max_x [U_s(x) - \xi^s(t)x]$.
  3. Communicates its new rate to all links on its path.

If γ is small enough and the problem is convex, this algorithm is convergent, even with modifications, ranging from asynchronous computation to multiple paths and limited communication ([1] and later papers). In reality, however, for many applications the utility function is unlikely to be concave (a good explanation can be found in [7]). In most cases this means, that utility grows rapidly around some preferred transmission rate, much higher than the minimum, and only slowly grows for higher rates. Eliminating such nonconcavities by raising lower limits to the "preferred" rates and using convex approximations, as suggested in [1], may lead to exceeding the limit defined in the feasibility assumption and other problems, explained in [2].

2.1. Proximal convexification

There are many available methods of convexification (a survey can be found in [3]). The augmented Lagrangian method [11, 12], although simple and effective, would destroy separability and the link prices would not be sufficient information for the sources. The proximal method (see [13]) deals with nonseparability at the cost of an additional level of iteration.
A nonconvex problem:

$$\min_x \; f(x) \qquad (2)$$
$$\text{subject to } h(x) = 0, \quad g(x) \le 0,$$

where all functions are twice continously differentiable, can be solved by iterative application of the multipliers method to a modified problem:

$$\min_x \; f(x) + (\theta/2)\|y - x\|^2 \qquad (3)$$
$$\text{subject to } h(x) = 0, \quad g(x) \le 0.$$

where y is a parameter approximating the optimal point and $\theta > 0$ is the convexification parameter. Large values of θ give a larger area of convergence, but its rate becomes slower. At a higher, additional level of iteration we treat the value of the optimised function in the above problem depending on the parameter y, $\phi(y)$ as the goal function and solve $\min_y \phi(y)$ using some simple iterative algorithm, usually a Jacobi type iteration $y_{k+1} = \hat{x}_k$, where $\hat{x}_k$ is the solution of (3) for $y = y_k$. The method can easily be adapted for problems with inequality constraints.

2.2. Partial convexification

The additional level of iteration cannot be implemented in a changing distributed environment. Finding the right value for the convexification parameter – not too large, not too small – is difficult too. Luckily, the strict convergence guarantees of the original method are not necessary for traffic control, neither is strict optimality, as long as "good" solutions are found. In a real network the algorithm may never have enough time to converge to a fixed solution – the state of the network changes all the time, even during optimization. Old connections terminate, new ones are created and even an existing connection may have changing requirements – the problem is not a stationary one. More important than optimality are nonoscillatory behaviour, ability to smoothen perturbations and a possibility of simple, efficient implementation. Therefore in [2] we proposed the following algorithm, merging the higher iteration levels and dropping the requirement for the convexification strength, represented by a parameter θ, to be sufficiently large to make the problem convex:

Algorithm 2 (convexified projection algorithm)
At times $t = 1, 2, \ldots$:

- Link (router) l (as in Algorithm 1):
  1. Collects the rates x_s of all sources using this link ($s \in S_l$) and computes $x^l = \sum_{s \in S_l} x_s$.
  2. Computes a new price $\xi_l(t+1) = [\xi_l(t) + \gamma(x^l(t) - c_l)]^+$, where $\gamma > 0$ is a stepsize, and $[z]^+ = \max\{z, 0\}$.
  3. Communicates new price $\xi_l(t+1)$ to all sources that use link l.
- Source s:
  1. Receives from the network the total price on its path $\xi^s(t) = \sum_{l \in L_s} \xi_l(t)$.
  2. Computes its new rate $x_s(t+1) = \arg\max_x \left[U_s(x) - \xi^s(t)x - (\theta/\rho)\,\|x_s(t) - x\|_\rho^\rho\right]$, where $\theta > 0$ and $\rho \ge 1$ are parameters.
  3. Communicates new rate to all links on its path.

Parameter ρ in this algorithm is equal to 2 by default, other values, although possible, are not recommended, as they additionally weaken the local convexification effect. Parameter θ should actually be a source-dependent value θ_s, so we use the parameter $\eta = \theta_s |M_s - m_s|^\rho$ as a global specification of convexification strength.

3. The ρ-proximal convexification

The ρ-proximal convexification method is a version of the proximal algorithm, developed in [3, 14]. It modifies the convexifying terms added to the goal function of the modified problem (3) in an attempt to attain faster convergence. The modified problem in the new version is as follows:

$$\min_x \; f(x) + (\theta/2)\|y-x\|^2 + (\theta/\rho)\|y-x\|_\rho^\rho \quad (4)$$

subject to $h(x) = 0, \quad g(x) \le 0.$

It is possible to weigh the two convexifying terms differently, but the general idea remains the same. It is important that the quadratic term is not completely removed, as it is necessary for local convexity near y, not provided by higher order terms. Value ρ should be greater than 2. For practical reasons integer values are preferred. The most probable choice is 4, as it is easy to compute from the quadratic term, 6 or 8 might also be used for more difficult problems. For further analysis see [3, 14].

4. Modified partial convexification

The modification proposed in Section 3 can also be applied to Algorithm 2, giving the following algorithm:

Algorithm 3 (modified convexified projection algorithm)
At times $t = 1, 2, \ldots$:

- Link (router) l (as in Algorithm 1):
  1. Collects the rates x_s of all sources using this link ($s \in S_l$) and computes $x^l = \sum_{s\in S_l} x_s$.
  2. Computes a new price $\xi_l(t+1) = [\xi_l(t) + \gamma(x^l(t) - c_l)]^+$, where $\gamma > 0$ is a stepsize, and $[z]^+ = \max\{z, 0\}$.
  3. Communicates new price $\xi_l(t+1)$ to all sources that use link l.
- Source s:
  1. Receives from the network the total price on its path $\xi^s(t) = \sum_{l\in L_s} \xi_l(t)$.
  2. Computes its new rate
  $$x_s(t+1) = \arg\max_x \Big[U_s(x) - \xi^s(t)x - \theta\Big((\alpha/2)\,\|x_s(t)-x\|_2^2 + ((1-\alpha)/\rho)\,\|x_s(t)-x\|_\rho^\rho\Big)\Big],$$
  where $\theta > 0$, $\alpha \in (0,1)$ (preferably not too small) and $\rho \ge 2$ are parameters.
  3. Communicates new rate to all links on its path.

The newly introduced parameter α can be chosen arbitrarily. For $\alpha = 0$ or $\alpha = 1$ the algorithm is identical to Algorithm 2. In the simulations we decided to make α a per-source parameter α_s, derived from the assumption that, when the argument is $M_s - m_s$, the quadratic term is four times smaller than the other one. This way near the center of convexification ($x(t)$ in this algorithm) the quadratic term still dominates, while at longer range the stronger convexifier is more important. Parameter θ is chosen the same way as before, ρ defaults to 4. The generally better convexification of the ρ-proximal method suggests, that the new algorithm may be more effective than the original.

The mixed quadratic – higher order concave term has the advantages of both the original and convexified algorithm. As in the Algorithm 2, there is a mechanism to prevent the sources from oscillatory behaviour – the higher order term adds a penalty for such big changes. At the same time, the quadratic term is smaller and doesn't affect the utility function more than necessary. This is similar to the simple convexified algorithm with ρ set to any value greater than 2. This method however does not completely eliminate the quadratic term, which results in better local concavity near $x_s(k)$. Through analogy to the family of proximal convexification methods for general optimization there are reasons to believe, that this may give better results for small steps. The simple convexified algorithm with $\rho \ge 3$ tended to cause oscillations for some types of utility functions and, although better than the original one, often gave worse results than with $\rho = 2$. The additional quadratic term is added to eliminate that effect.

The new algorithm is more complicated, but not significantly so. Adding one more multiplication (two, including a constant) and one addition doesn't have much effect on the calculation time for the utility function. The only real problem is the maximization of utility. In the original algorithm it was done by reversing the function. In the convexified algorithms, including the simple one, such a solution, while possible and suggested, is difficult to find. The convexification makes utility a function of two variables. Luckily only one of them is maximized, the other one is known in each iteration. For simplicity an optimization procedure was used in the experiment instead of an inverse function.

5. Computational results

The algorithms were tested on a single link with four sources, described by four different, S-shaped utility functions. For all functions $U(m_s) = 0$, $m_s = 1$ (not 0, to simulate the trickle of packets necessary to keep the connection to the router and identify the current price), $U(M_s) = 1$ and $M_s = 100$, except $M_4 = 120$. Another parameter is the rate at which utility is equal to 0.5, for these four sources those rates are 40, 60, 50 and 100. The last source also has a steeper gradient at this point, simulating a large, nonelastic connection.

We also ran tests with concave approximations of these functions, modifying the lower bound on sources' rates accordingly (if our algorithms failed in the simple convex case, their effectiveness against nonconvexity wouldn't matter). Tests were repeated with different capacities of the link and different parameters of the algorithms (Fig. 1). We set γ to 10^{-5}. Note, that for convex approximations the minimal bandwidth required for feasibility is about 190 units, so a problem with bandwidth of 150 units, more than enough to support any of the sources at full utility, does not have a feasible solution after the approximation. We will look at some of the results for a bandwidth of 150, 250 and 400 units. In the first case there is no feasible solution for the concave approximations, and the nonconvexity is significant. In the second case the approximation will work, but the original problem's nonconvexity can't be ignored. In the third case there is almost enough bandwidth and the nonconvexity should not have any effect on the calculation.

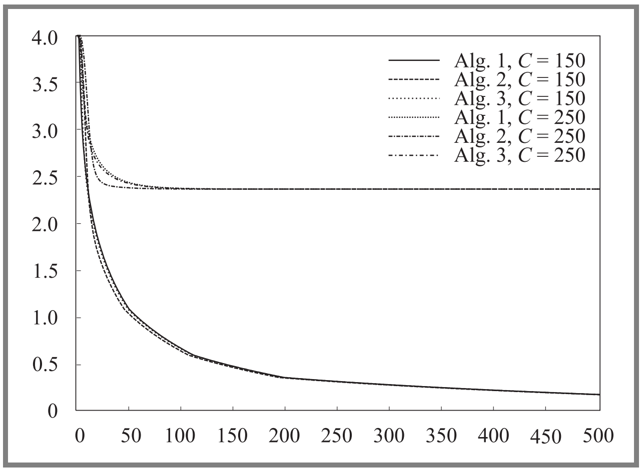

Fig. 1. The total utility functions for the concave approximation with link capacity of 150 and 250 units, different algorithms.

We analyzed the following results: the rates of all sources, the total utility of the sources, the price of the link and the difference between the total traffic and available capacity. Only some of those are shown and we concentrate on the total utility. The convexification parameter η is set to 2. Our tests have shown, that a value of 0.5 is enough for the convexification to be useful and reduce oscillations of the original algorithm, but 2 is better, as it is enough for the algorithm to converge to a stable solution. Another observation is that with little congestion ($C = 400$) the nonconvexity is not important, unless there is a huge number of small connections. The graphs in this case were almost identical, whether the original utility functions, or their concave approximations were used. Because of that we only present the graph for the nonconvex case (Fig. 2) – in the convex case all processes were about 20% slower, but very similar in shape, and total utility converged to 3.8891 instead of 3.9189, minor differences due to approximation error.

5.1. Concave approximation

As we can see in Figs. 1 and 2, all algorithms work well for the concave approximation, if a feasible solution exists. The convexified algorithms tend to react a little slower and overreact, causing a short oscillation – this is a predictable effect of the introduced inertia. The graph for the largest capacity shows the first advantage of the ρ-proximal algorithm over the quadratic one. Its reduced local effects reduce the oscillation and its results are close to the original algorithm's. The quadratic convexification doubles the time required to reach stable state. For $C = 250$ however, the proximal algorithm converged faster, although it reacted last to the changing price.

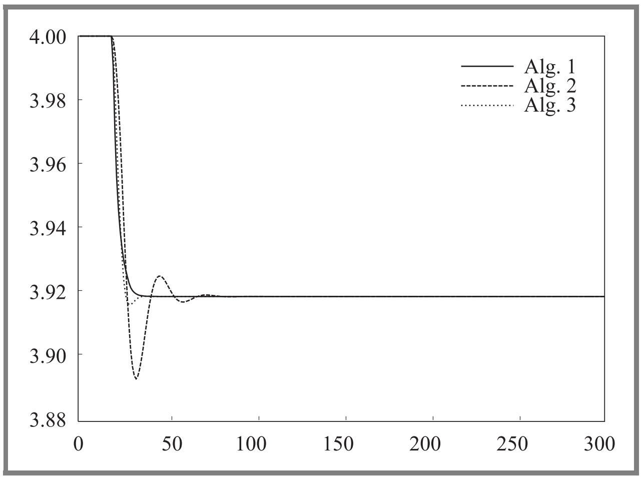

Fig. 2. The total utility functions for the nonconcave case with link capacity of 400 units, different algorithms. The graph for the concave approximation is very similar.

When no feasible solution exists, the total utility decreases exponentially, almost identically for all algorithms (Fig. 1, $C = 150$). This reduction corresponds to the logarithmic shape of the approximation and a constant, linear growth of the price, caused by an almost constant gap between the capacity (150) and the traffic, nearing the sum of lower bounds (about 191). When the price reaches the gradient of the total utility at the lower bound, the utility will achieve 0 and stay there, with the price growing idefinitely.

5.2. Original problem

With congested links and nonconcave utility functions the problem is more difficult. For both $C = 150$ and $C = 250$ there are feasible solutions if convex approximation is not used, but the original algorithm, as proposed by Steven H. Low does not reach one – instead, it oscillates indefinitely (Fig. 3). This confirms the expected behaviour for nonconvex systems. A smaller value of γ will not solve this problem, the algorithm will not converge. While the average utility will be near optimum, the oscillations may not be acceptable.

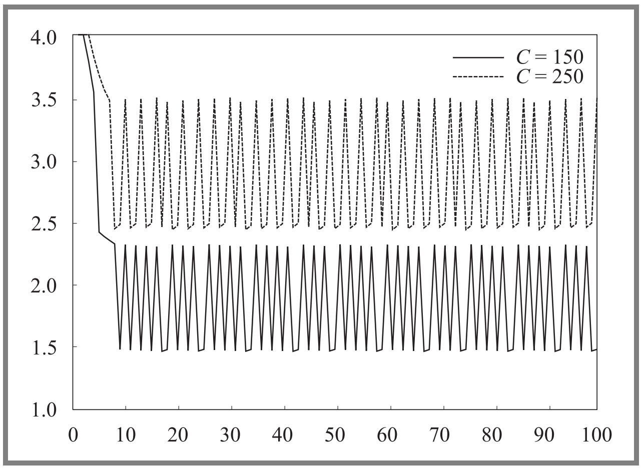

Fig. 3. The total utility functions for the concave approximation with link capacity of 150 and 250 units, Algorithm 1.

The convexified algorithms all converge to a solution (Fig. 4). The initial jumps are the effect of changing the convexification center to near 0 – the convexified function rapidly changes shape and the price has to be corrected. After one or two such rapid changes the algorithms converge. The discontinuities are less prominent with more available bandwidth, as the prices change less rapidly in this case.

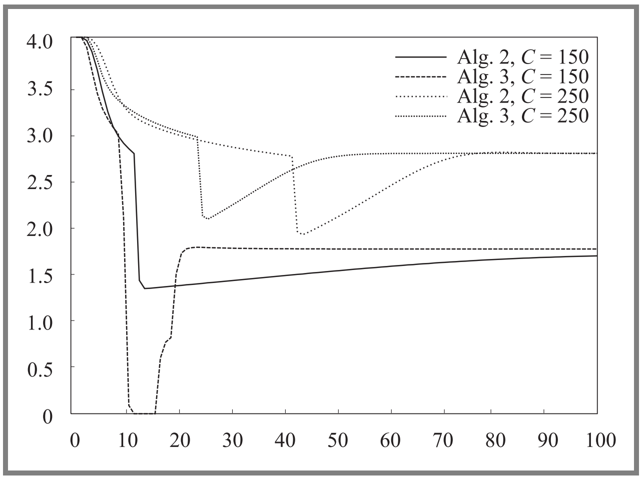

Fig. 4. The total utility functions for the concave approximation with link capacity of 150 and 250 units, convexified algorithms.

These graphs show the advantages of the ρ-proximal mode: the fourth-power term only works further from the convexification center (compared to the quadratic one), so as it moves to lower values, the convexification weakens and a jump occurs sooner. The local convexification with the quadratic term is weaker than in the quadratic-only version, so the algorithm converges faster. It is a general rule for convexification methods – the weaker the convexification, the faster the convergence, unless it is too weak, in which case the algorithm does not converge at all. The ρ-proximal algorithm is therefore the fastest one in this test. The quadratic term is unnecessarily slowed down, but also converges well.

5.3. Other results

The utility is not the only result we collected. In this section we will show, how it corresponds to more physical values. We will now focus on a system with 250 units of capacity and nonconcave utility functions, controlled by Algorithms 1 and 3. Its total utility can be found on Figs. 3 and 4. The sources' rates, price on the link, and the link's overuse (the difference between offered load and capacity) are depicted in Figs. 5, 6 and 7.

The Low's algorithm fails as soon as the price crosses a critical value for source number 4, which immediately jumps to its lower bound (an effect of nonconvexity). It then switches back and forth between the two ranges of rates, with price changing accordingly, as the total offered load is either significantly below or over the capacity.

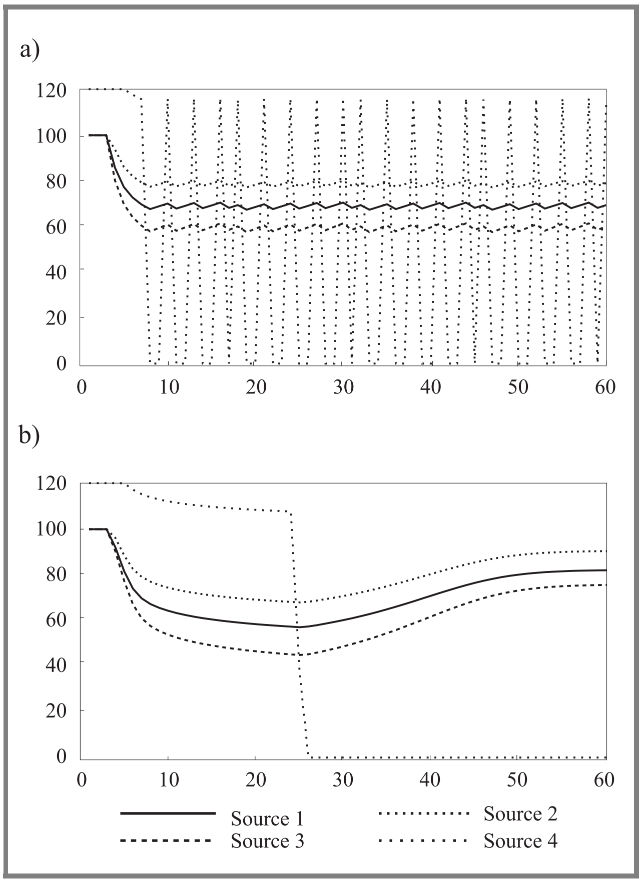

Fig. 5. Traffic rates for $C = 250$ and nonconvex utility functions: (a) Low's algorithm (Alg. 1); (b) ρ-proximal algorithm (Alg. 3).

Our ρ-proximal method also reaches the same point, a bit later, as the convexification slows it down. When the fourth source reduces its rate to minimum, however, it keeps that value, while the others increase their offered traffic to use up the free capacity, just as it should be. The price (Fig. 7) changes accordingly: first it grows to 0.0166 to defeat con-

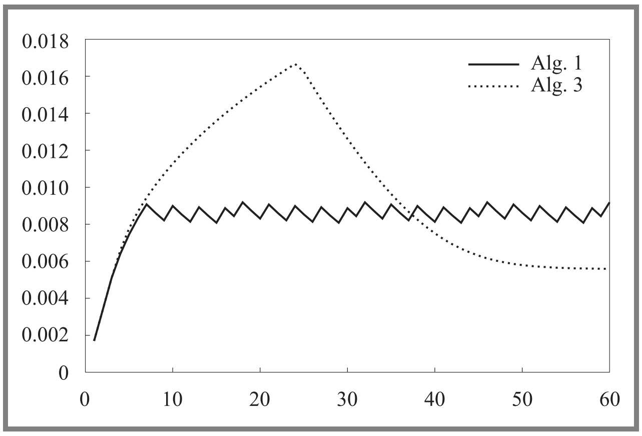

Fig. 6. Link price for $C = 250$, nonconvex utility functions, algorithms: Low's and ρ-proximal.

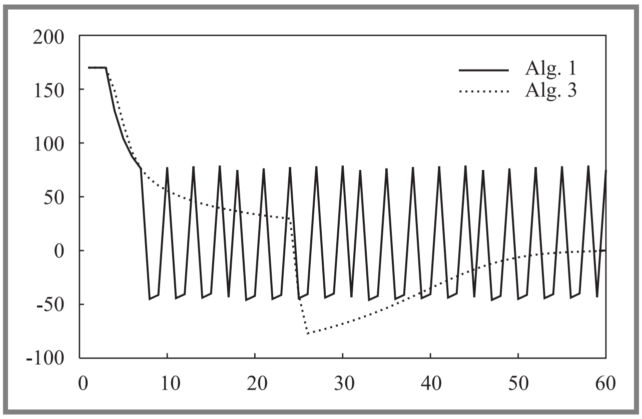

Fig. 7. Excess traffic/slack for $C = 250$, nonconvex utility functions, algorithms: Low's and ρ-proximal.

vexification, then, when the traffic suddenly drops and convexification keeps source 4 at low traffic rate, it returns to the proper level of 0.0056. Figure 7 shows how the excess traffic drops to the point where one of the connections has to switch to minimum traffic. Afterwards there is quite a lot of slack, and traffic grows to fill it.

6. Summary

In this paper we present the results of a new experiment with the convexified gradient projection algoritm and introduce a modification of this technique using ρ-proximal convexifying term.

The experiment for the older, quadratic variant confirmed the results of [2], using a very different approach – a single link with few, easily observable connections, instead of a network of 11 nodes with hundreds of connections and analysis limited to statistics. It is at the same time a complete reimplementation.

The new ρ-proximal variant proved to be as effective in stopping oscillations as the quadratic one and accellerates the convergence. Both work well for nonconcave utility functions, where the original Algorithm 1 fails.

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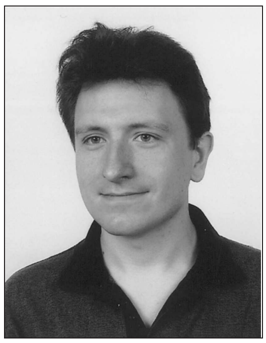

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Paper

Telecommunications network design and max-min optimization problem

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Abstract—Telecommunications networks are facing increasing demand for Internet services. Therefore, the problem of telecommunications network design with the objective to maximize service data flows and provide fair treatment of all services is very up-to-date. In this application, the so-called max-min fair (MMF) solution concept is widely used to formulate the resource allocation scheme. It assumes that the worst service performance is maximized and the solution is additionally regularized with the lexicographic maximization of the second worst performance, the third one, etc. In this paper we discuss solution algorithms for MMF problems related to telecommunications network design. Due to lexicographic maximization of ordered quantities, the MMF solution concept cannot be tackled by the standard optimization model (mathematical programme). However, one can formulate a sequential lexicographic optimization procedure. The basic procedure is applicable only for convex models, thus it allows to deal with basic design problems but fails if practical discrete restrictions commonly arriving in telecommunications network design are to be taken into account. Then, however, alternative sequential approaches allowing to solve non-convex MMF problems can be used.

Keywords*— *network design, resource allocation, fairness, lexicographic optimization, lexicographic max-min.

1. Introduction

Since the emergence of the Internet one has witnessed an unprecedented growth of traffic that is carried in the telecommunications networks. The pace at which the number of network users and the amount of traffic related to data-oriented applications are growing has been and still is much higher than several percent of growth that were typical for traditional voice-only networks; as a matter of fact data traffic almost doubles every year. It can also be observed that the distribution of traffic in data networks changes quickly, both – in the short and long time-scales, and is very difficult to predict. As a result, from the network operator's perspective the network extension process becomes very complicated – while it is not economically feasible to sufficiently over-dimension a network, it is also hard to decide when and where the network should be augmented. An inevitable effect of the situation that the capacity of a network does not match the traffic generated by network service users, is network overload – a phenomenon commonly encountered in current data-oriented networks.

Overloads influence the quality of service perceived by users – data transfer slows down because packet transfer delays increase and packet losses occur much more frequently. Overloads are one of the major concerns of network operators, because the guaranteed quality of service level is one of the basic elements of network operators' differentiation and a prerequisite of their success. In order to avoid overloads and provide the guaranteed quality of service level (instead of offering the so-called best-effort service) the network operator must control the amount of traffic that enters the network. The traffic admission control process is responsible for deciding how many users can be served and how much traffic each of these users can generate. What is important is that, in general, some users will be denied the service in order to reduce the overall stream of traffic that enters the network. Since the service denial probability is another important measure of the quality of service level, one of the primary objectives of the admission control process must be to guarantee that the users have fair access to network services. The most common "fairness-oriented" (as opposed to "revenue-oriented") approach is to admit equal amount of traffic from every stream – the amount being expressed in absolute or relative terms. Unfortunately, this approach can result in poor network capacity utilization, since for many streams much more traffic could still be admitted than this actual amount. Thus, one of the alternative approaches is to admit as much traffic as possible from every stream while making the smaller admitted amounts as large as possible.

The problem to determine how much traffic of every traffic stream should be admitted into the network, and how the admitted traffic should be routed through the network so as to satisfy the requirements of high network utilization and to guarantee fairness to the users, is one of the most challenging problems of current telecommunications networks design. In this paper we show how this problem is related to two well known OR problems – namely the max-min optimization problem and the lexicographic optimization problem. We study the general formulations of these problems and analyze how to use their notions to express the fairness of the traffic admission process. We go on to formulate basic network design problems and study the complexity of the obtained formulations. We analyze the methods of max-min and lexicographic optimization and examine how they can be applied to solve the presented network design problem.

The paper is organized as follows. In Section 2 we introduce the lexicographic max-min or the max-min fair (MMF) solution concept and summarize its major properties. In Section 3 we present details of three telecommunications problems leading to MMF formulations. Further in Section 4 we discuss solution algorithms for the lexicographic max-min optimization and analyze their applicability for telecommunications problems.

2. Max-min and the MMF concept

2.1. Max-min solution concepts

The problem we consider may be viewed in terms of resource allocation decisions as follows. Let us assume there is a set of m services. There is also a set Q of resource allocation patterns (allocation decisions). For each service j a function $f_j(\mathbf{x})$ of allocation pattern $\mathbf{x}$ has been defined. This function, called the individual objective function, measures the outcome (effect) $y_j = f_j(\mathbf{x})$ of the allocation pattern for service j. The outcomes can be measured (modeled) as service quality, service amount, service time, service costs as well as in a more subjective way the (client's) utility of the provided service. In typical formulations a greater value of the outcome means a better effect (higher service quality or client satisfaction); otherwise, the outcomes can be replaced with their complements to some large number. Therefore, without loss of generality, we can assume that each individual outcome y_j is to be maximized which results in a multiple criteria maximization model. The problem can be formulated as follows:

$$\max \{\mathbf{f}(\mathbf{x}) : \mathbf{x} \in Q\}, \tag{1}$$

where $Q \subseteq \Re^n$ is a feasible set and $\mathbf{f}(\mathbf{x}) = (f_1(\mathbf{x}), \ldots, f_m(\mathbf{x}))$ is a vector of real-valued functions $f_j : Q \to \Re, j = 1,2,\ldots,m$, where $\mathbf{x} = (x_1, x_2, \ldots, x_n)$ is an n-vector. We refer to the elements of the criterion space as outcome vectors. An outcome vector $\mathbf{y}$ is attainable if it expresses outcomes of a feasible solution $\mathbf{x} \in Q$ (i.e., $\mathbf{y} = \mathbf{f}(\mathbf{x})$). The set of all the attainable outcome vectors is denoted by Y. Note that, in general, convex feasible set Q and concave function $\mathbf{f}$ do not guarantee convexity of the corresponding attainable set Y. Nevertheless, the multiple criteria maximization model (1) can be rewritten in the equivalent form

$$\max \{\mathbf{y} : y_j \leq f_j(\mathbf{x}) \ \forall j, \ \mathbf{x} \in Q\}, \tag{2}$$

where the attainable set Y is convex whenever Q is convex and functions f_j are concave.

Model (1) only specifies that we are interested in maximization of all objective functions f_j for $j \in M = \{1,2,\ldots,m\}$. Each attainable outcome vector $\mathbf{y} \in Y$ is called *nondominated* if one cannot improve any individual outcome without worsening another one. Each feasible solution $\mathbf{x} \in Q$ generating the nondominated outcome is called an *efficient* (Pareto-optimal) solution of the multiple criteria problem (1). In other words, a feasible solution for which one cannot improve any outcome without worsening another is efficient [33]. In order to make model (1) operational, one needs to assume some solution concept specifying what it means to maximize multiple objective functions. Simple solution concepts are defined by achievement functions $\theta : Y \to \Re$ to be maximized. Thus the multiple criteria problem (1) is replaced with the aggregation $\max \{\theta(\mathbf{f}(\mathbf{x})) : \mathbf{x} \in Q\}$.

The most commonly used achievement function is the mean (or simply the sum) of individual performances; this defines the so-called maxsum solution concept. This solution concept is primarily concerned with the overall system efficiency. As based on averaging, it often provides a solution where some services are discriminated in terms of performances. An alternative approach depends on the so-called max-min solution concept, where the worst performance is maximized:

$$\max\{ \min_{j=1,\ldots,m} f_j(\mathbf{x}) \ : \ \mathbf{x} \in Q \}. \tag{3}$$

The max-min solution concept has been widely studied in the multi-criteria optimization methodology [33, 35]. The optimal set of the max-min problem (3) always contains an efficient solution of the original multiple criteria problem (1). Thus, if unique, the optimal max-min solution is efficient. In the case of multiple optimal solutions, one of them is efficient but also some of them may not be efficient. It is a serious flaw since practical large problems usually have multiple optimal solutions and typical optimization solvers generate one of them (essentially at random). Therefore, some additional regularization is needed to overcome this flaw of the max-min scalarization.

The max-min solution concept is regarded as maintaining equity. Indeed, in the case of a simplified resource allocation problem, the max-min solution

$$\max\{ \min_{j=1,\ldots,m} y_j \ : \ \sum_{j=1}^{m} y_j \leq b \} \tag{4}$$

takes the form $\bar{y}_j = b/m$ for all $j \in M$ thus meeting the perfect equity requirement $\bar{y}_1 = \bar{y}_2 = \ldots = \bar{y}_m$. In the general case, with possibly more complex feasible set structure, this property is not fulfilled [23]. Nevertheless, the following assertion is valid.

Theorem 1: If there exists a nondominated outcome vector $\bar{\mathbf{y}} \in Y$ satisfying the perfect equity requirement $\bar{y}_1 = \bar{y}_2 = \ldots = \bar{y}_m$, then $\bar{\mathbf{y}}$ is the unique optimal solution of the max-min problem

$$\max\{ \min_{j=1,\ldots,m} y_j \ : \ \mathbf{y} \in Y \}. \tag{5}$$

Proof: Let $\bar{\mathbf{y}} \in Y$ be a nondominated outcome vector satisfying the perfect equity requirement. This means, there exists a number α such that $\bar{y}_j = \alpha$ for $j = 1,2,\ldots,m$. Let $\mathbf{y} \in Y$ be an optimal solution of the max-min problem (5). Suppose, there exists some index j_0 such that $y_{j_0} \neq \bar{y}_{j_0}$. Due to the optimality of $\mathbf{y}$, we have:

$$y_j \geq \min_{1 \leq i \leq m} y_i \geq \min_{1 \leq i \leq m} \bar{y}_i = \alpha = \bar{y}_j \quad \forall \ j = 1,\ldots,m$$

which together with $y_{j_0} \neq \bar{y}_{j_0}$ contradicts the assumption that $\bar{\mathbf{y}}$ is nondominated. ■

According to Theorem 1, the perfectly equilibrated outcome vector is a unique optimal solution of the max-min problem if one cannot improve any of its individual outcome without worsening some others. Unfortunately, it is not a common case and, in general, the optimal set to the max-min aggregation (3) may contain numerous alternative solutions including dominated ones. While using standard algorithmic tools to identify the max-min solution, one of many solutions is then selected randomly.

Actually, the distribution of outcomes may make the max-min criterion partially passive when one specific outcome is relatively very small for all the solutions. For instance, while allocating clients to service facilities, such a situation may be caused by existence of an isolated client located at a considerable distance from all the location of facilities. Maximization of the worst service performances (equivalent to minimization of the maximum distance) is then reduced to maximization of the service performances for that single isolated client leaving other allocation decisions unoptimized. This is a clear case of inefficient solution where one may still improve other outcomes while maintaining fairness by leaving at its best possible value the worst outcome. The max-min solution may be then regularized according to the Rawlsian principle of justice. Rawls [30] considers the problem of ranking different "social states"which are different ways in which a society might be organized taking into account the welfare of each individual in each society, measured on a single numerical scale [30, p. 62]. Applying the Rawlsian approach, any two states should be ranked according to the accessibility levels of the least well–off individuals in those states; if the comparison yields a tie, the accessibility levels of the next–least well–off individuals should be considered, and so on. Formalization of this concept leads us to the lexicographic max-min concepts.

The lexicographic max-min solution is known in the game theory as the nucleolus of a matrix game. It originates from an idea, presented by Dresher [7], to select from the optimal (max-min) strategy set of a player a subset of optimal strategies which exploit mistakes of the opponent optimally. It has been later refined to the formal nucleolus definition [32] and generalized to an arbitrary number of objective functions [29]. The concept was early considered in the Tschebyscheff approximation [31] as a refinement taking into account the second largest deviation, the third one and further to be hierarchically minimized. Similar refinement of the fuzzy set operations has been recently analyzed [8]. Within the telecommunications or network applications the lexicographic max-min approach has appeared already in [3, 11] and now under the name max-min fair is treated as one of the standard fairness concepts. The approach has been used for general linear programming multiple criteria problems [1, 17], as well as for specialized problems related to (multiperiod) resource allocation [12, 16]. In discrete optimization it has been considered for various problems [4, 5] including the location-allocation ones [21].

2.2. Lexicographic optimization and MMF

Typical solution concepts for the multiple criteria problems are based on the use of aggregated achievement functions $\theta : Y \rightarrow \Re$ to be maximized, thus ranking the outcomes according to a complete preorder

$$\mathbf{y}' \succeq_\theta \mathbf{y}'' \quad \Leftrightarrow \quad \theta(\mathbf{y}') \geq \theta(\mathbf{y}''). \tag{6}$$

This allows one to replace the multiple criteria problem (1) with the maximization problem max $\{\theta(\mathbf{f}(\mathbf{x})) : \ \mathbf{x} \in Q\}$. However, there are well defined solution concepts which do not introduce directly any scalar measure, despite they rank the outcome vectors with a complete preorder. Especially, the lexicographic order is used for this purpose.

Let $\mathbf{a} = (a_1, a_2, \ldots, a_m)$ and $\mathbf{b} = (b_1, b_2, \ldots, b_m)$ be two m-vectors. Vector $\mathbf{a}$ is lexicographically greater than vector $\mathbf{b}$, $\mathbf{a} >_{lex} \mathbf{b}$, if there exists index k, $0 \leq k < m$, such that $a_j = b_j$ for all $j \leq k$ and $a_{k+1} > b_{k+1}$. Consequently, $\mathbf{a}$ is lexicographically greater or equal $\mathbf{b}$, $\mathbf{a} \geq_{lex} \mathbf{b}$, if $\mathbf{a} >_{lex} \mathbf{b}$ or $\mathbf{a} = \mathbf{b}$. Contrary to the standard vector inequality $\mathbf{a} \geqq \mathbf{b} \Leftrightarrow a_j \geq b_j \forall \ j$, the lexicographic order is complete which means that for any two vectors $\mathbf{a}$ and $\mathbf{b}$ either $\mathbf{a} \geq_{lex} \mathbf{b}$ or $\mathbf{b} \geq_{lex} \mathbf{a}$. Moreover, for any two different vectors $\mathbf{a} \neq \mathbf{b}$ either $\mathbf{a} >_{lex} \mathbf{b}$ or $\mathbf{b} >_{lex} \mathbf{a}$. Vector inequality $\mathbf{a} \geqq \mathbf{b}$ implies $\mathbf{a} \geq_{lex} \mathbf{b}$ but the opposite implication is not valid. The lexicographic order is not continuous and it cannot be expressed in terms of any aggregation function. Nevertheless, it is a limiting case of the order (6) for the weighting aggregation functions $\theta(\mathbf{y}) = \sum_{j=1}^{m} \ w_j y_j$ defined by decreasing sequences of positive weights w_j with differences tending to the infinity.

The lexicographic order allows us to consider more complex solution concepts defined by several (say m) outcome functions $\theta_k : Y \rightarrow \Re$ to be maximized according to the lexicographic order. Thus one seeks a feasible solution $\mathbf{x}^0$ such that for all $\mathbf{x} \in Q$

$$(\theta_1(\mathbf{f}(\mathbf{x}^0)), \ldots, \theta_m(\mathbf{f}(\mathbf{x}^0))) \geq_{lex} (\theta_1(\mathbf{f}(\mathbf{x})), \ldots, \theta_m(\mathbf{f}(\mathbf{x}))).$$

In other words, the multiple criteria problem (1) is replaced with the lexicographic maximization problem

$$\operatorname{lex\,max} \ \{(\theta_1(\mathbf{f}(\mathbf{x})), \theta_2(\mathbf{f}(\mathbf{x})), \ldots, \theta_m(\mathbf{f}(\mathbf{x}))) : \mathbf{x} \in Q\}. \tag{7}$$

Problem (7) is not a standard mathematical programme. Nevertheless, the lexicographic inequality defines a linear order of vectors an therefore the lexicographic optimization is a well defined procedure where comparison of real numbers is replaced by lexicographic comparison of the corresponding vectors. In particular, the basic theory and algorithmic techniques for linear programming have been extended to the lexicographic case [10]. Certainly, the lexicographic optimization may also be treated as a sequential (hierarchical) optimization process where first $\theta_1(\mathbf{f}(\mathbf{x}))$ is maximized on the entire feasible set, next $\theta_2(\mathbf{f}(\mathbf{x}))$ is maximized on the optimal set, and so on. This may be implemented as in the following standard sequential algorithm.

Algorithm 1: Sequential lexicographic maximization

Step 0: Put $k := 1$.
Step 1: Solve programme P_k:
$\max_{\mathbf{x}\in Q} \{\tau_k;\ \tau_k \leq \theta_k(\mathbf{f}(\mathbf{x})),\ \tau_j^0 \leq \theta_j(\mathbf{f}(\mathbf{x}))\ \forall j < k\}$
and denote the optimal solution by $(\mathbf{x}^0, \tau_k^0)$.
Step 2: If $k = m$, then stop ($\mathbf{x}^0$ is optimal solution). Otherwise, put $k := k+1$ and go to Step 1.

Note that directly from the properties of the lexicographic order it follows that for any achievement functions θ_k the lexicographic optimization problem always has unique values of those functions, as stated in the following assertion.

Theorem 2: For any two optimal solutions $\mathbf{x}^1, \mathbf{x}^2 \in Q$ of problem (7) the equalities $\theta_k(\mathbf{f}(\mathbf{x}^1)) = \theta_k(\mathbf{f}(\mathbf{x}^2))\ \forall\ k$ hold.

The most commonly used lexicographic models are based on simple functions $\theta_j(\mathbf{y}) = y_j$ thus introducing an hierarchy of original outcomes. In such a case, according to Theorem 2 the optimal solution is unique in the criterion space.

Theorem 3: In the case of problem (7) with $\theta_j(\mathbf{y}) = y_j \forall j \in M$, for any two optimal solutions $\mathbf{x}^1, \mathbf{x}^2 \in Q$ the equality $\mathbf{f}(\mathbf{x}^1) = \mathbf{f}(\mathbf{x}^2)$ holds and this unique outcome vector is nondominated.

Applying to achievement vectors $\Theta(\mathbf{y})$ a linear cumulative map one gets the cumulated achievements

$$\bar{\theta}_k(\mathbf{y}) = \sum_{j=1}^{k} \theta_j(\mathbf{y}) \quad \text{for } k = 1, 2, \ldots, m. \qquad (8)$$

Note that for any two vectors $\mathbf{y}', \mathbf{y}'' \in Y$ one gets

$$\Theta(\mathbf{y}') \geq_{lex} \Theta(\mathbf{y}'') \quad \Leftrightarrow \quad \bar{\Theta}(\mathbf{y}') \geq_{lex} \bar{\Theta}(\mathbf{y}''). \qquad (9)$$

Hence, the following assertion is valid.

Theorem 4: A feasible vector $\mathbf{x} \in Q$ is an optimal solution of problem (7), if and only if it is the optimal solution of the cumulated lexicographic problem

$$\operatorname{lex\,max}\ \{(\bar{\theta}_1(\mathbf{f}(\mathbf{x})), \ldots, \bar{\theta}_m(\mathbf{f}(\mathbf{x}))) : \mathbf{x} \in Q\}. \qquad (10)$$

The lexicographic order may also be used to construct refinements of various solution concepts [23]. We focus on application of the lexicographic optimization to refine the max-min solution concept according to the Rawlsian theory of justice. Let $\langle \mathbf{a} \rangle = (a_{\langle 1 \rangle}, a_{\langle 2 \rangle}, \ldots, a_{\langle m \rangle})$ denote the vector obtained from $\mathbf{a}$ by rearranging its components in the nondecreasing order. That means $a_{\langle 1 \rangle} \leq a_{\langle 2 \rangle} \leq \ldots \leq a_{\langle m \rangle}$ and there exists a permutation π of set M such that $a_{\langle i \rangle} = a_{\pi(i)}$ for $j = 1, \ldots, m$. Comparing lexicographically such ordered vectors $\langle \mathbf{y} \rangle$ one gets the so-called leximin order. The general problem considered in the balance of this paper depends on searching for the solutions that are maximal according to the leximin order. The problem called hereafter the max-min fair problem reads as follows:

P-MMF: Find $\mathbf{x}^0 \in Q$ such that $\langle \mathbf{f}(\mathbf{x}^0) \rangle \geq_{lex} \langle \mathbf{f}(\mathbf{x}) \rangle\ \forall\, \mathbf{x} \in Q$.

This problem may also be viewed as a standard lexicographic optimization (7) with the aggregation functions $\theta_j(\mathbf{y}) = y_{\langle j \rangle}$:

$$\operatorname{lex\,max}\ \{(\theta_1(\mathbf{f}(\mathbf{x})), \ldots, \theta_m(\mathbf{f}(\mathbf{x}))) : \mathbf{x} \in Q\}. \qquad (11)$$

Problem (11) represents the lexicographic max-min approach to the original multiple criteria problem (1). It is a refinement (regularization) of the standard max-min optimization, but this time, in addition to the smallest outcome, we also maximize the second smallest outcome (provided that the smallest one remains as large as possible), maximize the third smallest (provided that the two smallest remain as large as possible), and so on. Note that the lexicographic maximization is not applied to any specific order of the original criteria.

The lexicographic max-min is the only regularization approach of the max-min that satisfies the reduction (addition/deleting) principle [9]. Namely, if the individual outcome does not distinguish two solutions, then it does not affect the preference relation.

For the lexicographic max-min one may also take advantage of Theorem 4. Applying the cumulative map (8) to ordered outcomes $\theta_i(\mathbf{y}) = y_{\langle i \rangle}$ one gets $\bar{\theta}_k(\mathbf{y}) = \sum_{i=1}^{k} y_{\langle i \rangle}$ expressing, respectively: the worst (smallest) outcome, the total of the two worst outcomes, the total of the three worst outcomes, etc. Following Theorem 4, solution of the P-MMF is equivalent to the lexicographic problem

$$\begin{aligned} &\operatorname{lex\,max}\ \{(\bar{\theta}_1(\mathbf{y}), \ldots, \bar{\theta}_m(\mathbf{y})) : \mathbf{y} \leqq \mathbf{f}(\mathbf{x}),\ \mathbf{x} \in Q\}, \\ &\text{where } \bar{\theta}_k(\mathbf{y}) = \textstyle\sum_{j=1}^{k} y_{\langle j \rangle}. \end{aligned} \qquad (12)$$

Note that

$$\bar{\theta}_k(\mathbf{y}) = \sum_{j=1}^{k} y_{\langle j \rangle} = \min_{\pi \in \Pi} \sum_{j=1}^{k} y_{\pi(j)},$$

where the minimum is taken over all permutations of the index set M. Hence, $\bar{\theta}_k(\mathbf{y})$ is a concave piecewise linear function of $\mathbf{y}$ which, due to (12) guarantees several important properties of the lexicographic max-min solution itself.

Recall, that every optimal solution of the lexicographic max-min model is an efficient solution of the original multiple criteria optimization problem. Note that every lexicographic max-min solution is also an optimal solution of the standard max-min problem. Hence, by virtue of Theorem 1, the lexicographic max-min model, generates efficient solutions satisfying the perfect equity of individual outcomes, whenever such an efficient solution exists. When there does not exist any efficient solution with perfectly equal individual outcomes, then the lexicographic max-min model generates another efficient solution but, due to concave functions $\bar{\theta}_k(\mathbf{y})$, still providing equitability of individual outcomes with respect to the Pigou-Dalton principle of transfers [14]. The principle of transfers states, in the context considered here, that a transfer of small amount from an individual outcome to any relatively worse-off individual outcome results in a more preferred outcome vector. Indeed, the following assertion is valid.

Theorem 5: For any outcome vector $\mathbf{y} \in Y$, $y_{j'} < y_{j''}$ implies

$$\langle \mathbf{y} + \varepsilon \mathbf{e}_{j'} - \varepsilon \mathbf{e}_{j''} \rangle >_{lex} \langle \mathbf{y} \rangle \quad \forall\ 0 < \varepsilon < y_{j''} - y_{j'}\,, \tag{13}$$

where $\mathbf{e}_j$ denotes the jth unit vector.

Proof: Let $\mathbf{y}^\varepsilon = \mathbf{y} + \varepsilon \mathbf{e}_{j'} - \varepsilon \mathbf{e}_{j''}$ for $\varepsilon < y_{j''} - y_{j'}$ and let $y_{\langle k' \rangle} = y_{j'}$, $y_{\langle k'' \rangle} = y_{j''}$. Then, $y_{j'} < y_{\langle k'' \rangle}$ and $\sum_{j=1}^k y^\varepsilon_{\langle j \rangle} \geq \sum_{j=1}^k y_{\langle j \rangle}$ for all $k = 1, 2, \ldots, m$ with at least one strict inequality for some $k' \leq k < k''$. Hence, $\langle \mathbf{y}^\varepsilon \rangle >_{lex} \langle \mathbf{y} \rangle$, due to (9). ■

Following Theorem 2, any two optimal solutions $\mathbf{x}^1, \mathbf{x}^2 \in Q$ of problem (11) result in the same ordered outcome vectors $\langle \mathbf{f}(\mathbf{x}^1) \rangle = \langle \mathbf{f}(\mathbf{x}^2) \rangle$. Hence, all the optimal solutions have the same distributions of outcomes. Nevertheless, they may generate different (differently ordered) outcome vectors themselves. The unique outcome vector is guaranteed, however, in the case of convex problems. It follows from the alternative convex formulation (12) of the MMF problem.

Theorem 6: In the case of convex feasible set Q and concave objective functions $f_j(\mathbf{x})$, for any two optimal solutions $\mathbf{x}^1, \mathbf{x}^2 \in Q$ of problem P-MMF the equality $\mathbf{f}(\mathbf{x}^1) = \mathbf{f}(\mathbf{x}^2)$ holds.

Proof: First of all, let us notice that problem P-MMF is equivalent (in the criterion space) to the following:

$$\operatorname{lex\,max}\ \{\langle \mathbf{y} \rangle : y_j \leq f_j(\mathbf{x})\ \forall j,\ \mathbf{x} \in Q\} \tag{14}$$

and we need to prove that the problem has a unique optimal solution $\mathbf{y} \in Y$. Due to the convexity assumptions the attainable set Y is convex. Let, $\mathbf{y}^1 \neq \mathbf{y}^2 \in Y$ be optimal solutions of (14), thus $\langle \mathbf{y}^1 \rangle = \langle \mathbf{y}^2 \rangle$. Define $\mathbf{y}^\varepsilon = (1-\varepsilon)\mathbf{y}^1 + \varepsilon \mathbf{y}^2$ for some positive ε satisfying

$$0 < \varepsilon < \min_{y^1_{j'} \neq y^1_{j''}} |y^1_{j'} - y^1_{j''}| / \max_{y^1_{j'} \neq y^1_{j''}} |y^1_{j'} - y^1_{j''}|.$$

Due to the bound on ε, there exists a permutation π ordering both $\mathbf{y}^1$ and $\mathbf{y}^\varepsilon$, i.e., $y^1_{\pi(j)} \leq y^1_{\pi(j+1)}$ and $y^\varepsilon_{\pi(j)} \leq y^\varepsilon_{\pi(j+1)}$ for all $j = 1, \ldots, m-1$. Further, identifying the index j_o for which $y^1_{j_o}$ is the smallest value y^1_j such that $y^1_j \neq y^2_j$ one gets $y^\varepsilon_{\pi(j)} \geq y^1_{\pi(j)}$ for $j < j_o$ and $y^\varepsilon_{\pi(j_o)} > y^1_{\pi(j_o)}$ which contradicts optimality of $\mathbf{y}^1$. ■

The leximin order cannot be expressed in terms of any aggregation function. Nevertheless, it is a limiting case of the order (6) for the ordered weighted aggregation functions $\theta(\mathbf{y}) = \sum_{j=1}^m w_j y_{\langle j \rangle}$ defined by decreasing sequences of positive weights w_j with differences tending to the infinity [36, 38].

3. Telecommunications network design examples

Below we shall give three examples showing how the MMF concept can be used in formulations of multi-commodity network flow problems related to telecommunications applications.

3.1. Routing design for networks with elastic traffic

The first example is a problem of finding flows in a network with given link capacities so as to obtain the MMF distribution of flow sizes. This type of problem is applicable to networks carrying the so-called elastic traffic, which means that traffic streams can adapt their intensity to the available capacity of the network [28].

Problem 1: *Routing optimization for MMF distribution of demand volumes*

indices

$d = 1, 2, \ldots, D$ demands (pairs of nodes)
$p = 1, 2, \ldots, P_d$ allowable paths for demand d
$e = 1, 2, \ldots, E$ links

constants

δ_{edp} equals 1 if link e belongs to path p of demand d; 0, otherwise
c_e capacity of link e

variables

x_{dp} flow (bandwidth) allocated to path p of demand d (non-negative continuous)
X_d total flow (bandwidth) allocated to demand d (non-negative continuous), $X = (X_1, X_2, \ldots, X_D)$

objective

$$\operatorname{lex\,max}\ (X_{\langle 1 \rangle}, X_{\langle 2 \rangle}, \ldots, X_{\langle D \rangle}) \tag{15a}$$

constraints

$$\textstyle\sum_p x_{dp} = X_d \quad d = 1, 2, \ldots, D, \tag{15b}$$

$$\textstyle\sum_d \sum_p \delta_{edp} x_{dp} \leq c_e \quad e = 1, 2, \ldots, E, \tag{15c}$$

$$x_{dp} \geq 0 \qquad d = 1, 2, \ldots, D \quad p = 1, 2, \ldots, P_d. \tag{15d}$$

In the above formulation, Eq. (15b) defines the total flow, X_d, allocated to demand d, and constraint (15c) assures that the link load (left-hand side) does not exceed the link capacity. A solution of Problem 1 for an example network is discussed in Appendix A.

3.2. Restoration design for networks with elastic traffic

The second example corresponds to the problem of designing an optimal strategy of elastic traffic flows restoration in case of network failures ([27, Chapter 13]). It is assumed that a set of network failure situations have been identified. The adopted failure model is such that a failure may reduce the capacity of one or more network links. The design should determine optimal capacities of links and for each failure situation the optimal size and routing of every traffic flow so as to obtain the MMF distribution of revenue for all network failure situations. It is assumed that the revenue generated by a single traffic flow is proportional to the logarithm of this flow's size.

Problem 2: *Flow restoration optimization for MMF distribution of revenues*

indices

- $d = 1,2,\ldots,D$ demands
- $p = 1,2,\ldots,P_d$ allowable paths for demand d
- $e = 1,2,\ldots,E$ links
- $s = 1,2,\ldots,S$ states (including normal state)

constants

- δ_{ed} equals 1 if link e belongs to the fixed path of demand d; 0, otherwise
- r_{ds} revenue from demand d in situation s
- ξ_e unit cost of link e
- α_{es} fractional availability coefficient of link e in situation s ($0 \le \alpha_{es} \le 1$)
- B assumed budget

variables

- y_e capacity of link e (non-negative continuous)
- x_{dps} flow allocated to path p of demand d in situation s (non-negative continuous)
- X_{ds} total flow allocated to demand d in situation s (non-negative continuous)
- R_s logarithmic revenue in situation s (continuous), $R = (R_1, R_2, \ldots, R_S)$

objective

$$\operatorname{lex\,max}\ (R_{\langle 1\rangle}, R_{\langle 2\rangle}, \ldots, R_{\langle S\rangle}) \tag{16a}$$

constraints

$$X_{ds} = \textstyle\sum_p x_{dps} \qquad d = 1,\ldots,D;\ s = 1,\ldots,S, \tag{16b}$$

$$R_s = \textstyle\sum_d r_{ds} \lg X_{ds} \qquad s = 1,\ldots,S, \tag{16c}$$

$$\textstyle\sum_d \sum_p \delta_{edp} x_{dps} \le \alpha_{es} y_e \qquad e = 1,\ldots,E, \tag{16d}$$

$$\textstyle\sum_e \xi_e y_e \le B, \tag{16e}$$

$$x_{dps} \ge 0 \tag{16f}$$

$$d = 1,\ldots,D;\ p = 1,\ldots,P_d;\ s = 1,\ldots,S.$$

3.3. Capacity protection design

The last example corresponds to the problem of designing the protection of network links' capacity [20]. It is assumed that the capacity of network links and the size and routing of all network flows are given. The design should determine how much capacity of each link should be freed and reserved so in case of any single-link failure the capacity of the failed link could be restored using the reserved protection capacity. In order to free the capacity of links the size of traffic flows should be reduced in such a way so as to obtain the MMF distribution of traffic flow sizes.

Problem 3: *Protection capacity optimization for MMF distribution of flow sizes*

indices

- $d = 1,2,\ldots,D$ demands
- $p = 1,2,\ldots,P_d$ allowable paths for demand d
- $e,\ell = 1,2,\ldots,E$ links
- $q = 1,2,\ldots,Q_\ell$ candidate restoration paths for link ℓ

constants

- h_d "reference" volume of demand d
- δ_{edp} equals 1 if link e belongs to path p realizing demand d; 0, otherwise
- c_e total capacity of link e
- $\beta_{\ell eq}$ equals 1 if link ℓ belongs to path q restoring link e; 0, otherwise

variables

- y_e resulting normal capacity of link e
- x_{dp} normal flow realizing demand d on path p
- w_e protection capacity of link e
- z_{eq} flow restoring capacity of link e on path q
- X_d normalized realized demand volume for demand d, $X = (X_1, X_2, \ldots, X_D)$

objective

$$\operatorname{lex\,max}\ (X_{\langle 1\rangle}, X_{\langle 2\rangle}, \ldots, X_{\langle D\rangle}) \tag{17a}$$

constraints

$$X_d = \textstyle\sum_p x_{dp}/h_d \quad d = 1,\ldots,D, \tag{17b}$$

$$w_e + u_e \le c_e \quad e = 1,\ldots,E, \tag{17c}$$

$$\textstyle\sum_d \sum_p \delta_{edp} x_{dp} \le y_e, \quad e = 1,\ldots,E, \tag{17d}$$

$$y_e \le \textstyle\sum_q z_{eq} \quad e = 1,\ldots,E, \tag{17e}$$

$$\textstyle\sum_q \beta_{\ell eq} z_{eq} \le w_\ell \quad \ell, e = 1,\ldots,E;\ \ell \ne e, \tag{17f}$$

$$x_{dp} \ge 0 \qquad d = 1,\ldots,D \quad p = 1,\ldots,P_d. \tag{17g}$$

Note that the lexicographic max-min solution assures that all demand volumes will be in the worst case decreased by the same optimal proportion r^*, since in the optimal solution $\sum_p x^*_{dp} \ge r^* h_d$, $d = 1,2,\ldots,D$, for some number r^*, such that $\sum_p x^*_{dp} = r^* h_d$ for some d.

3.4. Non-convex extensions of the example problems

All three problems presented in the previous subsections have convex sets of feasible solutions. As we will see in Section 4, this property allows for efficient solution algorithms of the introduced problems, but, unfortunately, it is not always present in telecommunications problems. For instance, we may require that the demand volumes are realized only on single paths and that the choice of these single paths is subject to optimization. This requirement usually leads to mixed-integer programme (MIP) formulations. In particular, Problem 1 in the single-path version requires additional multiple choice constraints to enforce nonbifurcated flows. Assuming existence of some constants U_d upper bounding the largest possible total flows X_d, this can be implemented with additional binary (flow assignement) variables u_{dp} used to limit the number of positive flows x_{dp} with constraints:

$$x_{dp} \le U_d u_{dp} \qquad d = 1,\dots,D;\ p = 1,\dots,P_d, \tag{18a}$$

$$\sum_p u_{dp} = 1 \qquad d = 1,\dots,D, \tag{18b}$$

$$u_{dp} \in \{0,1\} \qquad d = 1,\dots,D;\ p = 1,\dots,P_d\ . \tag{18c}$$

In fact, as demonstrated in [13], such a modification makes Problem 1 *NP*-complete. The same requirement can be introduced to Problems 2 and 3 as well.
Another requirement leading to non-convex MIP problems is the modularity of the link capacity, which means that link capacities should be multiples of a given module C. Then, capacity variables become non-negative integers and respective constraints change. For example, for Problem 2 variables y_e are non-negative integers and constraints (16d) take the form

$$\sum_d \sum_p \delta_{edp} x_{dps} \le \alpha_{es} C y_e, \qquad e = 1,\dots,E. \tag{19}$$

Certainly, the capacity variables in Problem 3 can also be made integral.

4. MMF solution algorithms

4.1. Sequential max-min algorithms for convex problems

The (point-wise) ordering of outcomes causes that the lexicographic max-min problem (11) is, in general, hard to implement. Note that the quantity $y_{\langle 1\rangle}$ representing the worst outcome can be easily computed directly by the maximization:

$$y_{\langle 1\rangle} = \max\ r_1 \quad \text{subject to} \quad r_1 \le y_j \quad \text{for } j = 1,\dots,m.$$

Similar simple formula does not exist for the further ordered outcomes $y_{\langle k\rangle}$. Nevertheless, for convex problems it is possible to use iterative algorithms for finding the consecutive values of the (unknown) optimal unique vector $\mathbf{T}^0 = (T_1^0, T_2^0, \dots, T_m^0) = \langle \mathbf{f}(\mathbf{x}^0)\rangle$ by solving a sequence of properly defined max-min problems. Such algorithms are described below.
Suppose B is a subset of the index set M, $B \subseteq M$, and let $\mathbf{t}^B = (t_j :\ j \in B)$ be a $|B|$-vector. Also, let B' denote the set complementary to B: $B' = M \backslash B$. For given B and $\mathbf{t}^B$ we define the following convex mathematical programming problem in variables $\mathbf{x}$ and τ:

$\mathbf{P}(B,\mathbf{t}^B)$:

$$\begin{aligned} &\textbf{maximize} && \tau, && && (20a)\\ &\textbf{subject to} && f_j(\mathbf{x}) \ge \tau && j \in B', && (20b)\\ & && f_j(\mathbf{x}) \ge t_j^B && j \in B, && (20c)\\ & && \mathbf{x} \in \mathbf{X}. && && (20d) \end{aligned}$$

It is clear that the solution τ^0 of the convex problem $\mathbf{P}(\emptyset,\emptyset)$ (defined by (20) for empty set B and empty sequence $\mathbf{t}^B$) will yield the smallest value of $\mathbf{T}^0$, i.e., the value T_1^0 (and possibly some other consecutive entries of $\mathbf{T}^0$). This observation suggests the following algorithm for solving problem P-MMF specified by (11).

Algorithm 2: Straightforward algorithm for solving problem P-MMF

Step 0: Put $B := \emptyset$ (empty set) and $\mathbf{t}^B := \emptyset$ (empty sequence).

Step 1: If $B = M$ then stop ($\langle \mathbf{t}^B\rangle$ is the optimal solution of problem P-MMF, i.e., $\langle \mathbf{t}^B\rangle = \mathbf{T}^0$). Else, solve programme $\mathbf{P}(B,\mathbf{t}^B)$ and denote the resulting optimal solution by $(\mathbf{x}^0, \tau^0)$.

Step 2: For each index $k \in B'$ such that $f_k(\mathbf{x}^0) = \tau^0$ solve the following test problem $\mathbf{T}(B,\mathbf{t}^B,\tau^0,k)$:

$$\begin{aligned} &\textbf{max}, && f_k(\mathbf{x}), && && (21a)\\ &\textbf{s.t.} && f_j(\mathbf{x}) \ge \tau^0 && j \in B' \backslash \{k\}, && (21b)\\ & && f_j(\mathbf{x}) \ge t_j^B && j \in B, && (21c)\\ & && \mathbf{x} \in \mathbf{X}. && && (21d) \end{aligned}$$

If for optimal $\mathbf{x}^1$, while solving test $\mathbf{T}(B,\mathbf{t}^B,\tau^0,k)$ we have $f_k(\mathbf{x}^1) = \tau^0$, then we put $B := B \cup \{k\}$ and $t_k := \tau^0$.

Step 3: Go to Step 1.

It can happen that as a result of solving the test in Step 2 for some index $k \in B'$, it will turn out that $f_l(\mathbf{x}^1) > \tau^0$ for some other, not yet tested, index $l \in B'$ ($l \ne k$). In such an (advantageous) case, the objective function with index l does not have to be tested, as its value can be further increased without disturbing the maximal values $\mathbf{t}^B$. Observe that set B is the current set of blocking indices, i.e., the indices j for which the value $f_j(\mathbf{x}^0)$ is equal to t_j^B in every optimal solution of problem P-MMF. Note also, that although the tests in Step 2 are performed separately for individual indices $j \in B'$, the values of objective functions f_j for the indices $j \in B'$, where set B' is results from Step 2, can be simultaneously increased above the value of τ^0 in the next execution of Step 1. This follows from convexity of the set defined by constraints (21b–d): if $f_j(\mathbf{x}^j) = a^j > \tau^0$ and $\mathbf{x}^j$ satisfies (21b–d), then a convex combination of the points $\mathbf{x}^j$, $\mathbf{x} = \sum_{j\in B'} \alpha^j \mathbf{x}^j$ ($\sum_{j\in B'} \alpha^j = 1$, $\alpha^j > 0$, $j \in B'$) also satisfies (21b–d), and $f_j(\mathbf{x}) > \tau^0$ for all $j \in B'$.
Another version of Algorithm 2 may be more efficient, provided that the complexity of problems (20) and (21) is similar.

Algorithm 3: Algorithm for solving problem P-MMF

Step 0: Put $B := \emptyset$ and $\mathbf{t}^B := \emptyset$.

Step 1: If $B = M$ then stop ($\langle \mathbf{t}^B \rangle$ is the optimal solution of problem P-MMF, i.e., $\langle \mathbf{t}^B \rangle = \mathbf{T}^0$). Else, solve programme $\mathbf{P}(B, \mathbf{t}^B)$ and denote the resulting optimal solution by $(\mathbf{x}^0, \tau^0)$.

Step 2: Start solving the test problem $\mathbf{T}(B, \mathbf{t}^B, \tau^0, k)$ for all indices $k \in B'$ such that $f_k(\mathbf{x}^0) = \tau^0$. When the first $k \in B'$ with $f_k(\mathbf{x}^1) = \tau^0$ is detected, then put $B := B \cup \{k\}$ and $t_k := \tau^0$, and go to Step 3.

Step 3: Go to Step 1.

The idea behind the modification in Algorithm 3 is that in total it may involve solving less instances of problems $\mathbf{P}(B, \mathbf{t}^B)$ and $\mathbf{T}(B, \mathbf{t}^B, \tau^0, k)$ than Algorithm 2. If at optimum $\mathbf{x}^0$ all values $f_j(\mathbf{x}^0)$ are the same (equal to 0), then Algorithm 2 will require solving $m+1$ problems (problem $\mathbf{P}(\emptyset, \emptyset)$ and m tests $\mathbf{T}(\emptyset, \emptyset, \tau^0, k)$ for $k = 1, 2, \ldots, m$), whilst Algorithm 3 will require solving $2m+1$ problems (problem $\mathbf{P}(\emptyset, \emptyset)$, m tests $\mathbf{T}(B, \mathbf{t}^B, \tau^0, k)$ and m problems $\mathbf{P}(B, \mathbf{t}^B)$). Hence, in this case, Algorithm 3 requires solving $O(m)$ more problems than Algorithm 2. Now let us consider a somewhat opposite case where all values $f_j(\mathbf{x}^0)$ are different. Additionally, assume that all optimal solutions $\mathbf{x}$ of the consecutively solved problems $\mathbf{P}(B, \mathbf{t}^B)$ and $\mathbf{T}(B, \mathbf{t}^B, \tau^0, k)$ yield the same values $f_j(\mathbf{x})$ for $j \in B'$. In this case Algorithm 3 will require solving $O(m^2/4)$ problems, while Algorithm 2 – $O(m^2/2)$ problems. This means that Algorithm 2 requires solving $O(m^2/4)$ more problems than Algorithm 3; this is a substantial difference.

Both algorithms presented above can be time consuming due to excessive number of problems $\mathbf{P}(B, \mathbf{t}^B)$ and $\mathbf{T}(B, \mathbf{t}^B, \tau^0, k)$ that may have to be solved in the iteration process. Therefore, below we give an alternative algorithm which is very fast provided that dual optimal variables problems $\mathbf{P}(B, \mathbf{t}^B)$ can be effectively computed (this is for instance the case for linear programmes and the simplex algorithm).

Suppose $\lambda = (\lambda_j)_{j \in B'}$ denotes the vector of dual variables (multipliers) associated with constraints (20b). It leads to the following Lagrangian function for problem $\mathbf{P}(B, \mathbf{t}^B)$:

$$\begin{aligned} \mathrm{L}(\mathbf{x}; \tau; \lambda) &= -\tau + \textstyle\sum_{j \in B'} \lambda_j (\tau - f_j(\mathbf{x})) \\ &= (\textstyle\sum_{j \in B'} \lambda_j - 1)\tau - \textstyle\sum_{j \in B'} \lambda_j f_j(\mathbf{x}). \end{aligned} \tag{22}$$

The domain of Lagrangian (22) is defined by

$$\mathbf{x} \in \mathbf{Y}, \tag{23a}$$

$$-\infty < \tau < +\infty, \tag{23b}$$

$$\lambda \geq \mathbf{0}, \tag{23c}$$

where $\mathbf{Y}$ is determined by constraints (20c–d). Hence, the dual function is formally defined as

$$W(\lambda) = \min_{\tau, \mathbf{x} \in \mathbf{Y}} \mathrm{L}(\mathbf{x}, \tau; \lambda) \quad \lambda \geq \mathbf{0} \tag{24}$$

and the dual problem reads:

$$\textbf{maximize } W(\lambda) \textbf{ over } \lambda \geq \mathbf{0}. \tag{25}$$

The following theorem can be proved [27].

Theorem 7: Let λ^0 be the vector of optimal dual variables solving the dual problem (25). Then

1) $\sum_{j \in B'} \lambda_j^0 = 1$, (26)

2) if $\lambda_j^0 > 0$ for some $j \in B'$, then $f_j(\mathbf{x})$ cannot be improved, i.e., $f_j(\mathbf{x}^0) = \tau^0$ for every optimal primal solution $(\mathbf{x}^0, \tau^0)$ of (20).

Note that in general the inverse of (2) in Theorem 7 does not hold: $\lambda_j^0 = 0$ does not necessarily imply that $f_j(\mathbf{x})$ can be improved (for an example see [27, 28]). In fact, it can be proved [27, Chapter 13] that the inverse implication holds if and only if set B is regular (set B is called regular if for any non-empty proper subset G of B, in the modified formulation $\mathbf{P}(B \backslash G, \mathbf{t}^{B \backslash G})$ the value of $f_k(\mathbf{x})$ can be improved for at least one of the indices $k \in B \backslash G$).

Whether or not the consecutive sets B are regular, the following algorithm solves problem P-MMF.

Algorithm 4: Algorithm for solving problem P-MMF based on dual variables

Step 0: Put $B := \emptyset$ and $\mathbf{t}^B := \emptyset$.

Step 1: If $B = M$ then stop ($\langle \mathbf{t}^B \rangle$ is the optimal solution of problem P-MMF, i.e., $\langle \mathbf{t}^B \rangle = \mathbf{T}^0$). Else, solve programme $\mathbf{P}(B, \mathbf{t}^B)$ and denote the resulting optimal solution by $(\mathbf{x}^0, \tau^0; \lambda^0)$.

Step 2: Put $B := B \cup \{j \in B' : \lambda_j^0 > 0\}$.

Step 3: Go to Step 1.

Observe that if for some $j \in B'$ with $\lambda_j^0 = 0$, $f_j(\mathbf{x})$ cannot be further improved, then in Step 1 the value of τ^0 will not be improved; still at least one such index j will be detected (due to property (5)) and included into set B in the next execution of Step 2. The regularity of set B simply ensures that in each iteration at least one $f_j(\mathbf{x})$ ($j \in B'$) will be improved.

In the case of LP problems, the dual quantities used in Algorithm 4 can be obtained directly from the simplex tableau. Indeed, it was a basis of early implementations of the lexicographic max-min solution for LP problems [1, 2, 12].

4.2. Conditional means

The sequential max-min algorithms can be applied only to convex problems, because, in general, it is likely that there does not exist a blocking index set B allowing for iterative processing. This can be illustrated with the following small example. Problem

$$\text{lex}\max \{\langle\langle(x_1+2x_2, 3x_1+x_2)\rangle\rangle : x_1+x_2=1,\ x_1,x_2 \in \{0,1\}\}$$

has two feasible vectors $\mathbf{x}^1=(1,0)$, $\mathbf{x}^2=(0,1)$ and corresponding outcomes $\mathbf{y}^1=(1,3)$, $\mathbf{y}^2=(2,1)$. Obviously, $\mathbf{x}^1$ is the MMF optimal solution as $\langle\langle(1,3)\rangle\rangle >_{lex} \langle\langle(2,1)\rangle\rangle$. One can easily verify that both feasible solutions are optimal for max-min problem

$$\max \{\min\{x_1+2x_2, 3x_1+x_2\} : x_1+x_2=1,\ x_1,x_2 \in \{0,1\}\}$$

but neither f_1 nor f_2 is a blocking outcome allowing to define the second level max-min optimization problem to maximize the second worst outcome. For the same reason, the sequential algorithm may fail for the single-path version of the routing optimization for the MMF distribution of demand volumes and other discrete models (refer to Subsection 3.4).

Following Yager [37], a direct, although requiring the use of integer variables, formula can be given for any $y_{\langle k\rangle}$. Namely, for any $k=1,2,\ldots,m$ the following formula is valid:

$$\begin{aligned} y_{\langle k\rangle} = \quad & \max\ r_k \\ & \text{s.t.} \\ & r_k - y_j \le Cz_{kj},\ z_{kj}\in\{0,1\} \quad j=1,\ldots,m \\ & \sum_{j=1}^{m} z_{kj} \le k-1, \end{aligned} \tag{27}$$

where C is a sufficiently large constant (larger than any possible difference between various individual outcomes y_j) which allows us to enforce inequality $r_k \le y_j$ for $z_{kj}=0$ while ignoring it for $z_{kj}=1$. Note that for $k=1$ all binary variables z_{1j} are forced to 0 thus reducing the optimization in this case to the standard LP model. However, for any other $k>1$ all m binary variables z_{kj} are an important part of the model. Nevertheless, with the use of auxiliary integer variables, any MMF problem (either convex or non-convex) can be formulated as the standard lexicographic maximization with directly defined achievement functions:

$$\text{lex}\max\ (r_1, r_2, \ldots, r_m) \tag{28a}$$

s.t.

$$\mathbf{x}\in Q \tag{28b}$$

$$r_k - f_j(\mathbf{x}) \le Cz_{kj},\ z_{kj} \quad j,k=1,\ldots,m \tag{28c}$$

$$\in\{0,1\} \quad j,k=1,\ldots,m \tag{28d}$$

$$\sum_{j=1}^{m} z_{kj} \le k-1 \quad k=1,\ldots,m. \tag{28e}$$

Recall that one may take advantage of the formulation (12) with cumulated criteria $\overline{\theta}_k(\mathbf{y})=\sum_{i=1}^{k} y_{\langle i\rangle}$ expressing, respectively: the worst (smallest) outcome, the total of the two worst outcomes, the total of the three worst outcomes, etc. When normalized by k the quantities $\mu_k(\mathbf{y})=\overline{\theta}_k(\mathbf{y})/k$ can be interpreted as the worst conditional means [24]. The optimization formula (27) for $y_{\langle k\rangle}$ can easily be extended to define $\overline{\theta}_k(\mathbf{y})$. Namely, for any $k=1,2,\ldots,m$ the following formula is valid:

$$\begin{aligned} \overline{\theta}_k(\mathbf{y}) = \max\ & kr_k - \sum_{j=1}^{m} d_{kj} \\ \text{s.t.} & \\ & r_k - y_j \le d_{kj},\ d_{kj}\ge 0 \qquad j=1,\ldots,m \\ & d_{kj} \le Cz_{kj},\ z_{kj}\in\{0,1\} \qquad j=1,\ldots,m \\ & \sum_{j=1}^{m} z_{kj} \le k-1, \end{aligned} \tag{29}$$

where C is a sufficiently large constant. However, the optimization problem defining the cumulated ordered outcome can be dramatically simplified since all its binary variables (and the related constraints) turn out to be redundant. First let us notice that for any given vector $\mathbf{y}\in\Re^m$, the cumulated ordered value $\overline{\theta}_k(\mathbf{y})$ can be found as the optimal value of the following LP problem:

$$\begin{aligned} \overline{\theta}_k(\mathbf{y}) = \quad & \min \sum_{j=1}^{m} y_j u_{kj} \\ & \text{s.t.} \\ & \textstyle\sum_{j=1}^{m} u_{kj} = k,\ 0\le u_{kj}\le 1 \quad j=1,\ldots,m. \end{aligned} \tag{30}$$

The above problem is an LP for a given outcome vector $\mathbf{y}$ while it becomes nonlinear for $\mathbf{y}$ being a variable. This difficulty can be overcome by taking advantage of the LP dual to (30) as shown in the following assertion.

Theorem 8: For any given vector $\mathbf{y}\in\Re^m$, the cumulated ordered coefficient $\overline{\theta}_k(\mathbf{y})$ can be found as the optimal value of the following LP problem:

$$\begin{aligned} \overline{\theta}_k(\mathbf{y}) = \quad & \max\ kr_k - \sum_{j=1}^{m} d_{kj} \\ & \text{s.t.} \\ & r_k - y_j \le d_{kj},\ d_{kj}\ge 0 \quad j=1,\ldots,m. \end{aligned} \tag{31}$$

Proof: In order to prove the theorem it is enough to notice that problem (31) is the LP dual of problem (30) with variable r_k corresponding to the equation $\sum_{j=1}^{m} u_{kj}=k$ and variables d_{kj} corresponding to upper bounds on u_{kj}. ■

It follows from Theorem 8 that

$$\overline{\theta}_k(\mathbf{f}(\mathbf{x})) = \max\{\ kr_k - \textstyle\sum_{j=1}^{m} d_{kj} \ :\ \mathbf{x}\in Q;\ r_k - f_j(\mathbf{x}) \le d_{kj},\ d_{kj}\ge 0 \quad j\in M\}$$

or in a more compact form $\overline{\theta}_k(\mathbf{f}(\mathbf{x})) = \max\ \{kr_k - \sum_{j=1}^{m} (f_j(\mathbf{x})-r_k)_+ : \mathbf{x}\in Q\}$ where $(.)_+$ denotes the nonnegative part of a number and r_k is an auxiliary (unbounded) variable. The latter, with the necessary adaptation to the minimized outcomes in location problems, is equivalent to the computational formulation of the k–centrum model introduced in [26]. Hence, Theorem 8 provides an alternative proof of that formulation.

Due to Theorem 4, the lexicographic max-min problem (11) is equivalent to the lexicographic maximization of conditional means

$$\operatorname{lex\,max}\ \{(\mu_1(\mathbf{f}(\mathbf{x})), \mu_2(\mathbf{f}(\mathbf{x})), \ldots, \mu_m(\mathbf{f}(\mathbf{x})))\ :\ \mathbf{x} \in Q\}.$$

Following Theorem 8, the above leads us to a standard lexicographic optimization problem with predefined linear criteria:

$$\begin{aligned} \operatorname{lex\,max} \quad & (r_1 - \sum_{j=1}^{m} d_{1j}, \ldots, r_m - \frac{1}{m}\sum_{j=1}^{m} d_{mj}) \\ & \text{s.t.} \\ & \mathbf{x} \in Q \\ & d_{kj} \geq r_k - f_j(\mathbf{x}) \quad j,k = 1,\ldots,m \\ & d_{kj} \geq 0 \quad j,k = 1,\ldots,m. \end{aligned} \tag{32}$$

Note that this direct lexicographic formulation remains valid for nonconvex (e.g., discrete) feasible sets Q, where the standard sequential approaches [16, 17] are not applicable [21].

Model (32) preserves the problem convexity when the original problem is defined with convex feasible set Q and concave objective functions f_j. In particular, for an LP original problem it remains within the LP class while introducing $m^2 + m$ auxiliary variables and m^2 constraints. Thus, for many problems with not too large number of criteria m, problem (32) can easily be solved directly. Although, in general, for convex problems such an approach seems to be less efficient than the sequential algorithms discussed in the previous subsection. The latter may require m iterative steps only in the worst case (only one blocking variable at each step), while typically there are more than two blocking variables identified at each step which reduces significantly the number of steps. The direct model (32) essentially requires the sequential lexicographic Algorithm 1 with m steps.

Further research on the increase of computational efficiency of model (32) seems to be very promising. Note that all lexicographic criteria of this problem express the conditional means which are monotonic with respect to increasing k. While solving the lexicographic problem with the standard sequential Algorithm 1, one needs to solve at Step 2 the following maximization problem:

$$\begin{aligned} \max\ \{\tau_k\ :\quad & \tau_k \leq r - w\sum_{j=1}^{m} d_j; \mu_l(\mathbf{f}(\mathbf{x})) \geq \tau_l^0\ \forall l < k; \\ & \mathbf{x} \in Q;\ r - f_j(\mathbf{x}) \leq d_j,\ d_j \geq 0\ \forall j\}, \end{aligned}$$

where $w = 1/k$. It may occur that the optimal solution of the above problem remains also optimal for smaller coefficients $w = 1/\kappa$ thus defining conditional means for $\kappa > k$. In such a case, one may advance the iterative process to $\kappa + 1$ instead of $k + 1$. Hence, some parametric optimization techniques may allow us to reduce the number of iterations to the same level as in the sequential max-min algorithms.

Note that model (32) offers also a possibility to build some approximations to the strict MMF solution as it allows us to build lexicographic problems taking into account only a selected grid of indices k. In particular, the so-called augmented max-min solution concept, commonly used in the multiple criteria optimization [22, 35], is such an approximation, although very rough as based only on μ_1 and μ_k

$$\operatorname{lex\,max}\{(r_1,\ \frac{1}{m}\sum_{j=1}^{m} f_j(\mathbf{x})) :\ \ r_1 \leq f_j(\mathbf{x}) \quad j = 1,\ldots,m,\ \ \mathbf{x} \in Q\}.$$

4.3. Distribution approach

For some specific classes of discrete, or rather combinatorial, optimization problems, one may take advantage of the finiteness of the set of all possible values of functions f_j on the finite set of feasible solutions. The ordered outcome vectors may be treated as describing a distribution of outcomes generated by a given decision $\mathbf{x}$. In the case when there exists a finite set of all possible outcomes of the individual objective functions, we can directly describe the distribution of outcomes with frequencies of outcomes. Let $V = \{v_1, v_2, \ldots, v_r\}$ (where $v_1 < v_2 < \cdots < v_r$) denote the set of all attainable outcomes (all possible values of the individual objective functions f_j for $\mathbf{x} \in Q$). We introduce integer functions $h_k(\mathbf{y})$ $(k = 1,\ldots,r)$ expressing the number of values v_k in the outcome vector $\mathbf{y}$. Having defined functions h_k we can introduce cumulative distribution functions:

$$\bar{h}_k(\mathbf{y}) = \sum_{l=1}^{k} h_l(\mathbf{y})\ , \quad k = 1,\ldots,r. \tag{33}$$

Function $\bar{h}_k$ expresses the number of outcomes smaller or equal to v_k. Since we want to maximize all the outcomes, we are interested in the minimization of all functions $\bar{h}_k$. Indeed, the following assertion is valid [22]. For outcome vectors $\mathbf{y}', \mathbf{y}'' \in V^m$, $\langle \mathbf{y}' \rangle \geq \langle \mathbf{y}'' \rangle$ if and only if $\bar{h}_k(\mathbf{y}') \leq \bar{h}_k(\mathbf{y}'')$ for all $k = 1,\ldots,r$. This equivalence allows to express the lexicographic max-min solution concept for problem (1) in terms of the standard lexicographic minimization problem with objectives $\bar{\mathbf{h}}(\mathbf{f}(\mathbf{x}))$:

$$\operatorname{lex\,min}\ \{(\bar{h}_1(\mathbf{f}(\mathbf{x})), \ldots, \bar{h}_r(\mathbf{f}(\mathbf{x})))\ :\ \mathbf{x} \in Q\}. \tag{34}$$

Theorem 9: A feasible solution $\mathbf{x} \in Q$ is an optimal solution of the P-MMF problem, if and only if it is an optimal solution of the lexicographic problem (34).

The quantity $\bar{h}_k(\mathbf{y})$ can be computed directly by the minimization:

$$\begin{aligned} \bar{h}_k(\mathbf{y}) = \quad & \min \sum_{j=1}^{m} z_{kj} \\ & \text{s.t.} \\ & v_{k+1} - y_j \leq C z_{kj},\ z_{kj} \in \{0,1\} \quad j = 1,\ldots,m, \end{aligned}$$

where C is a sufficiently large constant. Note that $\bar{h}_r(\mathbf{y}) = m$ for any $\mathbf{y}$ which means that the rth criterion is always constant and therefore redundant in (34). Hence, the lexicographic problem (34) can be formulated as the following mixed integer problem:

$$\begin{aligned} &\operatorname{lex\,min} \left[\sum_{j=1}^{m} z_{1j},\ \sum_{j=1}^{m} z_{2j}, \ldots,\ \sum_{j=1}^{m} z_{r-1,j} \right] \\ &\text{s.t.} \\ &v_{k+1} - f_j(\mathbf{x}) \le C z_{kj} \;\; j=1,\ldots,m, \;\; k=1,\ldots,r-1, \\ &z_{kj} \in \{0,1\} \quad j=1,\ldots,m, \;\; k=1,\ldots,r-1, \\ &\mathbf{x} \in Q. \end{aligned} \tag{35}$$

Krarup and Pruzan [15] have shown that, in the case of discrete location problems, the use of the minisum solution concept with the outcomes raised to a sufficiently large power is equivalent to the use of the minimax solution concept. Formulation (34) allows us to extend such an approach to the lexicographic max-min solution concept. Note that the achievements functions in (34) can be rescaled with corresponding values $v_{k+1} - v_k$. When the differences among outcome values are large enough then the lexicographic minimization corresponds to the one-level optimization of the total of achievements which is equivalent to minimization of the sum of the original outcomes. In general, as shown by Burkard and Rendl [4], there is a possibility to replace then the lexicographic max-min objective function with an equivalent linear function on rescaled outcomes. Algorithms developed in [4, 5] take advantage of finiteness of the set of outcome values and they depend on making (explicitly or implicitly) differences among the outcomes larger (without changing their order) which does not affect the lexicographically maximal solutions of problem (11). When the differences are large enough the optimal solution of the maxisum problem is also the lexicographic max-min solution. In general, an unrealistically complicated scaling function may be necessary to generate large enough differences among different but very close outcomes. Therefore, the outcomes should be mapped first on the set of integer variables (numbered) to normalize the minimum difference, like in [4, 5] approaches. All these transformations are eligible in the case of finite outcome set. Nevertheless, while solving practical problems, large differences among coefficients may cause serious computational problems. Therefore, such approaches are less useful for large scale problems typically arriving in telecommunications network design.

Taking advantage of possible weighting and cumulating achievements in lexicographic optimization, one may eliminate auxiliary integer variables from the achievement functions. For this purpose we weight and cumulate vector $\overline{\mathbf{h}}(\mathbf{y})$ to get $\hat{h}_1(\mathbf{y}) = 0$ and:

$$\hat{h}_k(\mathbf{y}) = \sum_{l=1}^{k-1} (v_{l+1} - v_l)\overline{h}_l(\mathbf{y}) \quad k = 2,\ldots,r. \tag{36}$$

Due to Theorem 4 and positive differences $v_{l+1} - v_l > 0$, the lexicographic minimization problem (34) is equivalent to the lexicographic problem with objectives $\hat{\mathbf{h}}(\mathbf{f}(\mathbf{x}))$:

$$\operatorname{lex\,min} \{(\hat{h}_1(\mathbf{f}(\mathbf{x})), \ldots, \hat{h}_r(\mathbf{f}(\mathbf{x}))) \;:\; \mathbf{x} \in Q\} \tag{37}$$

which leads us to the following assertion.

Theorem 10: A feasible solution $\mathbf{x} \in Q$ is an optimal solution of the P-MMF problem, if and only if it is an optimal solution of the lexicographic problem (37).

Actually, vector function $\hat{\mathbf{h}}(\mathbf{y})$ provides a unique description of the distribution of coefficients of vector $\mathbf{y}$, i.e., for any $\mathbf{y}', \mathbf{y}'' \in V^m$ one gets: $\hat{\mathbf{h}}(\mathbf{y}') = \hat{\mathbf{h}}(\mathbf{y}'') \;\Leftrightarrow\; \langle \mathbf{y}' \rangle = \langle \mathbf{y}'' \rangle$. Moreover, $\hat{\mathbf{h}}(\mathbf{y}') \le \hat{\mathbf{h}}(\mathbf{y}'')$ if and only if $\Theta(\mathbf{y}') \ge \Theta(\mathbf{y}'')$ [22].

Note that $\hat{h}_1(\mathbf{y}) = 0$ for any $\mathbf{y}$ which means that the first criterion is constant and redundant in problem (37). Moreover, putting (33) into (36) allows us to express all achievement functions $\hat{h}_k(\mathbf{y})$ as a piece wise linear functions of $\mathbf{y}$:

$$\hat{h}_k(\mathbf{y}) = \sum_{j=1}^{m} (v_k - y_j)_+ = \sum_{j=1}^{m} \max\{v_k - y_j, 0\} \qquad k = 1,\ldots,r. \tag{38}$$

Hence, the quantity $\hat{h}_k(\mathbf{y})$ can be computed directly by the following minimization:

$$\begin{aligned} \hat{h}_k(\mathbf{y}) = \quad &\min \sum_{j=1}^{m} t_{kj} \\ &\text{s.t.} \\ &v_k - y_j \le t_{kj},\; t_{kj} \ge 0 \quad j = 1,\ldots,m. \end{aligned} \tag{39}$$

Therefore, the entire lexicographic model (37) can be formulated as follows:

$$\begin{aligned} &\operatorname{lex\,min} \left[\sum_{j=1}^{m} t_{2j},\ \sum_{j=1}^{m} t_{3j}, \ldots,\ \sum_{j=1}^{m} t_{rj} \right] \\ &\text{s.t.} \\ &v_k - f_j(\mathbf{x}) \le t_{kj},\; t_{kj} \ge 0 \quad j = 1,\ldots,m, \;\; k = 2,\ldots,r \\ &\mathbf{x} \in Q. \end{aligned} \tag{40}$$

Note that the above formulation, unlike the problem (35), does not use integer variables and can be considered as an LP modification of the original multiple criteria problem (1). Thus, this model preserves the problem's convexity when the original problem is defined with a convex feasible set Q and a concave objective functions f_j. The size of problem (40) depends on the number of different outcome values. Thus, for many problems with not too large number of outcome values, the problem can easily be solved directly and even for convex problems such an approach may be more efficient than the sequential algorithms discussed in the previous subsection. Note that in many problems of telecommunications network design, the objective functions express the quality of service and one can easily consider a limited finite scale (grid) of the corresponding outcome values. Similarly, in the capacity protection design (Subsection 3.3), one may focus on a finite grid of demand volumes. One may also notice that model (40) opens a way for the fuzzy representation of quality measures within the MMF problems.

5. Concluding remarks

Today, the major objective of telecommunications network design for Internet services is to maximize service data flows and provide fair treatment of all services. Fair treatment of services can be formalized through the MMF solution concept, which assumes that the worst service performance is maximized and the solution is additionally regularized with the lexicographic maximization of the second worst performance, the third one, etc. We have argued that the MMF solution concept is tightly related to the Rawlsian principle of justice and is equivalent the lexicographic max-min concept.

We have shown that with respect to telecommunications networks carrying the so-called elastic traffic, the problems of routing design, restoration design and protection capacity design are examples of important design problems that can be formulated with the use of the MMF notion to express design objectives. We have presented and evaluated several general efficient sequential algorithms that can be used to solve the basic variants of these problems as well as many other MMF problems. These algorithms are based on the idea to solve a sequence of properly defined max-min subproblems. The algorithms differ with respect to the strategy of choosing this sequence. We have shown that the efficiency of different strategies depends on the distribution of outcome values of the optimal solution to the original problem. Since the algorithms can still be time-consuming due to excessive number of subproblems that have to be solved in the iteration process, the values of subproblems' dual variables can be used to considerably reduce the number of solved subproblems. In the case of LP problem formulations the values of dual variables can be obtained directly from the simplex tableau.

Unfortunately, sequential algorithms are only applicable to convex problems. Hence if network design problems are augmented with the requirements that data flows are to be routed along single paths or that link capacity is modular, these algorithms cannot be applied any more. However, we have shown that the original problem of lexicographic maximization of the solution vector can be replaced with the lexicographic minimization of the vector that describes the distribution of outcome values, which, fortunately enough, is convex as long as an original problem is defined with a convex feasible set Q and a concave objective functions f_j. The complexity of the transformed problem is directly related to the number of different outcome values. As far as telecommunications network design is concerned, this number can be pretty small, for example if the objective functions express the quality of service. Therefore, further research on application of distribution approach to various classes of telecommunications MMF problems seems to be very promising.

Appendix A. Numerical example

In this appendix we present a numerical example of Problem 1 (Subsection 3.1). The structure of the considered network is shown in Fig. 1; c_e denotes the capacity of link e. We assume that the set of demands corresponds to the set of all pairs of nodes.

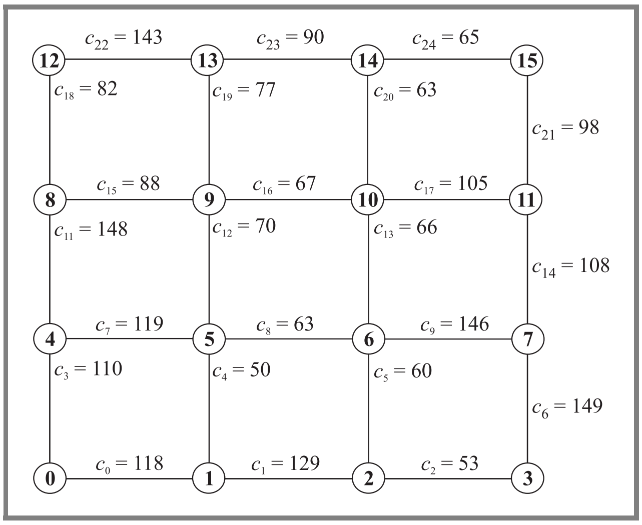

Fig. 1. 16-node square network.

The results of applying Algorithm 4 (Subsection 4.1) to Problem 1 are presented in Table 1. The table contains information pertaining to consecutive iterations of the algorithm. The information includes the number of demands blocked in an iteration and their flow size. To effectively solve the problem we applied a path (column) generation technique [27, Subsection 8.2.1] allowing for problem decomposition. The overall number of paths used in each iteration is presented in Fig. 2. The LP subproblems were

Table 1
Consecutive values of τ_n^0 and number of blocked demands in MMF allocation procedure

Iteration n	Value τ_n^0	Blocked demands
1	5.286	63
2	6.625	8
3	7.013	28
4	10.214	8
5	14.606	4
6	16.115	1
7	25.362	1
8	29.908	2
9	30.962	1
10	35.093	2
11	49.288	1
12	82.145	1

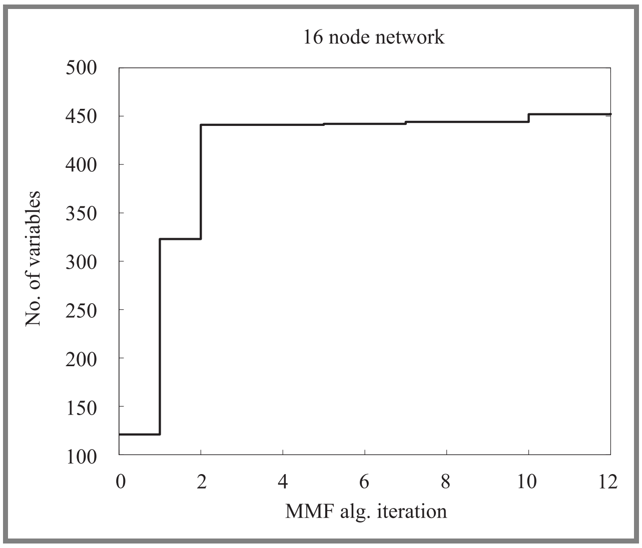

Fig. 2. Number of problem columns in function of MMF algorithm iterations.

solved with the use of the CPLEX 9.0 optimization package. Solving the problem on a PC-class computer equipped with a 2.4 GHz P4 HT processor required 0.2 s of the processor time, of which only 0.03 s in total was spent on solving the LP subproblems.

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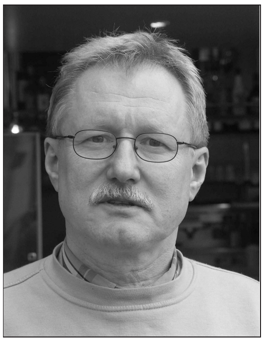

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Paper

Software tools for a multilayer network design

Holger Höller and Stefan Voß

Abstract—Today's long haul and metro high-speed networks are mainly based on synchronous digital hierarchy (SDH) or its American equivalent synchronous optical network (SONET) and wavelength division multiplex (WDM). On the other hand, the large amount of traffic growth during the last years has been caused mainly by Internet protocol (IP) traffic. Traditionally, the IP-router based networks and the cross-connect based synchronous networks are often planned and operated separately. However, in line with new developments such as generalized multiprotocol label switching (GMPLS), network providers begin to realize that the convergence of these two worlds promises significant benefits. A set of software tools to support the network designer has been developed and used on various kinds of real world network planning problems arising in the SDH/WDM context. This includes, among others, 1+1 protection planning, static restoration and dual homing issues. These tools are extended with additional features to handle aspects of the IP/SDH interplay in a GMPLS environment. The two main components are an AMPL based integer model (solved via CPLEX) and a heuristic implemented in C++.

Keywords— network design, GMPLS, SDH, WDM.

1. Introduction

The term *layer* may be used with different meanings in the context of telecommunication networks. First, there is the well-known ISO/OSI model that divides the communication process into seven distinct layers. The physical layer at the bottom describes the technological means how to transport the data from one place to another while the higher layers handle connections, routing, error-processing, etc. The meaning of the word layer throughout this paper resembles this definition but is more based on the client relationships of the different technologies and their protocols. The lowest layer is constituted by the dark fibers. Especially in core networks, WDM systems are usually installed at least on the main routes; they constitute the second layer. SDH/SONET signals are multiplexed in the WDM systems, thus they are clients of WDM and form the next higher layer. IP, as a possible client to SDH/SONET, is the next layer.

Also the hierarchical structure of a network, i.e., the division into access, regional and backbone networks, can be viewed as different layers. Though the software tools are able to support some hierarchical network structures, including dual homing at the borders between regions and the backbone, this kind of layering will not be the focus of this article.

In this paper we aim at describing our current developments in modeling and solution approaches for multilayer network design. The focus is on integrated planning of different layers whose planning was usually performed separately in the past. The technologies and network structures discussed throughout this article are based on the systems that are currently installed or scheduled for large-scale use in the near future. Thus switching in the cross-connects takes places in the electrical domain and WDM is simply used as a high-capacity point-to-point connection. Future developments such as all-optical cross-connects and thus the routing and wavelength assignment problem (RWA) are not considered.

The structure of this article is as follows: Section 2 treats the network design problems and their technological and economical context. Section 3 describes the planning tools that we have developed to cope with these problems and Section 4 sums the article up with some conclusions.

2. The network design problem

2.1. Network structure and demands

The traditional organizational and technological structure of many carriers consists of a transport network department (with its roots in the telephone network) and a separate IP network department (with its roots in packet data networks). They both tend to optimize "their own network" according to their own needs. The result might be two local minima instead of a global one for the combined networks. Even if this description is a bit over-exaggerated, the mutual understanding of people working in the "IP world" and those working in the "transport world" should improve in order to design and operate an integrated multi service network.

In recent years, there has been a tremendous shift in the demands towards IP; meanwhile data traffic has superseded voice traffic. The Internet and together with it also IP are becoming the predominant means for communication, even voice traffic is beginning to migrate to IP. The two technologies are converging on the side of the customers and now also in the core networks. On the other hand, when asking network operators, they often state that most demands in the core networks are leased lines with a fixed capacity. This seeming contradiction might be solved when considering what data is actually transported over these lines. A 155 Mbit/s leased line for a virtual private network (VPN) of a company looks like an ordinary switched circuit demand. However, one can ask whether it is really used for time-critical circuit switched data or merely for IP-packets groomed in a virtual container (VC) at the customer side. On the other hand, the question arises, if an IP VPN in the core still can benefit from statistical multiplexing. Maybe the traffic between the two customer locations is already so much aggregated before it enters the core network that the statistical gain for the network provider would be close to zero. This would mean that one of the big advantages of IP, the better network usage due to statistical multiplexing, might not work any more. At least during the busy hour, the actual capacity needed for such IP traffic might be almost identical to switched circuit traffic.

Capacity planning for switched circuit networks is comparatively easy, since the demands have a fixed size that does not change over time. Once the appropriate container size is determined, a deterministic routing and capacity planning for these containers can take place. However, IP traffic has a stochastic behavior. A demand cannot be entirely described by a single value for the required bandwidth; its size may change from one second to another. A simple way for capacity planning would be just to take the peak value (or the average value) of the bandwidth requirement and treat it like an ordinary switched circuit demand. For a conventional IP over SDH network with a fixed mapping of an incoming IP stream into SDH containers, this might be suitable. But of course, this does not necessarily lead to an efficient use of the resources since large parts of the containers might be empty most of the time.

A more flexible and resource saving possibility might be achieved by the use of virtual concatenation in combination with the link capacity adjustment scheme (LCAS) (cf. [4, 12]). An IP demand is mapped into a number of virtually concatenated SDH containers and that number is varying over time according to the actual size of the IP demand. Of course, the capacity has to be available in the underlying transport network, but this time it is not wasted if a demand requires less capacity than on average, but it can be dynamically used by other demands that need above average capacity at that moment. However, this requires more knowledge about the IP demands than a traditional static planning. Along with the peak bandwidth requirements, a distribution of the requirements over time has to be known. Otherwise, it is not possible to route the demands such that demands with peaks at different times use the same routes so that they can actually share capacities. If all or at least most of the demands have their peaks at the same time of the considered period (e.g., a day), even the dynamic use of virtual concatenation would not help to save resources. This is a point that has to be considered generally in the discussion about more flexibility and bandwidth on demand in transport networks. The capacity and thus the equipment have to be available in the first place in order to be assigned dynamically. But if this capacity cannot be shared with other demands, a small line with additional bandwidth on demand is not cheaper than a large dedicated line. In both cases, the extra capacity is wasted outside the busy hour and the costs for the carrier are the same.

2.2. Generalized multiprotocol label switching

An important part in the future integration of transport and IP networks is probably played by GMPLS. Banerjee *et al.* [2, 3] give a versatile description of GMPLS. For the official standards refer to the respective requests for comments from the Internet Engineering Task Force (IETF)[1]. The goal is a common control plane for the entire network. Routing, resilience, monitoring, etc., are all performed by a unified management system. It may give the network operator the opportunity to provide new kinds of services and an integrated planning process may reduce the overall network costs while guaranteeing the required service level (cf. [11]). GMPLS can be implemented in two different scenarios called overlay model and peer-to-peer model. The overlay model can be seen as an intermediate step between the current separation of layers or networks and an all-integrated network. The network consists of different clouds that hide their inner structure and communicate via specific interaction points. These clouds can be different layers like SDH and IP. But also the layers themselves can be divided into separate entities, e.g., devices of different vendors or different network providers. The peer-to-peer model opens the entire internal structure of a network to the outside. Thus the edge devices of the adjacent layers or networks mutually know their topologies. Only this knowledge enables a truly common control plane for the different parts of the network. However, the peer-to-peer model should not be seen as the ultimate goal for all networks. There are good reasons to use an overlay approach for some scenarios. For example, the GMPLS standard leaves room for vendor specific features, which of course only work inside its own equipment cloud. Thus it might be impossible to use a common control plane that has all the features the provider needs for the different parts of the network unless the equipment comes from one vendor only. On the other hand, different parts of the network might be operated by different providers, which do not want to reveal their inner structure to a (possible) competitor. While the overlay model can be handled well with existing planning tools, the peer-to-peer model poses a much more complicated task.

2.3. Resilience

Another important aspect in multi-layer networks is the resilience. The question is not just which mechanism should be used, as, e.g., 1+1 protection versus restoration in SDH networks, but also on which layer it should be implemented. Demeester *et al.* [5] discuss this topic for a similar problem, the integrated planning of asynchronous transfer mode (ATM) and SDH/WDM networks. Pongpaibool [10]

[1] See, http://www.ietf.org/rfc.html

extensively treats survivability with respect to GMPLS networks and gives many useful references. Traditionally, SDH and WDM core mesh networks are often 1+1 protected, which means a distinct backup path for every demand. IP-networks have the inherent ability to reroute the traffic in case of failures, provided that enough spare capacity is available. If the IP network now acts as a client of the transport network, this might lead to a double fold protection and therefore to a massive waste of capacities.

Different approaches might be taken in order to prevent this problem. Two obvious but may be too extreme possibilities are to use only the resilience on one layer. If the IP layer has enough spare capacity, then it might not be necessary to use any kind of protection on the optical layer. The feasibility of this approach, however, depends on the size and structure of the network. It is doubtful whether an IP-router network could handle the simultaneous rerouting of thousands of demands if a WDM link fails. At least it would take a while before the normal state of operation would be reached again. On the other hand, protection could take place only in the optical domain. Fast protection switching would establish a route around the failure before the IP layer would notice it. No spare capacity for rerouting would be needed in the IP layer. A drawback of this approach is the probably high resource consumption. Also the lower layer cannot sense router failures without any additional signaling from the IP layer.

A good solution in the long run might be in between. Both layers, the IP as well as the optical, begin to adopt the advantages of the other one. On one hand, SDH and WDM get fast rerouting or dynamic restoration in order to save resources. On the other hand, multiprotocol label switching (MPLS) introduced circuit like traffic flows for IP, which (among other features) enables a quick rerouting. This might ultimately converge in GMPLS peer to peer networks where a unified resilience mechanism is coordinated between the layers. Low layer failures are directly handled on the optical layer (SDH or WDM) before the IP layer notices the outage. IP layer failures are handled on the IP layer, but the IP layer has knowledge of the available backup resources on the optical layer and can use them for its own recovery operations, e.g., after a router failure. Of course, this mechanism, like many others, works only reliably for single failures. Once the IP layer occupies capacity on layer 1 and layer 2 systems for its backup routes, they are blocked and can no longer be used for recovery of failures on the optical layer.

3. The software tools

The SDH/WDM network design problem, as treated, e.g., in [6, 9], is to decide which combination of equipment and routing will be able to carry the given demands at the lowest cost. It is important to realize that the routing and the equipment assignment cannot be separated. Due to the strong economies of scale, the shortest path is not always the cheapest, it might often be useful to accept a detour and even additional hops in order to fill large long-haul systems that are very expensive as a whole but very cheap per bit.

The models presented here rely on a set of common network equipment with certain user-adjustable parameters. The network has to carry a certain set of (protected) demands with the objective of minimizing the investment in the equipment. The different layers considered are the fiber-layer, 2.5 Gbit/s SDH-, 10 Gbit/s SDH- and WDM-systems. The integration of IP introduces an additional layer on top of the SDH layer. The tools consist of two major parts, a mixed integer model solved by CPLEX and a heuristic implemented in C++.

3.1. Data and preprocessing

Most realistic network planning scenarios are not a green-field study but they are rather based at least on an existing fiber graph. The laying of new fiber lines is a very expensive task and therefore is avoided whenever possible. Thus the set of fiber lines is considered as static. It is given by the network provider along with its estimated figures for the future demands and the costs for the possible equipment choices. The main data considered for the planning process is the following:

- Fiber lengths and maybe quantities if they are restricted.
- Demands in VC-x units for SDH and Mbit/s for IP (IP demands might also be asymmetric).
- Equipment specifications like capacities, ranges and prices of WDM multiplexers, transponder, amplifiers, regenerators, cross-connects, IP routers, port cards and so on.

Our tools have no direct access to the databases of the network provider; they import the data from EXCEL via the Windows open database connectivity (ODBC) interface or custom ASCII files. AMPL/CPLEX are compatible to these two formats as well. For details on the AMPL ODBC interface refer to the documentation of AMPL optimization LLC [1]. Both, the custom heuristic and AMPL, allow a batch processing in order to quickly evaluate different scenarios (demand matrices, parameter sets, cost functions, etc.) of the same problem instance without any intermediate user interaction.

This common interface also enables an interworking of the two tools without any further data conversion, which opens some interesting opportunities. First of all, it allows an easy use of the same data sets to compare both approaches, which simply saves time. Second it is possible that the output of one method is used as input for the other one. The best solution found by the heuristic may serve as an upper bound for CPLEX. This helps to reduce the run-time and the memory consumption and thus leads to better solutions for some problem instances, especially to those that previously were aborted due to a lack of memory. But interaction may also take place the other way round. For networks where CPLEX was not able to find an optimal solution, the best feasible solution that it has actually found may be used as an alternative starting solution for the heuristic. However, the latter possibility has not yet been tested.

An important aspect for the input of the data is a sensitivity analysis. Past experience shows, that this data is often erroneous or incomplete due to various reasons. Duplicate links with different lengths and missing links are perhaps among the most common problems. The requirements for the planning tools are twofold. First they should identify such problems if possible and second they should nevertheless try to proceed as normal. While a detailed error reporting is of course helpful, maybe even indispensable, dozens or even hundreds of pop-up windows because of duplicate links are certainly not welcomed by the user. Maybe he is aware of duplicate links in his database and just does not bother if the program chooses any one of them because the length differences are negligible. However, more important is a check for bi-connectivity in case a protection planning should be carried out.

While the AMPL import does not support “silent” sensitivity analysis for such cases, the heuristic contains preprocessing steps where such things are handled and reports may be generated if selected by the user. This might, e.g., include a list of articulation points or a list of duplicate links. The user might choose to completely ignore such things and then the algorithm deletes duplicate edges and adds additional ones in the case of missing connectivity according to simple rules. At this point some more sophisticated algorithm to add links might be included but that is at least currently beyond the scope of our approaches.

3.2. The algorithms and their implementation

The heuristic is a custom development programmed in C++. Figure 1 shows a flow diagram of the core algorithm. It processes the input data as described in Subsection 3.1 and, starting from a shortest path solution, iteratively improves the routing and the equipment assignment according to the cost and capacity constraints. A detailed description of the algorithms can be found in [7]. Table 1 summarizes some results in comparison to a shortest path routing and a provably optimal solution of the respective integer programming formulation.

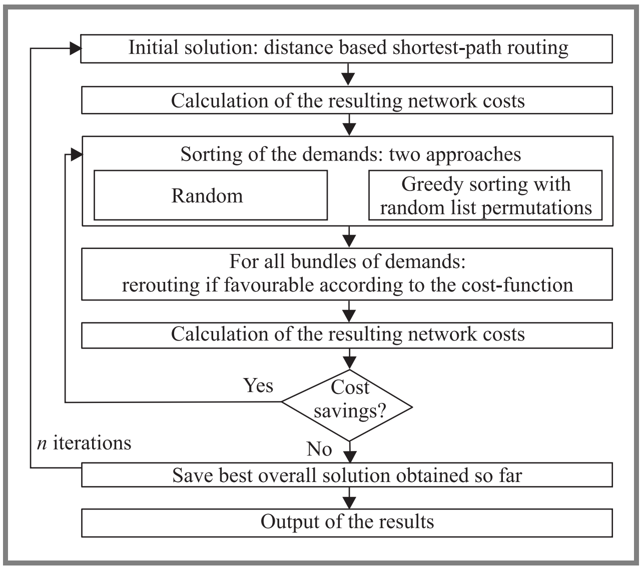


Fig. 1. Flow diagram of the heuristic part.

Table 1
Comparative results of the best objective value that was found for different networks

Nodes/edges/ demands	CPLEX 8.1	Shortest path	Heuristic 2000 runs
11/16/19 VC4 grooming	1231	1347	1244
20/33/84	9426	9728	9604
111/160/243 1+1 protection	–	333 909	313 145

One fundamental difference between IP and circuit switched traffic is that IP is unidirectional, thus in principle the size and the routing of the traffic from *A* to *B* can be totally different from the size and routing from *B* to *A*, the different routings and bandwidths are aggregated for the calculation of the port-cards, they can be freely rearranged in transit nodes as long as the minimum switching granularity is respected. Therefore, the algorithm is able to work with bi-directional demands as well as with arbitrary demand granularity. This can be, e.g., a granularity of 155 Mbit/s for a VC4 switched network or 1 kbit/s for the IP-routers. The statistical multiplexing gain is not yet implemented, largely due to a lack of appropriate data, but is scheduled for the near future.

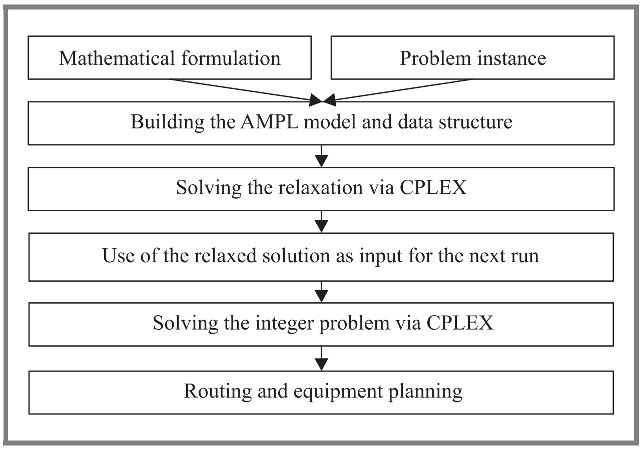


Fig. 2. Flow diagram of the AMPL/CPLEX part.

The sources of the tools are open to the network operator so that the planner knows what he is doing and what the effect of different parameters is. This is an advantage over most commercial tools, which are usually a black box to

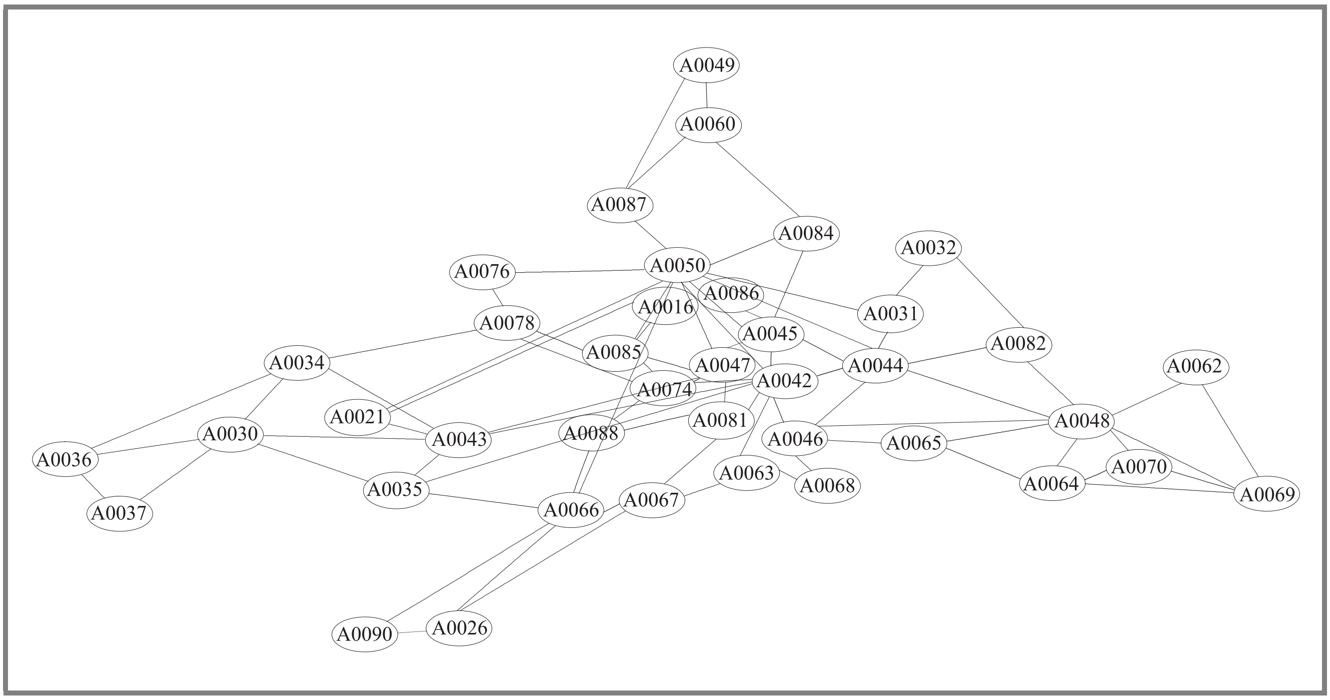


Fig. 3. Graphical output of a network structure.

Table 2
Runtimes for different network topologies

Runtime [s] for the problem instance, Δ between the best solution that was found and the lower bound						
Number of nodes/links/demands in the network						
17/26/28	20/22/46	20/33/84	24/48/36	24/48/60	27/32/45	30/45/60
16	4.5	12 300	4840	36 160 $\Delta = 1.78\%$	17	5900

the network designer. Of course, the companies that develop these tools want to protect their intellectual property and a black box might be suitable for some standard planning or re-planning issues that arise in certain intervals and have therefore been established as well-known routine. But for more advanced problems and for more advanced users, knowledge of the algorithms and their software implementations can be a great help in order to guide the planning process in the desired direction.

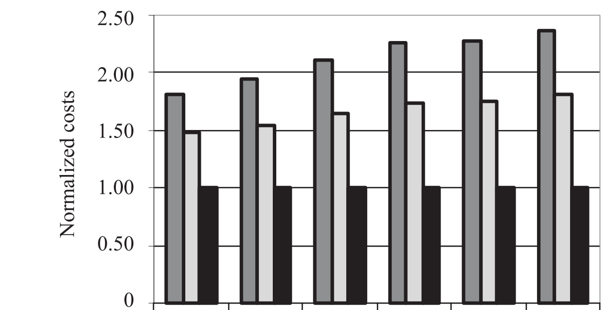
The heuristic works with up to 1000 nodes on state-of-the-art desktop PCs. Although run-time is usually not a major concern for such a strategic network planning process, fast processing is necessary for an interactive planning process, where the planner "plays" with different scenarios or configurations. For this purpose, mechanisms like the greedy randomized adaptive search procedure (GRASP) metaheuristic (cf. [7]) have been included that may be used to find good solutions in a short amount of time when it is more important to get results fast than to get a 1% better objective value. Given that it is partly based on forecasts, the general question arises if the input data is accurate enough to justify a comparison of different scenarios with a precision of a few percent more or less overall investments. This could be pseudo-accuracy.

The integer model is based on classic flow formulations and processes almost the same data as the heuristic. A drawback is the fact that this formulation currently does not support path restoration but only path protection. Figure 2 shows the steps for the use of this approach. First, the mathematical formulation is adjusted to the current real world problem. Then a linear relaxation of the problem is solved. This intermediate step is basically a means to reduce the memory consumption of the following integer calculation that produces the routing and the equipment plan.

The CPLEX calculation works for some dozens of nodes, which restricts it to comparatively small problem instances. However, the maximum network size that can be solved is very dependent on the actual structure of the problem. Specific relationships of the equipment costs or special fiber topologies might be very easy to solve, while other networks that are much smaller cannot be handled. Table 2 gives the runtime of some problem instances and clearly shows that they depend on the specific graph and not on the size alone. It is always a good idea to give the exact approach a try even for larger problem instances. Furthermore, it is useful as a reference for the heuristic.

3.3. Results of the planning process

The output of the software is a plan of the network that contains the routings for all demands and the equipment that is needed to carry them and of course the overall investment that is necessary for the equipment. Initially, these are large lists and tables, e.g., with load and equipment lists for all nodes and links. It gives the planner a full picture of the scenario but it is not easy to get an overview and draw conclusions from a scenario just with lots of numbers. Some kind of graphical output has been an often-requested feature during the development of the tools. The EXCEL part of the output can easily be converted into diagrams that already give a quick overview of the results. Yet besides these possibilities, a basic graphical output of the network structure (static pictures) with the nodes, the links and their loads can be displayed with the help of the free software package Graphviz[2]. Figure 3 shows such a topology graph. However, full interactive graphical output is beyond the scope of this development and too time-consuming to implement.



Demand scenario	0.50	0.67	1.00	1.50	2.00	4.00
1+1 protection	1.82	1.94	2.11	2.26	2.28	2.36
restoration	1.48	1.54	1.65	1.73	1.76	1.81
unprotected	1.00	1.00	1.00	1.00	1.00	1.00

Fig. 4. Scenarios with different demand forecasts.

In principle, this output is a (near) optimal solution of the initial network planning problem with respect to the overall investment in new equipment. However, there are too many simplifications and uncertainties in the underlying data and models, so that these tools should not be misused as an automatic network planning system. The results require an experienced planner for thorough examination and interpretation. In most cases, the planning process will probably be interactive and iterative. A specific scenario is calculated with the help of the tools, the planner analyses the results and then changes some parameters for the next run.

[2]See, http://www.research.att.com/~north/graphviz/

A comfortable feature is the batch mode, which allows predefining a set of scenarios and doing the calculations in one run. This is especially helpful for input data with intrinsic uncertainty, e.g., the demand distribution, and may lead to best case/worst case/average case results instead of just a single network design. Figure 4 shows an example from a past study (cf. [8]), where different demand forecasts were compared for a given network topology and three resilience mechanisms.

4. Conclusions

In this article, we have given some background on current multilayer planning problems for IP/SDH/WDM networks, including recent developments like GMPLS. We have presented two complementary tools, a heuristic and an integer programming approach, that started as classic switched circuit network planning tools and are now evolving towards the integration of IP networks. They provide a flexible and promising basis for further developments and have been already successfully used on several different planning problems. However, an integrated planning process that covers all the important aspects like, e.g., unified resilience mechanisms and statistical multiplexing effects, while simultaneously optimizing the routing and the necessary equipment, is still some way to go.

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Paper

Structural modeling and systems analysis of uneasy factors for realizing safe, secure and reliable society

Hiroyuki Tamura and Katsuhiro Akazawa

Abstract— In this paper we try to extract various uneasy factors in our life. Then, we try to construct structural models among these factors using Decision Making Trial and Evaluation Laboratory (DEMATEL). For the purpose of analyzing priority among these factors we revised the DEMATEL and found effective factors to be resolved in order to realize future safe, secure and reliable (SSR) society.

Keywords— safe, secure and reliable society, DEMATEL, structural modeling, systems analysis.

1. Introduction

Any people are living with some anxiety in any society. For example, people have had traditional anxiety for fire, natural disaster, human relations, responsibility in their work, etc. So in a long history, people have advanced science and technology and developed modern social systems to decrease these anxiety. On the other hands, in addition to this kind of traditional anxiety, it is said that vague uneasiness on the socio-economic situation, educational systems, safety in our life, etc., is expanding in Japan, in the environment of economic stagnation after the collapse of the "bubble economy" [1]. Since new anxiety are generated in addition to traditional anxiety, anxiety become complex and with wide variety. Accordingly, it is difficult to find how to decrease these anxiety. Therefore, it is worthwhile to try to decrease anxiety of people by extracting and analyzing various uneasy factors in order to create future safe, secure and reliable (SSR) society.

In this paper after finding various uneasy factors in our life we try to construct a structural model among these factors. For the purpose of structural modeling and systems analysis we use a system methodology called Decision Making Trial and Evaluation Laboratory (DEMATEL) [2–5]. By using DEMATEL we could extract mutual relationships of interdependencies among various uneasy factors and the strength of interdependence as well. However, the original DEMATEL cannot reflect the importance or seriousness of each factor to the result. In this paper we propose a revised method of DEMATEL to overcome this difficulty in the original DEMATEL and try to extract effective factors to be resolved in order to realize future SSR society.

2. DEMATEL

2.1. Outline

DEMATEL was developed in Battelle Geneva Institute, to analyze complex "world problematique" dealing mainly with interactive man-model techniques and to evaluate qualitative and factor-linked aspects of societal problems. The applicability of the method is widespread ranging from industrial planning and decision making to city planning and design, regional environmental assessment, analyzing global world problematique, and so forth.

2.2. Methodology

DEMATEL will try to get a weighted hierarchical structural model by analyzing quantitative data on the strength of binary relations on every two factors.

First of all, we extract all the factors that belong to the problematique. Suppose the problematique is composed of n factors. Next, we pay attention to the strength of some relation between two factors, and we try to find the strength of relations for all the pairs (i, j) of all the n factors such that "How much would it help in order to resolve factor j by resolving factor i?"

Suppose x^*_{ij} which is (i, j) element of $n \times n$ matrix $\mathbf{X}^*$, denotes the strength of relation from factor i to factor j, and suppose

- $x^*_{ij} = 0$: if by resolving factor i it would not help to resolve factor j at all;
- $x^*_{ij} = 1$: if by resolving factor i it would help to resolve factor j a little bit;
- $x^*_{ij} = 2$: if by resolving factor i it would help to resolve factor j very much.

Matrix $\mathbf{X}^*$ is called the direct matrix and the element $x^*_{i,j}$ denotes the strength of the direct influence from factor i to factor j.

Then suppose we obtain a direct matrix $\mathbf{X}^*_e$ concerning factor a, b and c as

$$\mathbf{X}^*_e = \begin{pmatrix} 0 & 2 & 1 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \end{pmatrix}. \qquad (1)$$

Figure 1 shows this structure. Factor a and factor b are mutually influenced and factor a affects factor c. In addition factor c affects factor b. Therefore, factor a affects factor b

directly and indirectly. In this case the strength of influence is the largest from factor a to factor b.

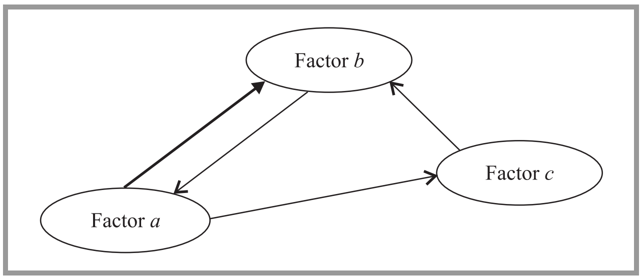

Fig. 1. Directed graph.

In DEMATEL we could further evaluate the other quantitative influence by simple matrix operations. Suppose we normalize the direct matrix $\mathbf{X}^*$ as

$$\mathbf{X} = \lambda \cdot \mathbf{X}^*, \tag{2}$$

where

$$\lambda = 1/(\text{the largest row sum of } \mathbf{X}^*).$$

In this case, $\mathbf{X}$ is called the normalized matrix. Since

$$\lim_{\theta \to \infty} \mathbf{X}^\theta = [0] \tag{3}$$

then we obtain

$$\mathbf{T} = \mathbf{X} + \mathbf{X}^2 + \cdots = \mathbf{X}(\mathbf{I} - \mathbf{X})^{-1}. \tag{4}$$

Matrix $\mathbf{T}$ is called the direct/indirect matrix. The $(i,\ j)$ element t_{ij} of matrix $\mathbf{T}$ denotes the direct and indirect influence from factor i to factor j. For example, direct/indirect matrix $\mathbf{T}_e$ concerning direct matrix $\mathbf{X}_e^*$ is obtained as follows:

$$\mathbf{T}_e = \begin{pmatrix} 0.35 & 1.05 & 0.45 \\ 0.45 & 0.35 & 0.15 \\ 0.15 & 0.45 & 0.05 \end{pmatrix}. \tag{5}$$

Suppose D_i denotes the row sum of ith row of matrix $\mathbf{T}$. Then, D_i shows the sum of influence dispatching from factor i to the other factors both directly and indirectly. Suppose R_i denotes the column sum of ith column of matrix $\mathbf{T}$. Then, R_i shows the sum of influence that factor i is receiving from the other factors. Furthermore, the sum of row sum and column sum $(D_i + R_i)$ shows the index representing the strength of influence both dispatching and receiving, that is, $(D_i + R_i)$ shows the degree of central role that the factor i plays in the problematique. If $(D_i - R_i)$ is positive, then the factor i is rather dispatching the influence to the other factors, and if $(D_i - R_i)$ is negative, then the factor i is rather receiving the influence from the other factors. For example, calculating these values concerning direct/indirect matrix $\mathbf{T}_e$, then we obtain $D_a + R_a = 2.8$, $D_b + R_b = 2.8$, $D_c + R_c = 1.3$, $D_a - R_a = 0.9$, $D_b - R_b = -0.9$ and $D_c - R_c = 0$. These results suggest factor a plays a central role and is dispatching factor, factor b plays a central role and is receiving factor.

3. Composite importance of each factor

3.1. Definition

In the original DEMATEL method we could evaluate the quantitative strength of each relation between each pair of factors. Therefore, it is possible to find factors that are resolved slightly or enormously when some factor was resolved. In the case of a direct matrix $\mathbf{X}_e^*$, by resolving the factor a which plays a central role and is dispatching factor, many factors in problematique are encouraged to be resolved enormously. However, this analysis is based only on the relations among factors. That is, the original DEMATEL is not taking into account the importance of each factor itself. Hence, it is not possible to evaluate the priority among the factors. For example, if the importance of factor c is high, then it may be efficient to resolve factor c.

To overcome this difficulty we introduce a new measure called the composite importance $\mathbf{z}$ into the original DEMATEL. The composite importance $\mathbf{z}$ is evaluated as follows: suppose the problematique is composed of n factors. We ask the respondent on the importance of each element. Based on the answers of the respondent we obtain n dimensional column vector $\mathbf{y}^*$. When we ask the importance of each factor to the respondent with 5-grade evaluation, the ith element y_i^* of n dimensional vector $\mathbf{y}^*$ is determined based on the answer of the respondent as

- $y_i^* = 0$: if factor i is not important at all;
- $y_i^* = 1$: if factor i is not important so much;
- $y_i^* = 2$: if factor i is important;
- $y_i^* = 3$: if factor i is very important;
- $y_i^* = 4$: if factor i is quite important.

Let normalize $\mathbf{y}^*$ as

$$\mathbf{y} = \mu \cdot \mathbf{y}^*, \tag{6}$$

where

$$\mu = 1/(\text{the largest element of } \mathbf{y}^*).$$

The ith element of the column vector obtained by multiplying the direct/indirect matrix $\mathbf{T}$ by $\mathbf{y}$, denotes the importance of factors resolved by resolving factor i. Then, taking into account the importance of factor i itself the composite importance of each element could be evaluated as

$$\mathbf{z} = \mathbf{y} + \mathbf{T}\mathbf{y} = (\mathbf{I} + \mathbf{T})\mathbf{y}. \tag{7}$$

3.2. Numeric examples

We show the numeric examples of the composite importance in the case of foregoing section. The composite importance is calculated from the strength of influence and the importance of factor. The strength of influence is given by direct/indirect matrix $\mathbf{T}_e$. As concerns the importance of factor, three cases are provided.

Suppose that the importance of each factor is same. This is equivalent to the case that the importance isn't taken into account. Let normalize importance be ${}^t\mathbf{y}_{e1} = (0.5\ \ 0.5\ \ 0.5)$, then the composite importance is

Table 1
The factors that prevent safety and security

Respondents	Private factors	Societal factors
University students	Career to pursue, scholastic performance, finance, health of one's own, health of family, marriage, looks, ability/character, human relations, part time/full time job	Traffic accident, fire disaster, natural disaster, recession, pension system, national debt, terrorism, war, public peace, child-abuse incident, BSE, decline in academic achievement, environmental destruction, radioactive leakage, depletion of natural resources
Unmarried adults	Finance, health of one's own, health of family, unemployment, marriage, looks, ability/character, human relations, part time/full time job	
Married adults	Finance, health of one's own, health of family, unemployment, children, looks, ability/character, human relations, part time/full time job	

calculated as ${}^t\mathbf{z}_{e1} = (1.425 \;\; 0.975 \;\; 0.825)$. This value suggests that resolving factor a is most effective and resolving factor b is more effective. As just described, in taking no thought of importance, the composite importance reflects only strength of influence.

In the next place, let normalize importance be ${}^t\mathbf{y}_{e2} = (0.3 \;\; 0.1 \;\; 0.8)$, then the composite importance is calculated as ${}^t\mathbf{z}_{e2} = (0.870 \;\; 0.390 \;\; 0.930)$. In this example, the priority corresponds to the importance of each factor. In addition, let normalize importance be ${}^t\mathbf{y}_{e3} = (0.1 \;\; 0.4 \;\; 0.8)$, then the composite importance is calculated as ${}^t\mathbf{z}_{e3} = (0.915 \;\; 0.705 \;\; 1.035)$. In this case, the priority doesn't correspond to both the strength of influence and the importance. That is the reason, why the priority of factor c is highest is that the importance of this factor is highest. Also factor a, which has lowest importance, has second priority, the reason is because the strength of influence of factor a is highest. As described above, the composite importance is the measure that reflects both the strength of influence and the importance of each factor. Therefore, this measure provides useful information in fixing an order of priority.

4. Data obtained from the respondents

We asked respondents to answer two kinds of questionnaire: questionnaire A and questionnaire B, for extracting and analyzing factors that prevent safety and security in our life. In questionnaire A we tried to extract the factors that prevent safety and security. In questionnaire B we asked the questions on binary relations on each pair of factors. Questionnaire B is designed based on the factors extracted in questionnaire A.

In questionnaire A we asked questions to 42 respondents on the factors that prevent safety and security where we let them answer without any restraint. As the result, we could extract two kinds of factors: one kind is private factors of respondents and the other kind is societal factors. We found that private factors depend upon the respondents' social standing: university students, unmarried adults and married adults. Table 1 shows the factors extracted.

In questionnaire B the importance of each factor is asked to the respondents by 5-grade evaluation as shown in Fig. 2, where we adopted 10 people each for university students, unmarried adults and married adults. In this questionnaire

Do you feel uneasy for your career to pursue?

1. I feel quite uneasy.
2. I feel very uneasy.
3. I feel uneasy.
4. I do not feel uneasy so much.
5. I do not feel uneasy at all.

Fig. 2. An example of questions on the importance of each factor.

How much would it help in order to resolve the anxiety for pension system by resolving the anxiety for your career to pursue?

1. I think it would help very much.
2. I think it would help fairly well.
3. I do not think it would help at all.
4. I do not know.

Fig. 3. An example of questions on the strength of relation between two factors.

the importance of each factor implies the degree of feeling uneasy for each factor. Then, the strength of relation is asked by 3-grade evaluation. In detail, we obtained information on the binary relations between two private factors, between two societal factors and a societal factor to a private factor. Figure 3 shows an example of questions on the strength of relation between two factors.

5. Results

5.1. Structural models among uneasy factors

5.1.1. Private factors

Structural models for private factors are shown in Figs. 4–6. In these figures thick arrow is drown from factor i to factor j

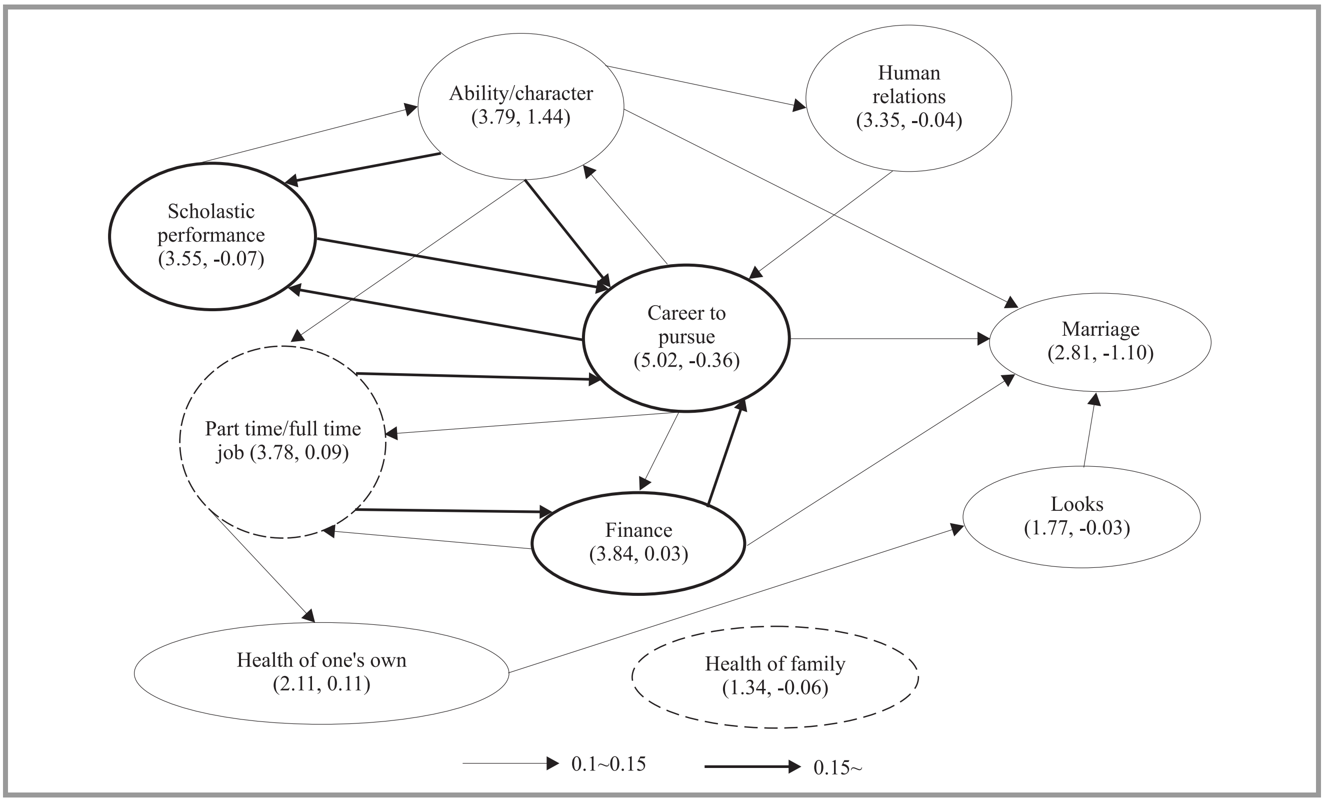

Fig. 4. Structural model for private factor of university students.

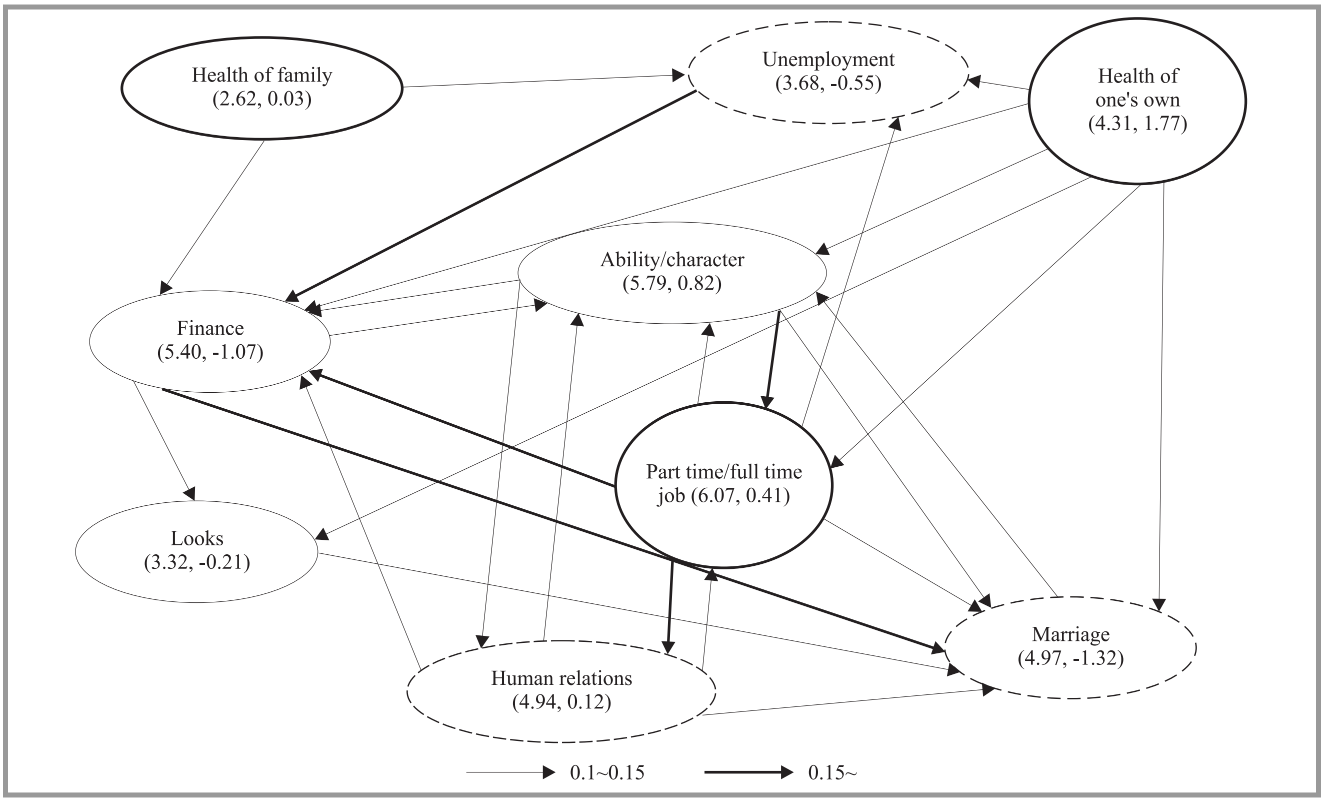

Fig. 5. Structural model for private factor of unmarried adults.

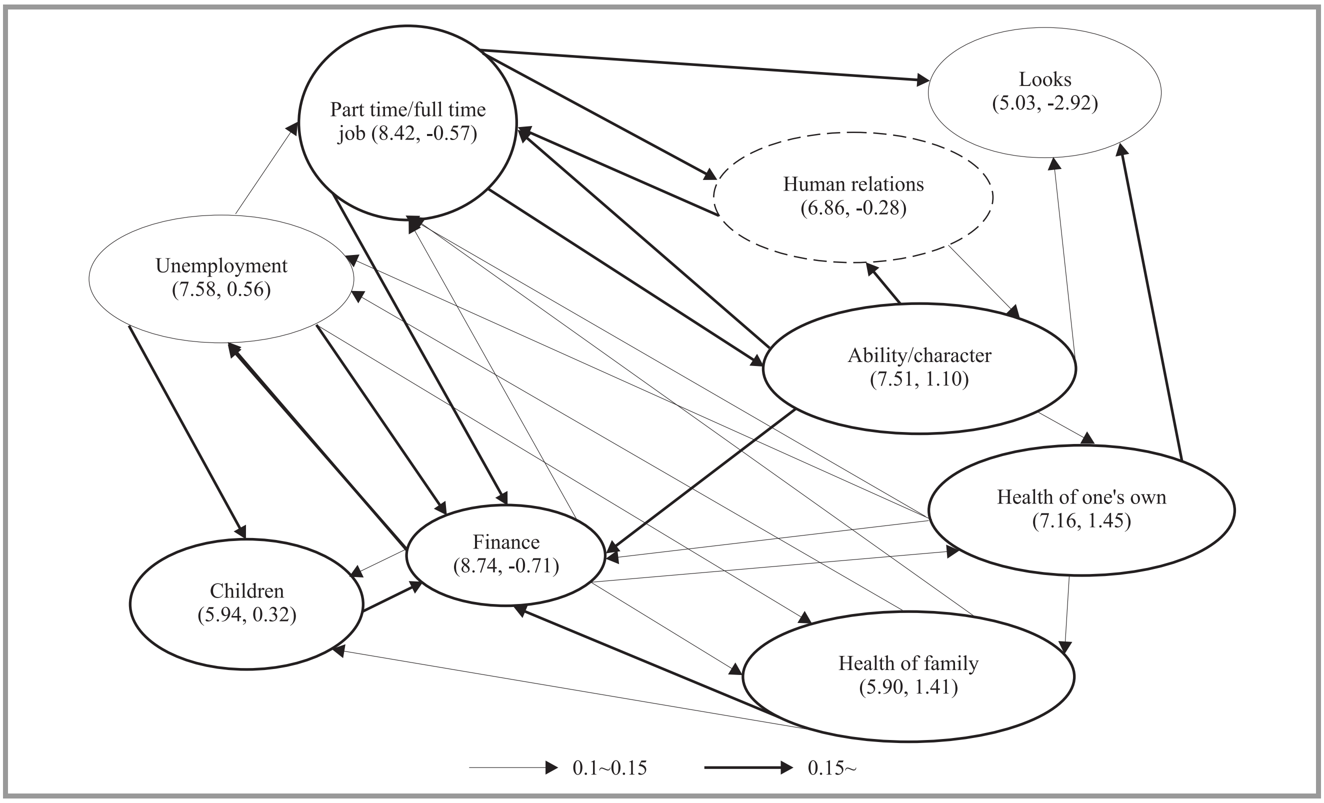

Fig. 6. Structural model for private factor of married adults.

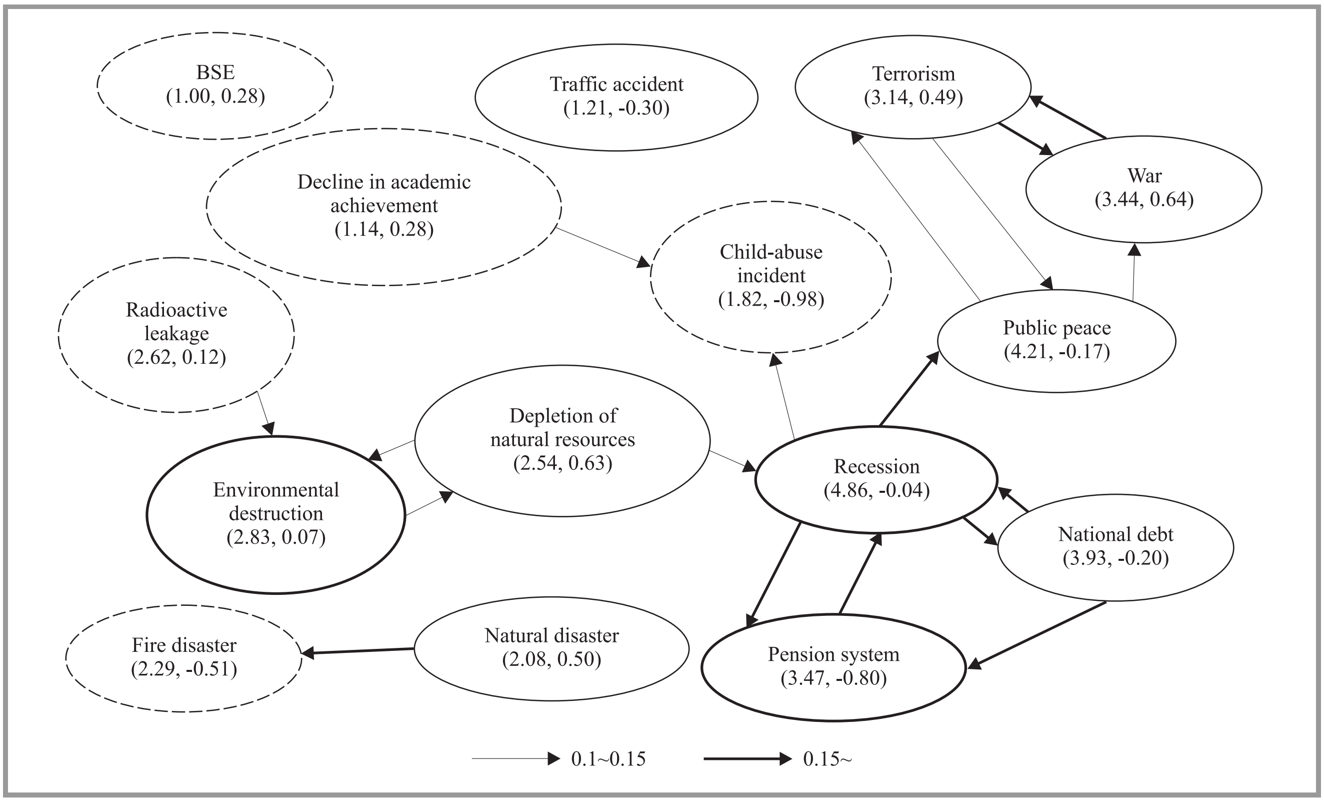

Fig. 7. Structural model for societal factors.

if x_{ij} is greater than or equal to 0.15, thin arrow if x_{ij} is between 0.1 and 0.15, and no arrow if x_{ij} is less than 0.1. Each factor is circled by a thick line if the importance y_i of factor i is greater than or equal to 0.55, by a thin line if y_i is between 0.45 and 0.55, and by a dotted line if y_i is less than 0.45. Under each factor the values of $(D_i + R_i)$ and $(D_i - R_i)$ are shown.

From Fig. 4 we found the following information for private factors of university students.

- "Ability/character" got the highest $(D-R)$ value, that is, this is the main influence dispatching factor that will affect other factors. This means that by resolving the anxiety on "ability/character" the anxiety on "scholastic performance", "career to pursue", "human relations", "marriage" and "part time/full time job" will be improved very much. That is, "ability/character" plays central role among many other factors.

- "Marriage" got the lowest $(D-R)$ value, that is, this is the main factor of receiving influence from other factors. This means that by resolving anxiety on "marriage" it will not affect other factors, but resolving the anxiety on "career to pursue", "ability/character", "looks" and "finance" will help to resolve the anxiety on "marriage".

- $(D+R)$ value of "career to pursue" is high. This means that "career to pursue" has strong connection with other factors and plays central role. Especially, it will receive big influence from "scholastic performance", "finance" and "part time/full time job", and it will affect "scholastic performance".

- $(D+R)$ value of "health of family" is low. This means that "health of family" is neither an influence dispatching factor nor an influence receiving factor.

From Fig. 5 we found the following information for private factors of unmarried adults.

- "Health of one's own" got the highest $(D-R)$ value, that is the most influence dispatching factor. This means that by resolving the anxiety on "health of one's own" the anxiety on "ability/character", "part time/full time job", "finance", "marriage" and "looks" will be improved very much.

- Just like university students "marriage" is a factor that will receive influence from the other factors for unmarried adults as well.

- $(D+R)$ value of "part time/full time job" is high. This means that "part time/full time job" has strong connection with the other factors and plays central role. Especially, it will receive big influence from "ability/character", and it will affect "finance" and "human relations".

- Just like university students "health of family" is neither an influence dispatching factor nor an influence receiving factor.

From Fig. 6 we found the following information for private factors of married adults.

- $(D-R)$ value of "ability/character" is high for married adults just like university students and unmarried adults. $(D-R)$ value of "health of one's own" is also high just like unmarried adults. Significant feature of married adults is that $(D-R)$ value of "health of family" is high. This means that by resolving the anxiety on "health of family" the anxiety on "finance" and "unemployment" will be improved very much. This structural model reflects the feeling of anxiety of married adults that the finance is supported by the family and "health of family" is one of the most important factor.

- $(D-R)$ value of "looks" is the lowest, that is, this is the main factor of receiving influence from the other factors.

- Compared with university students and unmarried adults $(D+R)$ value of almost all the factors is high and has strong relation with the other factors. Especially, $(D+R)$ value of "finance" is the highest, and "finance" has strong connection with the other factors. Especially, it will receive big influence from "ability/character", "children", "health of family", "unemployment" and "part time/full time job". $(D+R)$ value of "part time/full time job" is high just like unmarried adults.

5.1.2. Societal factors

From Fig. 7 we found the following information for societal factors.

- $(D+R)$ values of "recession", "public peace", "national debt" and "pension system" are high and these factors play central role. These four factors and "terrorism" and "war" are influencing each other.

- In general interrelations among various factors such as "bovie spongiform encephalopathy (BSE)", "traffic accident" and others are weak.

5.2. Composite importance of each factor

In Subsection 5.1 we obtained structural models of uneasy factors, and found factors with high $(D+R)$ value that play central role, factors with high $(D-R)$ value that mainly dispatch influence to the other factors, factors with low $(D-R)$ value that mainly receive influence from the other factors, and so forth. However, from these discussions we cannot find effective factors to be resolved in order to create future safe, secure and reliable society. For this purpose we

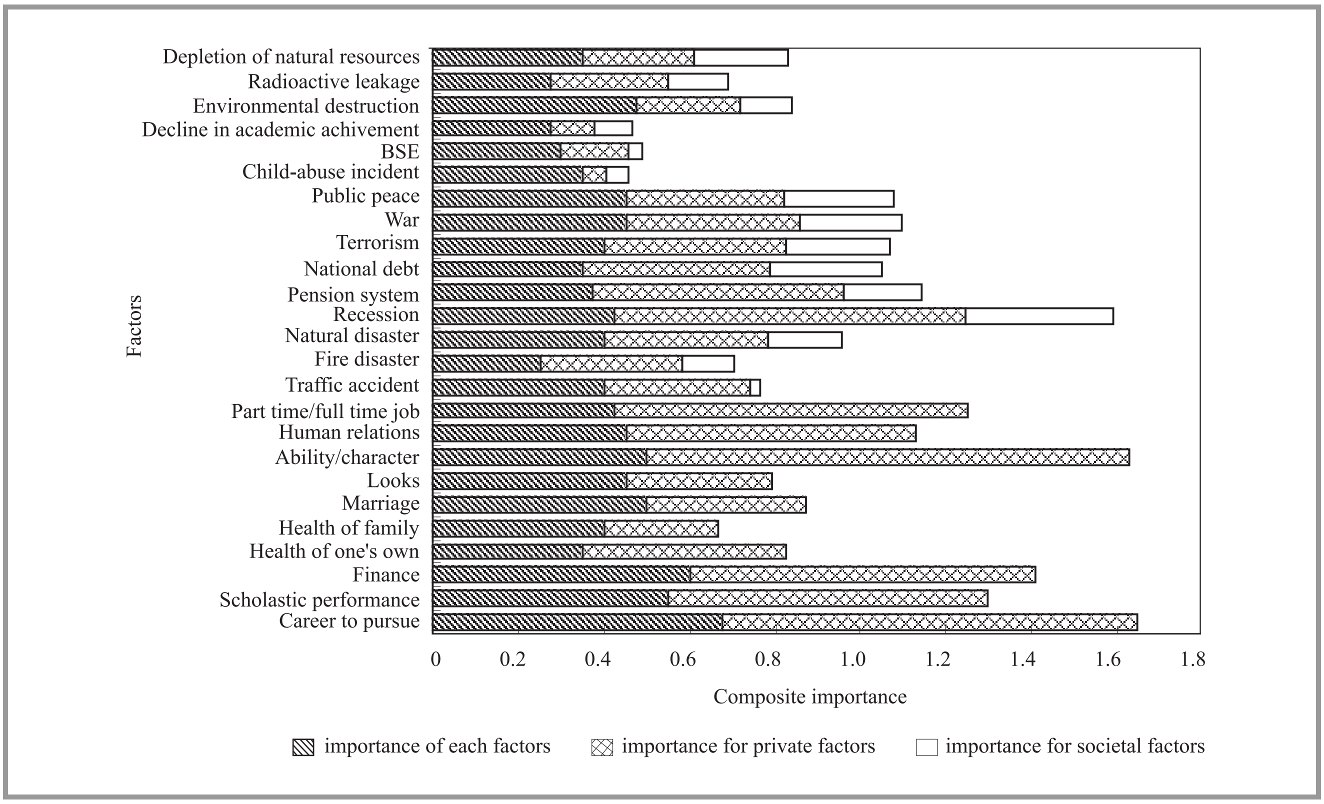

Fig. 8. Composite importance of university students.

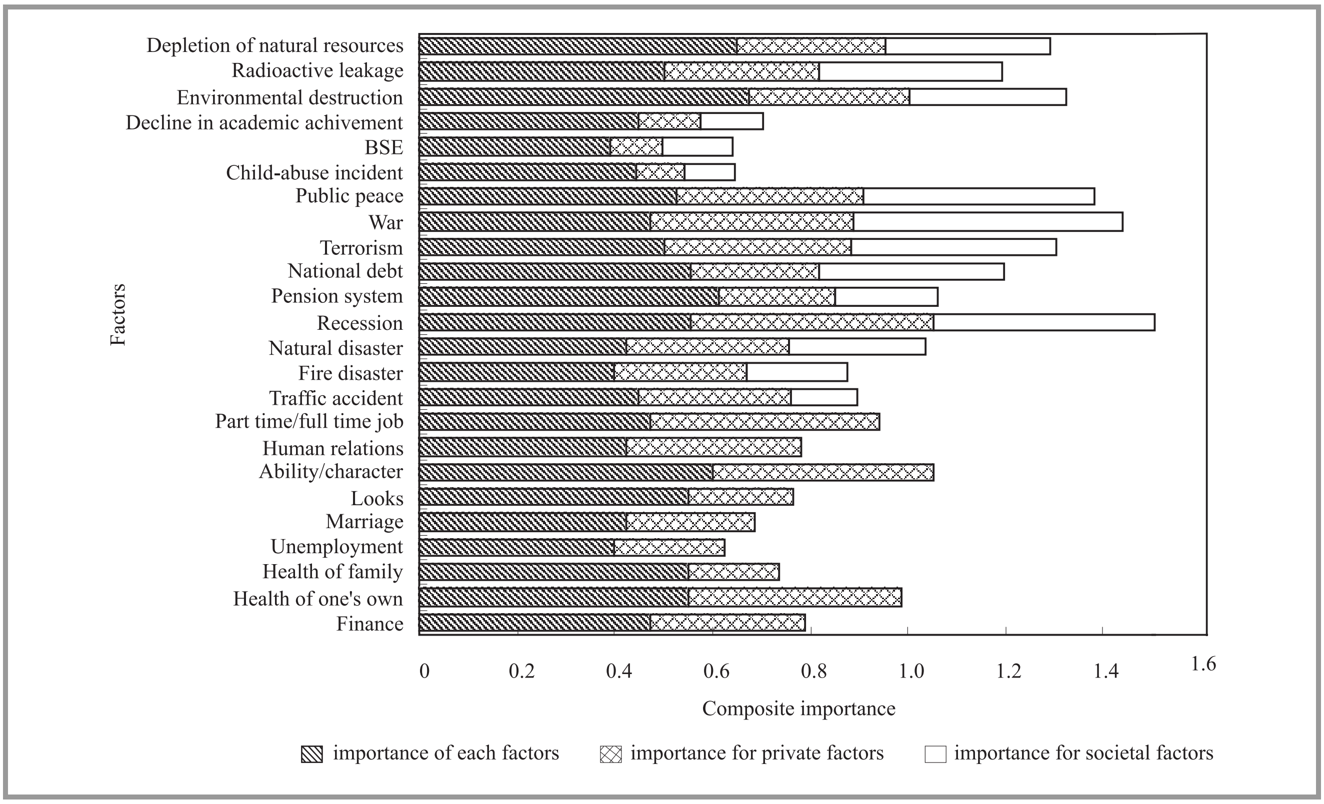

Fig. 9. Composite importance of unmarried adults.

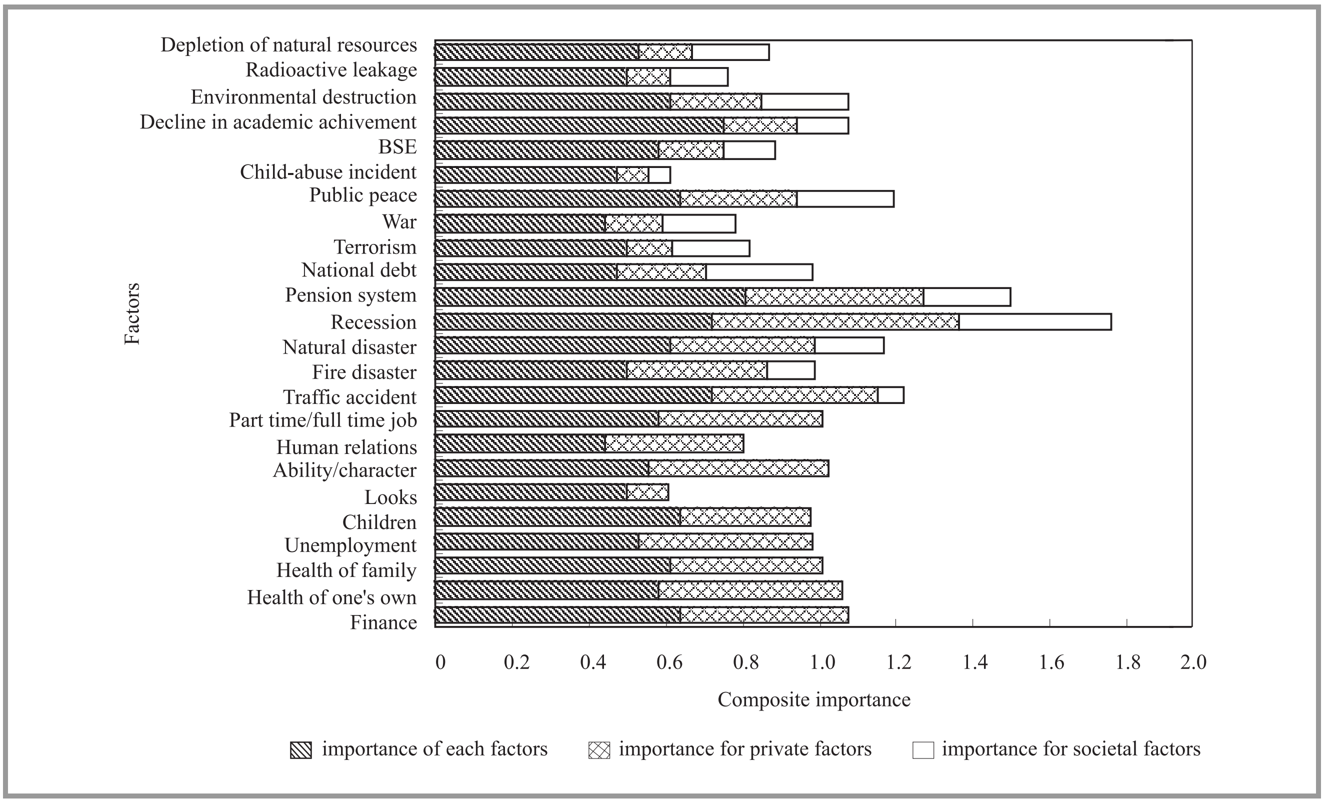

Fig. 10. Composite importance of married adults.

evaluated Eq. (7) the composite importance of each factor to be resolved in order to realize SSR society. Figures 8–10 show the results obtained for this purpose. In these figures the importance of each factor itself, composite importance of private factors, and composite importance of societal factors, are shown.

For university students, the composite importance of "career to pursue", "ability/character" and "recession" is high. Therefore, resolving anxiety for these factors is effective to resolve the anxiety for the other factors for them.

For unmarried adults, the composite importance of "recession", "war", "public peace", "environmental disruption", "terrorism" and "resource shortage" is high. Comparing with university students, unmarried adults feel more importance for societal factors. In addition, focusing attention to private factors, then factors that play central role and/or dispatch influences to the other factors doesn't necessary have high composite importance value. For example, though the factor that plays central role is "part time/full time job" and the most influence dispatching factor is "health of one's own", the composite importance of these factors is not so high. The composite importance of "ability/character" is high.

Finally, for married adults the composite importance of "recession" and "pension system" is high. As seen so far the composite importance of "recession" is high for all the people. That is, to resolve the anxiety for "recession" is the most effective means to improve the anxiety of other factors.

6. Concluding remarks

In this paper after finding various factors that prevent safety and security in our life we constructed structural models among these factors by using DEMATEL. From these models we found interdependencies among these factors and the strength of interdependencies. Furthermore, as a revised DEMATEL we proposed a new measure to show the composite importance of each factor and found the important factor to be resolved in order to resolve anxiety of the other factors effectively. This result may suggest to find effective policy to realize future SSR society.

For further research we may try to do more realistic questionnaire survey in order to propose effective policies to realize future SSR society. In this paper we studied abstract anxiety in general arising in Japanese society. For further research we may study in more specific field, e.g., the anxiety for buying food, anxiety to work in a specific environment, and so forth. If we could evaluate the cost, labor, etc., for resolving factors that prevent safety and security in our life we could get more useful results for realizing future SSR society.

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Paper

Intelligent decision system based on the evidential reasoning approach and its applications

Dong-Ling Xu and Jian-Bo Yang

Abstract—Intelligent decision system (IDS) is a window-based software package that has been developed on the basis of the evidential reasoning (ER) approach, a recent development in handling hybrid multiple criteria decision analysis (MCDA) problems with uncertainties. In this paper, the evidential reasoning approach will be briefly described first, and its major differences from and the relationships with conventional MCDA methods will also be discussed. Then the main features, advantages and benefits of IDS will be demonstrated and explained using two application examples: supplier pre-qualification assessment and customer satisfaction survey analysis, which have been investigated as part of the research projects led by the authors and funded by the UK government and the EC. It is concluded in the paper that the ER approach can be used not only to deal with problems that traditional methods can solve, but also to model and analyse more complicated decision problems that traditional methods are incapable of handling.

Keywords— multicriteria decision support systems, knowledge management, intelligent decision system, the evidential reasoning approach.

1. Introduction

In increasingly competitive, demanding and hostile business environments, many organisations are under pressure to cut costs and improve quality of their services and products. During the past several years, we have been in close collaboration with a number of companies in applying multicriteria decision analysis methods to help them achieve those goals. Assessing suppliers systematically in e-procurement processes and conducting quality and service surveys among customers are two of the areas where many companies have asked us to provide support.

Such assessments and surveys are normally based on specially designed models and can be regarded as a typical type of multiple criteria decision analysis (MCDA) problems [1, 13], which normally include a large number of criteria having both a quantitative and qualitative nature. Traditional ways of conducting such assessments and surveys include the use of average scores as performance indicators. The advantage of such methods is their simplicity and practicality. However, an average score does not provide sufficient information on the diversity of the performances of a business, nor can it indicate where the business is doing well and where it needs to improve if its average performance is acceptable. Therefore strengths and weaknesses need to be identified separately to supplement average scores. However, questions have been raised as to the accuracy of average scores generated and the consistency between average scores and strengths and weaknesses identified [6, 8].

Recently, significant effort has been made by the authors and their colleagues to introduce a new MCDA method, the evidential reasoning (ER) approach into such assessment exercises [6, 12, 13]. Several projects have been funded by the UK Engineering and Physical Science Research Council (EPSRC) and the European Commission (EC) to conduct research in applying the ER approach to support such assessments. A number of papers and research reports have been generated and published as the results of the research projects. These results show that the ER approach can help to reduce subjectivity in the assessment processes and generate a range of useful information for an organisation in question. This paper will describe how the ER approach and its software realisation intelligent decision system (IDS) [9] can be applied to support supplier assessment and customer quality survey analysis.

In the following section, the ER approach and its development history will be described first and the IDS software will be introduced as well. A supplier pre-qualification assessment model and its implementation will then be discussed, followed by the description of a customer quality survey analysis using the IDS software. The paper will conclude in Section 5.

2. The evidential reasoning approach and its software realisation – IDS

The evidential reasoning approach uses an evidence-based reasoning process to reach a conclusion, which differs from traditional MCDA methods. The motivation of developing the ER approach originates from the authors' experiences of working with industry in developing decision support systems [16], in particular to deal with MCDA problems having both quantitative and qualitative information with uncertainties and subjectivity. The ER approach has been developed using the concepts from several disciplines, including decision sciences (in particular utility theory), artificial intelligence, statistical analysis, fuzzy set theory, and computer technology [10–12, 14–16].

The development of the ER approach has experienced five major stages. The first stage was the introduction of a belief structure into a decision matrix [16]. This provides a novel way to model MCDA problems, in particular those having both quantitative and qualitative criteria with uncertainties. In conventional methods, a MCDA problem is modelled using a decision matrix, with each criterion assessed at each alternative decision by a single value. In the ER approach, a MCDA problem is described using a belief decision matrix, with each criterion assessed at each alternative by a two-dimensional variable: possible criterion referential values (assessment grades) and their associated degrees of belief.

Mathematically, in the ER approach a MCDA problem with L criteria A_i $(i = 1,\ldots,L)$, K alternatives O_j $(j = 1,\ldots,K)$ and N evaluation grades H_n $(n = 1,\ldots,N)$ for each criterion is represented using a belief decision matrix with $S\big(A_i(O_j)\big)$ as its element at the ith row and jth column, where $S\big(A_i(O_j)\big)$ is given as follows:

$$S\big(A_i(O_j)\big) = \Big\{\big(H_n, \beta_{n,i}(O_j)\big),\ n = 1,\ldots,N\Big\}\quad i = 1,\ldots,L,\ j = 1,\ldots,K,$$

where $\beta_{n,i}(O_j)$ is the degree of belief to which the alternative O_j is assessed to the nth grade of the ith criterion. It should be noted that a criterion could have its own set of evaluation grades that may be different from those of other criteria and also criteria could consist of a hierarchy [12].

The above ER framework allows more information to be contained in the model where the decision maker is no longer forced to pre-aggregate decision information into a single value when the original information is truly two-dimensional. In this context, the ER framework not only provides flexibility in describing a MCDA problem, it also prevents any loss of information due to the conversion from two-dimensional to one-dimensional values in the modeling process.

The second stage was the introduction of the Dempster-Shafer theory [2, 5] into the ER approach so that the two-dimensional information contained in the belief decision matrix could be aggregated to produce rational and consistent assessment results. For years, the authors have been searching for appropriate theoretical frameworks to fulfil such a task and the Dempster-Shafer theory has been chosen because of its unique capacity of dealing with ignorance which is inherent in subjective assessments, its powerful evidence combination rules and the reasonable requirements to apply the rules [2, 3, 10, 11].

Instead of aggregating average scores, the ER approach employs an evidential reasoning algorithm to aggregate belief degrees, which has been developed on the basis of the belief decision matrix, decision theory and the evidence combination rule of the Dempster-Shafer theory [10–12, 14]. Thus, scaling grades is not necessary for aggregating criteria in the ER approach and it is in this way that the ER approach is different from other MCDA approaches, most of which aggregate average scores or utilities.

The ER aggregation process is briefly described as follows. The following descriptions are of a mathematical nature and may be skipped until the end of the last set of equations. First, the degrees of belief $\beta_{n,i}(O_j)$ (or $\beta_{n,i}$ for short) for all $n = 1,\ldots,N$, $i = 1,\ldots,L$ are transformed into basic probability masses [12, 14]. Let ω_i be the weight of the ith criterion, $m_{n,i}$ a basic probability mass representing the degree to which the ith criterion is assessed to the nth evaluation grade H_n. Let $m_{H,i}$ be a remaining probability mass unassigned to any individual grade after the ith criterion has been assessed; $m_{n,i}$ and $m_{H,i}$ are calculated as follows:

$$m_{n,i} = \omega_i \beta_{n,i} \quad n = 1,\ldots,N,$$

$$m_{H,i} = 1 - \sum_{n-1}^{N} m_{n,i} = 1 - \omega_i \sum_{n=1}^{N} \beta_{n,i},\quad i = 1,\ldots,L,$$

$$\overline{m}_{H,i} = 1 - \omega_i \quad \text{and} \quad \widetilde{m}_{H,i} = \omega_i\Big(1 - \sum_{n=1}^{N}\beta_{n,i}\Big)$$

with $m_{H,i} = \overline{m}_{H,i} + \widetilde{m}_{H,i}$ for all $i = 1,\ldots,L$ and $\sum_{i=1}^{L}\omega_i = 1$. The probability mass assigned to the whole set of grades $H = [H_1, H_2, \ldots, H_N]$, which is unassigned to any individual grade H_n, is split into two parts, one caused by the relative importance of the ith criterion or $\overline{m}_{H,i}$ and the other by the incompleteness of the ith criterion or $\widetilde{m}_{H,i}$.

Then, all L criteria are aggregated to generate the combined degree of belief for each possible grade H_n. Let $m_{n,I(1)} = m_{n,1}$ $(n = 1,\ldots,N)$, $\overline{m}_{H,I(1)} = \overline{m}_{H,1}$, $\widetilde{m}_{H,I(1)} = \widetilde{m}_{H,1}$ and $m_{H,I(1)} = m_{H,1}$. The combined probability assignments $m_{n,I(L)}$ $(n = 1,\ldots,N)$, $\overline{m}_{H,I(L)}$, $\widetilde{m}_{H,I(L)}$, and $m_{H,I(L)}$ can be generated by aggregating all the basic probability masses using the recursive evidential reasoning algorithm [14]:

$$\{H_n\}: \quad m_{n,I(i+1)} = K_{I(i+1)}\big[m_{n,I(i)}m_{n,i+1} + m_{H,I(i)}m_{n,i+1} + m_{n,I(i)}m_{H,i+1}\big] \quad n = 1,2,\ldots,N$$

$$\{H\}: \quad m_{H,I(i)} = \widetilde{m}_{H,I(i)} + \overline{m}_{H,I(i)}$$

$$\widetilde{m}_{H,I(i+1)} = K_{I(i+1)}\big[\widetilde{m}_{H,I(i)}\widetilde{m}_{H,i+1} + \overline{m}_{H,I(i)}\widetilde{m}_{H,i+1} + \widetilde{m}_{H,I(i)}\overline{m}_{H,i+1}\big]$$

$$\overline{m}_{H,I(i+1)} = K_{I(i+1)}\big[\overline{m}_{H,I(i)}\overline{m}_{H,i+1}\big]$$

$$K_{I(i+1)} = \Bigg[1 - \sum_{t=1}^{N}\sum_{\substack{i=1\\ j\neq t}}^{N} m_{t,I(i)}m_{j,i+1}\Bigg]^{-1}$$

$$i = \{1,2,\ldots,L-1\}$$

$$\{H\}: \quad \beta_H = \frac{\widetilde{m}_{H,I(L)}}{1 - \overline{m}_{H,I(L)}}$$

$$\{H_n\}: \quad \beta_n = \frac{m_{n,I(L)}}{1 - \overline{m}_{H,I(L)}} \quad n = 1,2,\ldots,N$$

Parameter β_n denotes the degree of belief to which the L criteria are assessed to the grade H_n and β_H represents the remaining belief degrees unassigned to any H_n. It has been proved that $\sum_{n=1}^{N}\beta_n+\beta_H=1$ [14]. The final distribution assessment for O_j generated by aggregating the L criteria can be represented as follows:

$$S(O_j)=\left\{\left(H_n,\beta_n(O_j)\right),\quad n-1,\ldots,N\right\}.$$

Suppose the utility (or score) of an individual output term H_n is denoted by $u(H_n)$. The average utility of $S(O_j)$ can be given as follows [12]:

$$u(O_j)=\sum_{n=1}^{N}\beta_n(O_j)u(H_n).$$

Note that β_n denotes the lower bound of the likelihood that the alternative O_j is assessed to H_n. The upper bound of the likelihood is given by $(\beta_n+\beta_H)$. Complementary to the above distribution assessment, a utility interval can also be established [12] if the assessment is incomplete or imprecise, characterized by the maximum, minimum and average utilities of $S(A^*)$ defined as follows given $u(H_{n+1})\geq u(H_n)$:

$$u_{\max}(O_j)=\sum_{n=1}^{N-1}\beta_n(O_j)u(H_n)+\left(\beta_N(O_j)+\beta_H(O_j)\right)u(H_N),$$

$$u_{\min}(O_j)=\left(\beta_1(O_j)+\beta_H(O_j)\right)u(H_1)+\sum_{n=2}^{N}\beta_n(O_j)u(H_n),$$

$$u_{avg}(O_j)=\frac{u_{\max}(O_j)+u_{\min}(O_j)}{2}.$$

Note that if all original assessments $S\left(A_i(O_j)\right)$ in the belief decision matrix are complete, then $\beta_H(O_j)=0$ and $u\left(S(O_j)\right)=u_{\max}(O_j)=u_{\min}(O_j)=u_{avg}(O_j)$. It should also be noted that the above utilities are only used for characterizing an assessment but not for criterion aggregation.

The computational complexity using the combination rule of the Dempster-Shafer theory could be one of the major points of criticism if the combination rule is not used properly. In fact, Orponen [4] showed that the combination of mass functions or basic probability assignments (BPAs) using Dempster's rule is #P-complete (the class #P is a functional analogue of the class NP of decision problems). But the computational complexity of reasoning using Dempster's rule based on the above specific ER framework becomes linear rather than #P-complete [10–12]. It should also be noted that conflicting information can be explicitly modelled using the ER framework with the normalized ω_k and logically processed using the ER algorithm, thereby overcoming another drawback of the original combination rule of the Dempster-Shafer theory in dealing with conflicting evidence.

The third stage was the development of the rule and utility-based information transformation techniques to transform various types of evaluation information to a unified framework so that all criteria of both a quantitative and qualitative nature can be assessed in a consistent and compatible manner in the ER framework [12]. This to certain extent mirrors the traditional normalisation techniques used to handle quantitative criteria with different units in MCDA problems. The key difference is that in the ER framework the new techniques can in a sense preserve the two-dimensional information represented in the belief structure. It has been proved that by using the developed information transformation techniques not only the expected utilities of the original and the transformed assessments are equivalent but the degrees of incompleteness or completeness in the original assessments are also preserved.

The fourth stage is the enhancement of the approximate reasoning process of the original ER approach. Although the Dempster-Shafer theory has been used as the theoretical framework for information aggregation in the ER approach, its original evidence combination rule would generate irrational synthesis results if there is conflicting evidence. Significant modifications have been made since the theory was first introduced into the ER approach to deal with MCDA problems. It is proved that the new reasoning process of the ER approach satisfies the following common sense synthesis rules (CSSR) [14]:

CSSR 1: *If no sub-criterion is assessed to an evaluation grade at all then the upper-level criterion should not be assessed to the same grade either.*

CSSR 2: *If all sub-criteria are precisely assessed to an individual grade, then the upper-level criterion should also be precisely assessed to the same grade.*

CSSR 3: *If all sub-criteria are completely assessed to a subset of grades, then the upper-level criterion should be completely assessed to the same subset as well.*

CSSR 4: *If sub-criterion assessments are incomplete, then an upper-level assessment obtained by aggregating the incomplete basic assessments should also be incomplete with the degree of incompleteness properly expressed.*

The fifth stage is the implementation of the ER approach by developing a Windows based software package, the intelligent decision system [9, 12, 14, 15]. As mentioned earlier, the ER approach models a MCDA problem using a belief decision matrix with two-dimensional values, so inevitably the calculations involved in the aggregation processes could be more complicated than some traditional methods such as the additive utility function approach. Without a user-friendly computer interface to facilitate information collection, processing and display, the task could be rather difficult to accomplish by hands, even for a relatively small scale MCDA problem.

Although the ER approach involves relatively complicated calculations, its computational requirements are linearly proportional to the scale of a MCDA problem, namely the numbers of criteria and alternatives in a problem. IDS has been used in a variety of applications, such as motorcycle assessment [10], general cargo ship design selection (or assessment), marine system safety analysis and synthesis, executive car assessment, project management and organizational self-assessment [6, 13]. The experiences gained from these applications indicate that for MCDA problems with up to a few thousands of criteria and many alternatives, the calculation time using a PC is unnoticeable. It has also been proved in these applications that the ER approach not only produces consistent and reliable results for problems that can be solved using conventional MCDA methods, but also is capable of dealing with MCDA problems of the following features, which are difficult to handle using conventional methods without making further assumptions:

– mixture of quantitative and qualitative information,

– mixture of deterministic and random information,

– incomplete (missing) information,

– vague (fuzzy) information,

– large number (hundreds) of criteria in a hierarchy,

– large number of alternatives.

In addition to its mathematical functions, IDS is also a knowledge management tool. It records assessment information including evidence and comments in organised structures, provides systematic help at every stage of the assessment process including guidelines for grading criteria, and at the end of an assessment generates a tailored report with strengths and weaknesses highlighted at a click of a button. In the following sections, two application examples are to be examined to demonstrate some of the features of the IDS software package.

3. Supplier pre-qualification assessment

World markets have become increasingly competitive and integrated. In a global marketplace, a good supplier appears to be an invaluable resource for a buying organisation. The selection and management of right suppliers is the key element for a company to achieve its own performance targets.

Supplier pre-qualification assessment is considered as the critical step of a supplier selection process. Its objective is to screen out supply applicants who do not meet the basic requirements to such a degree that any further detailed assessment of their applications would be unnecessary. It also aims to provide feedback information to an applicant about where it should improve in order to be qualified as a supplier. It thus consists of both establishing minimal capacities below which a vendor will not be considered and determining whether an applicant can fulfil these basic requirements.

In recent years, the authors have established and supervised a number of summer consultancy projects for supplier assessment together with the Purchase Department of the Shared Service Ltd of Siemens UK, a global leading company in communications, electronics and electrical engineering. The objectives of such projects are to help investigate existing supplier assessment models, develop new models and realise them using the IDS software package both online and offline. One of such projects was dedicated to developing a supplier pre-qualification assessment model for the company [7]. The model has a hierarchy of criteria as shown in Fig. 1, which is the IDS main window for displaying an assessment model.

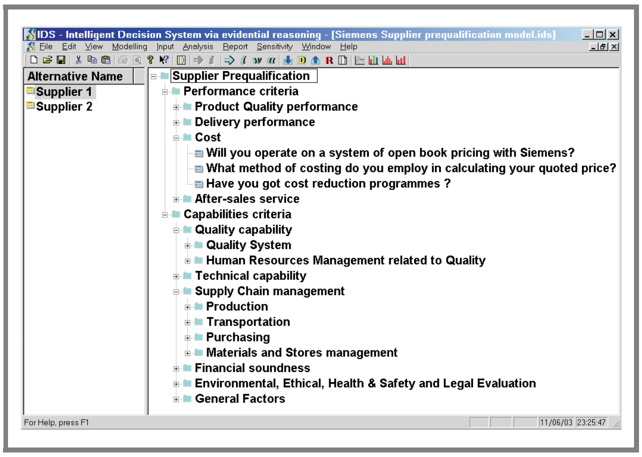

Fig. 1. IDS main window for Siemens UK supplier pre-qualification model.

The IDS main window consists of a tree view on the right side to display the names of a hierarchy of criteria; a list view on the left side to show the names of alternative suppliers to be assessed; a menu bar where all IDS functions can be assessed for model building, data input, result analysis, reporting and sensitivity analysis; and a short cut bar for easy access to frequently used IDS functions.

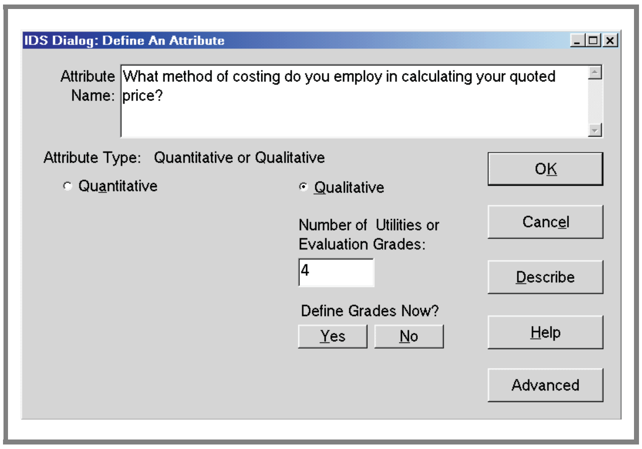

Fig. 2. Define a qualitative criterion using IDS dialog.

The criteria hierarchy can be fully expanded in the same way as in Window Explorer. A criterion can be defined as a quantitative, qualitative or uncertain criterion using the IDS dialog windows [9]. For example, Fig. 2 shows the IDS dialog window for defining a qualitative criterion where the user can enter the name of the criterion, choose the number of assessment grades and provide a description about the criterion. Many of the criteria in the Siemens pre-qualification model are of a similar qualitative nature.

Not only can the user define the number of assessment grades, but they can describe and define each grade as well. Figure 3 shows the IDS dialog window for this purpose. Guidelines about how each grade could be chosen can be described by clicking the Define button. The utilities of grades are determined by both the utilities of the grades of high-level criteria and the propagation rules from lower level criteria to high level criteria.

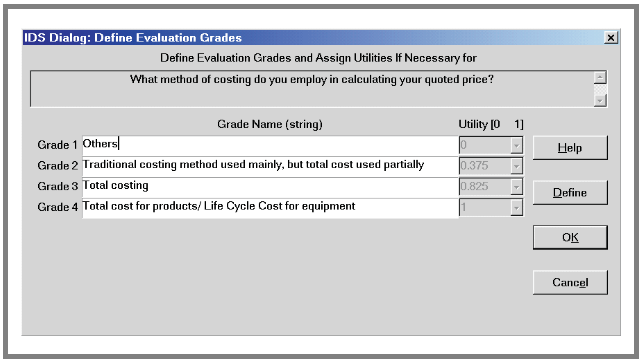

Fig. 3. Define assessment grades using IDS dialog.

A qualitative criterion can be assessed using the grades and a degree of belief to which each grade is assessed. Figure 4 shows an IDS input data dialog window where the user can choose one or more answers with different degrees of belief. The grade definition provides guidelines and/good practices about what a grade actually means, in what circumstances a grade (or answer) should be selected and to what degree a grade could be assessed to. Furthermore, the user can collect evidence to support an assessment and also provide comments on why the assessment is given this way. Such an assessment process is referred to as an evidence-based mapping process, which is designed to improve the objectivity and accuracy of the inherent subjective process. This also provides a structured knowledge base which is easy to access and could be used to support the assessment in subsequent discusses.

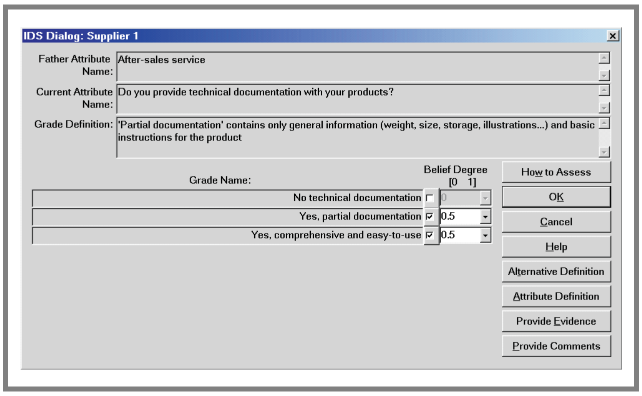

Fig. 4. Enter subjective assessment using IDS dialog.

Quantitative criteria can also be defined and used together with qualitative criteria for assessment. Figure 5 shows the numerical data input window. The best value and the worst value define the range of data that can be entered, which is

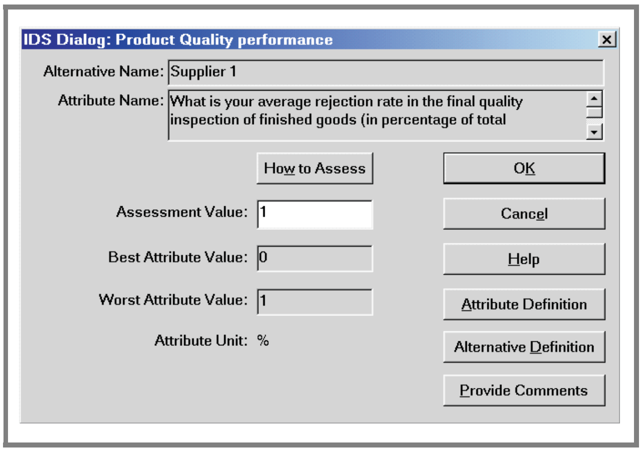

Fig. 5. Enter numerical data using IDS dialog.

defined by the user and between which an assessment figure can be assigned. Random numbers with various probabilities can also be defined and both the possible values and the likelihood can be entered as well, though this model does not contain such criteria.

Apart from screening out poor supply applicants, the main purpose of such assessment includes the identification of strengths and weaknesses of an applicant, which could form a basis for subsequent detailed assessments and for creating action plans to address the weaknesses identified. As such, the concept of the distribution assessment developed in the ER approach would be helpful in identifying strengths and weaknesses. For example, Fig. 6 shows the final distribution assessment for a Siemens supplier "Supplier 1", which provides a panoramic view about the overall

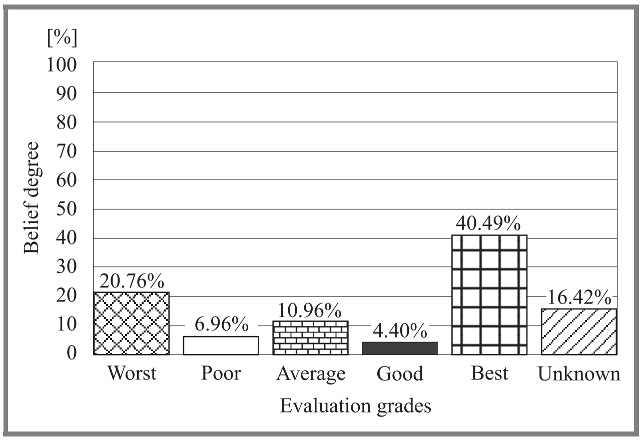

Fig. 6. Distribution assessment generated by IDS.

performance of the supplier in all areas. Clearly, the company has achieved the best performance in many areas, as over 40% of the areas are assessed to be "Best". However, the company does need to improve in nearly 21% of all assessed areas. Also, the company was unable to answer some of the questions put forward by Siemens. In other words, over 16% of the areas need to be further investigated. On the whole, the average percentage score that the company has achieved is just below 60% with a variation between 51% and 57% (Fig. 7). The variation is caused due to the unanswered questions.

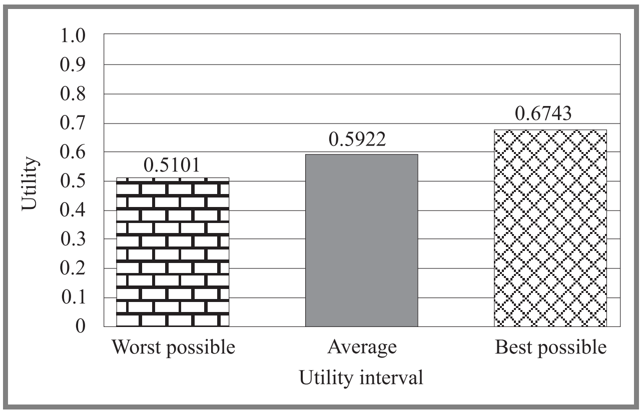

Fig. 7. Average assessment generated by IDS.

Using IDS, the performance distributions of the company on any criterion can be examined in a similar way. This enables Siemens to investigate the areas where the supplier has done well as well as the areas where the supplier has to improve. For example, the company received a zero score or the 100% "Worst" grading on the *product quality performance* criterion because of two problems. The first problem is that "*its average rejection rate in the final quality inspection of finished goods in percentage of total production*" is 3% whilst the lowest acceptable rate by Siemens is only 1%. The second problem is that "*its average return rate from customers in percentage of products or services delivered*" is 7% whilst the lowest acceptable rate by Siemens is only 1% as well. Such investigations provide both sides, Siemens on the customer side and Supplier 1 on the supplier side, with a clear objective view about what Supplier 1 needs to improve to achieve the standards required by Siemens.

The managers of Siemens UK and Supplier 1 both took part in the modeling process, the data collection and the result analysis. They are satisfied with the accuracy and objectivity of the investigation conducted using the ER approach supported by the IDS software.

4. Customer quality and service survey analysis

Customer quality and service survey can provide useful information for a company to improve the quality of its services and products. Silcoms is a medium manufacturing company, located in North West England and specialised in supplying components to aerospace industry among other businesses. The company faces tough competition from overseas in particular Asian companies which can supply cheap products. The management of Silcoms are aware of the competition and are totally committed to improving quality not only for the products they manufacture but also for the services they provide. The company has been given quality awards by the Excellence North West of England. The authors have collaborated closely with the company management and have been very much impressed by their desire to improve their products and services, which have already achieved high standards.

The company, together with the help of external consultants and academics including the authors, has developed a model for conducting quality and service survey among its customers. Figure 8 shows the model structure having four major areas each of which is addressed using a number of questions. To facilitate data collection, the answers to the questions adopt a five-grade scale.

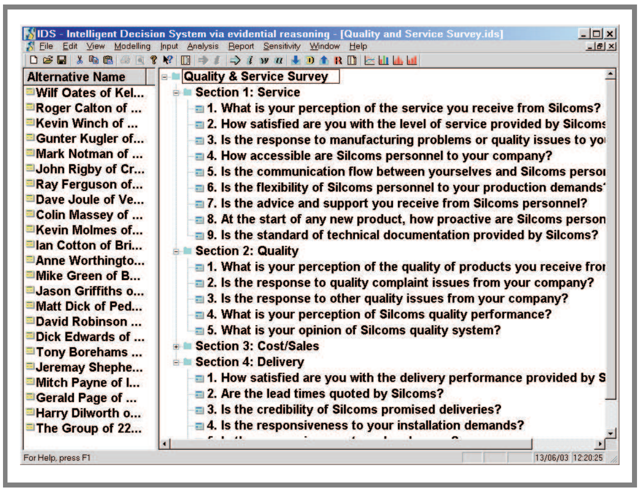

Fig. 8. Questions numbered in four major sections.

Data were collected using a paper version of the model which was nicely bound together and individually sent to each customer. Figure 9 shows a typical answer window and no definition for the grades is provided as the question (criterion) and the answers (grades) are regarded to be straightforward. The customers often chose one answer and occasionally opted to not answer some questions either because the questions are irrelevant to their companies or there may be a lack of information.

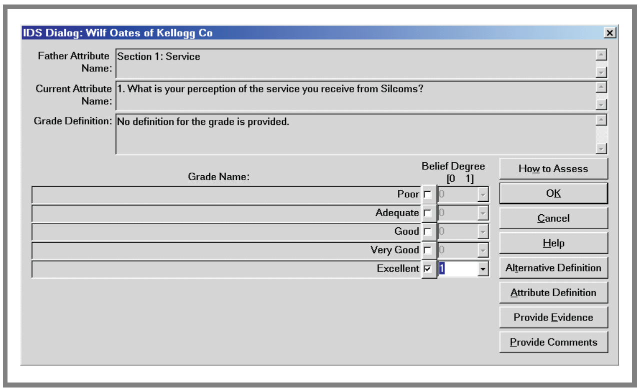

Fig. 9. Original answer provided by a customer.

In Fig. 8, the last alternative *The group of 22 customers* was generated by averaging the answers given by the 22 customers. In Fig. 10, the belief degree assigned to an answer was therefore the percentage of the 22 customers who had chosen this answer. IDS provides a function to combine the original answers provided by individual customers for group analysis. In Fig. 10 it is clear that nearly 80% of the customers have graded their perception of Silcoms service to be *Very good* or *Excellent*, which is an impressive result, considering that most customers were randomly selected with two known "critical" customers chosen deliberately.

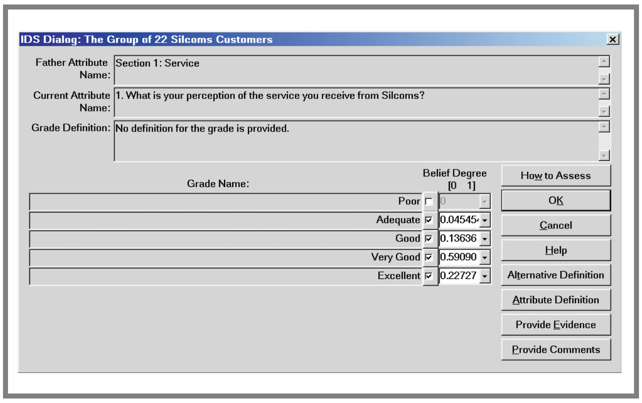

Fig. 10. Degrees of belief assigned by a group of customers.

The IDS provides a range of functions to support the analysis of such surveys, including the analyses of the individual customers' responses on any criteria and the comparison of results provided by the customers. Different groups of the customers can also be combined to show the collective opinions of these groups on any criteria. For example, Fig. 11 shows the collective assessment of the 22 customers on the quality and services provided by Silcoms. The distribution assessment shown in Fig. 11 provides a holistic

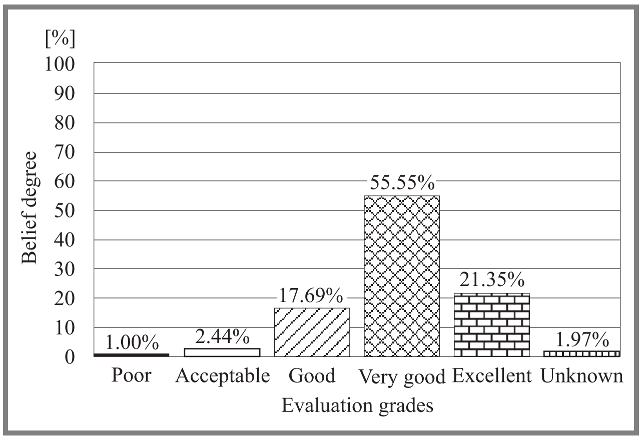

Fig. 11. Collective assessment of Silcoms quality and services.

view of the overall performances of Silcoms. The majority of the customers graded Silcoms at the *Very good* and *Excellent* grades in most areas with the combined belief degree of over 76%. This is a very good result for Silcoms, supporting the company's policy of placing services and quality in the first priority of their policies and strategies. However, there are a couple of customers who did provide critical assessment in some areas, which is clearly displayed. Unlike an average score, this panoramic view will not hide any unsatisfied areas for the good average assessment, thereby preventing the company from missing the opportunity of further improvement.

5. Conclusions

In this paper, the evidential reasoning approach and the intelligent decision system were briefly introduced. Their applications to supplier assessments and the customer surveys of quality and service for two companies in the North West England were reported in some detail. The main feature of this kind of decision problems is that both quantitative and qualitative assessments are included and need to be treated both simultaneously and rationally. Using conventional decision methods, one may need to provide precise number for each assessment, which could be difficult from time to time. Also, assumptions may need to be made in cases where there are missing data or other uncertainties. Traditional methods may only be able to generate average numbers, where bad performances may be averaged out by good performance thereby missing opportunities to identify areas for improvement, which is indeed the very purpose of conducting such assessments in most cases. The IDS software provides easy to use functions to build assessment models, organise and manage knowledge, conduct analysis and generate results.

Acknowledgement

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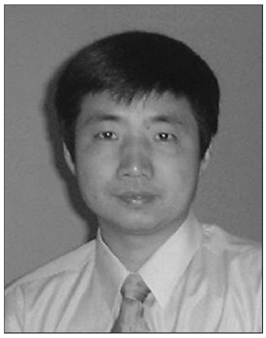

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Paper

Structural representations of unstructured knowledge

Wiesław Traczyk

Abstract—Knowledge should be represented in a formal, structured manner if we want to process and manage it. Unfortunately a source knowledge presented in many documents has informal, unstructured shape. The goal of these considerations is to present the methods of translation from the textual, unstructured knowledge to the structured knowledge, preserving textual form.

Keywords— textual knowledge, knowledge representation languages, ontology.

1. Introduction

Knowledge is used in all areas of human activities but there are domains, relevant to computer applications, in which the word *knowledge* is located in the name of a branch. Three more popular such fields, with their links to data, knowledge and human agents, are presented in Fig. 1.

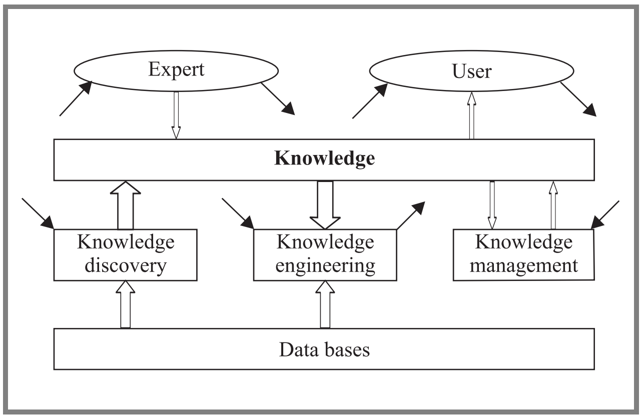

Fig. 1. Knowledge generation, processing and utilization.

Knowledge discovery looks for specific, previously unknown but important, regularities in large data bases. These regularities (or patterns) determine a new knowledge gained from data. Different forms of knowledge obtained from data mining are adapted to the goals and dictated by experts. Usually patterns are described by association rules, decision trees, regression functions and neural networks.

Knowledge engineering constructs knowledge-based systems used for reasoning, intelligent search and decision support. Knowledge, related to particular domain, is built into a system and utilized for data transformation (e.g., from *conditions* to *conclusions*).

Knowledge management consists in organization and facilitation of knowledge generation and utilization, with a main task – increase of institution profit.

In the last two domains a form of primary knowledge can be very diversified. The low and company regulations, institution rules, medical procedures, web pages, fragments of books and other documents written in a natural language are frequently the main sources of knowledge. But, if the knowledge is to be processed by computers, its form has to be converted to a formal shape, with precisely defined syntax and semantics. Then, the proper *knowledge representation* (KR) is needed, easy to obtain from natural language descriptions and easy to understand by computers. General demands for such a representation are usually summarized by *the ontology*: a set of formally specified concepts (such as things or events) and their properties and relations that describe a domain of interest, in order to create an agreed-upon vocabulary for exchanging information.

Since the goal of KR is to express knowledge in computer-tractable form, the two questions should be answered before discussing the possible solutions:

– what does it mean *"knowledge"*?

– what are the demands from *languages* used for knowledge representation?

There is no universally accepted definition of knowledge. Most likely the cause of it lies in very big diversity of the notion. Knowledge (in its intuitive meaning) can be *descriptive* or *procedural*, *explicit* or *tacit*, *qualitative* or *quantitative*, *individual* or *collective*, and so on (even *eastern* or *western*). It is essential to determine the need that the knowledge must fulfill. It can be used for problem solving and decision support, for inference, as an interlingua for cooperating agents or software modules, to define semantics for natural language interpretation, etc.

If the range of knowledge application is limited to knowledge engineering and management, the following definition seems to be appropriate: *knowledge is a special kind of information able to transform the source information to the information in demand.*

In another words – it is a mapping:

$$\text{KNOWLEDGE: } \textit{information} \rightarrow \textit{information}.$$

The definition of *information* can be taken from C. Shannon, the father of information theory: *information is everything that decreases uncertainty.*

It is assumed that the meaning of *uncertainty* is well known.

Knowledge as a means of information transformation is well suited to different domains:

– knowledge of problem solving: *problem* → *solution*,

– knowledge of decision support:
problem + *criteria* → *decision*,

– knowledge of inference: *conditions* → *conclusion*,

– knowledge of search: *demands* → *goal*, etc.

Source and resulting information can be interpreted as input and output data, and knowledge – as a program used for data processing. Such a program should be described by specialized languages of knowledge representation.
Demands for representation language depends on the layer of processing [1, 2].

- The *domain layer* concerns declarative knowledge about the domain of application. Such knowledge describes the objects of discourse in a particular domain, facts that hold about these objects, and relationships among them. This type of knowledge is often represented by concepts, relations, hierarchies, properties, rules, etc. A crucial property of this category of knowledge is that it is represented as much as possible independently from how it will be used.

  The domain layer describes the formal model of a domain.
- The *inference layer* describes the roles of domain expressions and specifies how these expressions are to be used in the inference steps. One can say that the inference layer is a meta-layer of the domain layer.
- The *task layer* enforces control over the inference steps specified at the inference layer. Here it is decided in which order the inferences should be executed. The procedural knowledge is used on this layer and is concerned with sequences, actions, iterations, etc.

In this hierarchy of layers only the domain layer formally describes a particular domain of application and should be adapted to textual form of knowledge sources. That is why only languages of the first layer will be considered below.

Knowledge has many forms that have to be represented by appropriate languages but the most important and frequently used are three groups of demanded dependencies between elements of representation.

- *Logical dependencies* between *statements* with values TRUE or FALSE determine principles for rule based reasoning, the most popular method of inference. The long history of logics and its established position as a tool for formal analysis help to solve numerous problems.
- *Hierarchical dependencies* organize *objects* of a domain into class taxonomy. The ability to represent the relationships between an object and its class or between a class and its superclass has proven to be very useful in many applications. In some cases properties of a class are passed on to its subclasses, simplifying representation.
- *Relational dependencies* between *concepts* represent connections linking together either concrete or abstract elements of the domain. This can help in searching for complex dependencies between given concepts.

There are numerous languages describing some or all of these dependencies. Short presentation of selected, representative examples will help in farther considerations.

2. Languages for knowledge representation

A common problem during the development of ontologies and languages is a range of their applications. It is more costly to create representations that will be reusable across multiple domains than it is to create a language that is suitable for just one application. Languages described below attempt to be more or less universal. The original terminology from the source papers is preserved.

(ML)². A formal language (ML)² [2] has been developed for the representation of KADS [1] models of expertise. It uses an extended first order predicate calculus. Concepts are represented by constants, i.e., nameable and distinguishable entities. All constants and variables have associated sorts (types, classes), organized in a subsort hierarchy. Predicates and functions are also typed. The relation between constants and sorts is the equivalent of the IS-A relation. A knowledge base is divided into *theories*, consisting of import relations, signature (sorts, constants, models of functions and predicates), variables and axioms (instantiations of predicates, production rules and functions).
The main ideas can be illustrated on the following example:

theory carFailure
    signature
        sorts number (vehicle (bus car (station-car limousine))); *the subsorts tree*
        constants
            myCar, yourCar : car;
            bus-412 : bus;
        functions
            noOfWheels : vehicle → number;
        predicates
            sameBrand : car × car;
            greater : number × number;
    variables a, b, c : number;
    axioms
        same Brand(myCar, yourCar);
        greater(a, b) ∧ greater(b, c) → greater(a, c);
        noOfWheels(yourCar) = 4;
endtheory

Telos. A hybrid language Telos [3] supports three different representation formats: a logical, a graphical (semantic network with relations) and a frame representation. A Telos knowledge base is a finite set of interrelated *propositions* presented as vectors: $\langle oid, x, l, y, tt\rangle$. Their meaning is as follows:

> The object x has a relationships called l to the object y. This relationship has identifier *oid* and is believed by the system for the time tt.

A standard way to describe objects together with their classes, subclasses and attributes is the frame syntax. It groups the labels of propositions around the label of common source. If $l \in \{$**instanceof, *isa, ...*$\}$ then the triplet $\langle x, l, y\rangle$ is interpreted as $\langle object, relation, class\rangle$. All other propositions describe aggregates $\langle object, attribute, value\rangle$.

Logical operations are performed on such aggregates. There is also a natural interpretation of a set of propositions as a directed graph (semantic network).

CycL. It is a formal language [4] whose syntax derives from first-order predicate calculus and from Lisp. The vocabulary of CycL consists of *terms*, combined into meaningful *expressions (sentences)*. Constants are terms that can denote collections of other concepts such as individual things, classes, predicates and functions. Variables (e.g., ?X) stand for constants whose identities are not specified. Sentences have the structure of a Lisp list and consist of objects: predicates and their arguments, logical connectives and quantifiers. Predicates describe taxonomy relations (e.g., *isa*), attribute names and many other relations or properties.
The following is a example of CycL expression:

```
(implies
   (isa ?A Animal)
   (thereExists ?M
      (and (mother ?A ?M)
           (isa ?M FemaleAnimal))))
```

ICONS. The intelligent content management system (ICONS) [5] describes an ontology by *the data model*, that allows to depict the "world", and *the knowledge representation language*, through which one can extend the entities to the schema, infer new knowledge and make some reasoning. The data model contains a set of class names, described by attribute-class pairs and linked to another class name by the *isa* or *partOf* relation, for example:

> *student(name : string, age : integer, enrol : faculty) isa person.*

The language contains appropriate atoms (facts) with the concrete values:

> *student(name : peter, age : 21, enrol : cs).*

Internal class names can be substituted by the full atoms, giving a sequence of dependencies. In production rules with such atoms disjunctions are allowed in the rules' heads (so called disjunction logic programming), for modeling incomplete knowledge. ICONS introduces also reasoning under uncertainty (using Dempster-Shafer theory) and XML as a serial syntax definition language.

F-Logic. In a deductive, object oriented language F-Logic [6, 7] *objects* are organized in *classes* and *methods* represent relationships between objects. Facts are collections of objects with classes and appropriate methods, e.g.,

```
peter:student[father->john:man].
```

Rules use similar forms, often with quantifiers and variables:

```
FORALL X,Y  X[son->Y] <-Y:man[father->X].
```

A query to the object base is formulated as follows:

```
FORALL X,Y   X:women[son->Y[father->john].
```

Properties of objects can be described by predicates, objects denoting numbers are processed with comparison and several arithmetic operators, special path expressions are used to navigate through the large sets of objects.
The presented examples of languages for knowledge representation, and many not presented here (e.g., SILO, CLASSIC, ALLNR, Gellish, GOL, EULE, KIF, ALUNI) show that:

- object-based approach is very popular,
- predicates and first-order logic are basic descriptions of relations and properties,
- the forms of description are diversified,
- not all systems use a taxonomy,
- there are no special approaches simplifying conversion from the unstructured textually-represented knowledge to the formal representation.

The next section tries to collect more important properties of languages and to add textually-oriented elements, in order to construct a useful system of representation.

3. The ontology for textual knowledge

3.1. Compositional representations

The main task of these considerations is to find a way for conversion from unstructured, textual knowledge description to structured, formal knowledge representation. Such a structured form enable processing by computers, reuse in many applications and formal validation.
The conversion should not be to complicated and the structured, final representation should be comprehensible. Therefore a textual form of information should be preserved, if possible, through the whole process of conversion.
Representations are easier for processing and more reusable if they are compositionally constructed. A representation is *compositional* if it describes each individual concept in the domain of discourse, and the representation of complex

concepts is obtained by composing representations of individual concepts. The following example illustrates this, starting from the text:

```
The form PIT is intended for tax-payers
that obtain salaries, pensions
or scholarships.
```

The noncompositional representation of this might be

THE FORM PIT IS INTENDED FOR TAX-PAYERS THAT OBTAIN SALARIES, PENSIONS OR SCHOLARSHIPS.

This statement can be interpreted as an individual concept or as a logical *proposition* with the truth value (e.g., TRUE or FALSE).

The simple decomposition gives

"THE FORM PIT IS INTENDED FOR *tax-payer*"
if "*tax-payer* OBTAINS SALARY, PENSION
OR SCHOLARSHIP".

Now the two *propositions with variables* are connected by the logical operator **if**, giving *decision rule* that can be used for *backward reasoning*. This form is equivalent to the logical rule

$$Q(x) \Leftarrow P(x).$$

Another possible compositional representation can be described, using the model

$$Q(x) \Leftarrow P_1(x) \vee P_2(x) \vee P_3(x),$$

by the following two statements:

THE FORM PIT is intended for TAX-PAYERS GROUP,
person is a member of TAX-PAYERS GROUP **if**
[(*person* obtains SALARY) **or** (*person* obtains PENSION)
or (*person* obtains SCHOLARSHIPS)].

The elements of these statements have the special meanings:

THE FORM PIT, SALARY, PENSION, SCHOLARSHIPS – *object names* of the domain,
person – *variable* representing an object,
TAX-PAYERS GROUP – *class name* indicating a class of objects,
is intended for, obtains – *roles* played by objects,
is member of – *relation* between object and class,
if, or – logical *operators*.

This representation of knowledge, contained in the source text, can easily be used for reasoning, formally performed by any agent. Having information (*the fact*):

```
Mr Brown obtains pension
```

and substituting MR BROWN for *person,* one can get the conclusion:

MR BROWN IS A MEMBER OF TAX-PAYERS GROUP

and information that the form PIT is for him.

The second example shows some other elements of structural representation.

```
Wanted is employee from Warsaw, 20 to 30
years old, with MCS degree.
```

The compositional description of this is as follows:

"WANTED EMPLOYEE" [address: WARSAW, age: 20-30, degree: MCS].

This structure contains *object name* and its *properties* in the form of *attribute-name: attribute-value* pairs. It can be considered as a special case (or *instantiation*) of a more general description:

EMPLOYEE [address: TOWN, age: NUMERICAL INTERVAL, degree: DEGREES].

Now the properties of a *class* are presented as *attribute-name: attribute-type* pairs, showing some restrictions for attributes. The object "WANTED EMPLOYEE" *inherits* attribute names and types from the class EMPLOYEE.
The inheritance is sometimes continued, involving more general classes. For example the class EMPLOYEE may be considered as a subclass of the superclass PERSONS, inheriting wanted attributes: general (e.g., nationality: COUNTRY or concrete (e.g., number of legs: 2).
Presented examples show some possible structures, applicable to a knowledge in textual form. To be useful these structures should be described more formally.

3.2. Principles of structuring

Basic elements. Knowledge description language, as all other languages, has many levels of a syntax. The lower one (neglecting numerals and letters) usually contains numbers and strings, but here we will introduce more specialized elements:

- *numbers*;
- *symbols* – like $x, P_1(x), \leq$, USA – used in expressions, as abbreviation or shortening;
- *names* – e.g., employee, Mr Brown – short descriptions, more informative than symbols;
- *texts* – as "MR BROWN OBTAINS PENSION" – easy to understand descriptions of concepts, comments, source knowledge, etc., of special importance in this paper.

Symbol, name and text can represent *constant, variable, relation* and *function*.

Statements. Textual variable with a value from the set of truth values will be called *proposition*. Textual function with such values is known as *propositional function*. Sometimes propositions describe relations, but it is easier to represent relations between constants, variables and functions using conventional symbols ($<, \leq, \subset, \ldots$) or names (*lower, cheaper*).

Some propositions have internal structure enabling shorter and more elastic representation. For example the same meaning as the proposition

MR BROWN HAS VERY HIGH INCOME

has the triplet ⟨MR BROWN, *income*, "very high"⟩

with a general structure

⟨ *object-name*[1], *attribute-name, attribute-value* ⟩.

Such an *aggregate* has truth value (as a proposition), its components can exist as variables, and the form can be modified.

A truth value is a common property of proposition, relation and aggregate, which together will be called *statements*. They constitute basic elements of *logical expressions*: statements connected by logical operators.

Frames. *An object name* (or shortly *object-n*) denotes an element of the domain, physical (as BUILDING) or abstract (as PROGRESS). Properties of an object are described individually in *aggregates* or collectively in *frames*. A simple *object-frame* contains information about an individual concept and has a structure:

⟨*object-n* [*attr-name*$_1$: *attr-value*$_1$,..., *attr-name*$_k$: *attr-value*$_k$]⟩

and all components defined as constant names and values. These names and values belong to certain families, that should be defined.

The family of objects with common properties is called *class* and is described by a *class-frame* (or *prototype frame*) with a structure as follows:

⟨*class-n* [*attr-name*$_1$: *attr-type*$_1$,..., *attr-name*$_k$: *attr-type*$_k$]⟩.

The *type* of attribute denotes a category of its values as:

– the set of concrete values, e.g., 5-9, Mon.-Fri., {white, red}, 2004;

– the class of values, e.g., CITIES, "ALL EVEN NUMBERS", strings.

An object-frame is *instance-of* a class-frame with the same set of attribute names, what is indicated by the structure:

⟨*object* [...] ⟩ *instance-of class*.

Usually each class is a *subclass* of one or more classes higher up in the hierarchy, called its *superclass(es)*. The relations between hierarchically organized classes, showing specialization and generalization, can be presented in the structure:

⟨ *class-a* [...] ⟩ ρ *class-b*

with $\rho \in$ {*is-a, part-of, member-of, ...*}.

[1]In all cases described below the word *text* can be used instead of *name*.

Relation *is-a* causes *inheritance* of all properties from superclass to subclass. In the case of *part-of* relation only some special attributes are inherited (e.g., possession).

Properties of class-frames sometimes include also additional information concerning attributes (units of measure, procedures used for value calculation, etc.).

Dependencies. In many cases objects and classes are related, but not organized in taxonomy. Such "horizontal" relations are known as *roles* and described in the following form, called *dependence*:

⟨*object-a - role - object-b* ⟩.

For example, if A. Smith is a manager of B. White then the appropriate dependence will be:

⟨B. WHITE – manager – A. SMITH ⟩.

Dependencies describe relations, therefore have also truth values and should be included in the set of statements.

Dependencies can be joined in *path expressions* in the form:

⟨*object-a - role-k - object-b - role-l - ... - object-z* ⟩,

as in the example:

⟨B. WHITE – manger – A. SMITH – english-teacher – C. BLACK ⟩.

The set of path expressions with some common objects describes a graph of dependencies, equivalent (if names of objects and roles are properly chosen) to *semantic net* or *conceptual graph*, typical tools of knowledge representation.

If dependencies describing the domain are stored in the knowledge base, path expression with a variable (in the place of an object or a role) can be used for navigation and searching for appropriate value.

Objects and roles in dependencies are presented as names or texts, directly describing some parts of a source document.

Rules. The *logical expression* contains statements connected by logical operators **and, or, not**. The special case of it – *and-expression* – contains only **and** operator. *Production rules* are described by one of the following forms:

if *logical-expression* **then** *and-expression*
if *logical-expression* **then** *and-expression-A* **else** *and-expression-B*
and-expression **if** *logical-expression*.

Different types of a statement (proposition, relation, aggregate, dependence) help in the rule adaptation to various applications.

4. Conclusion

Since statements obtained from the source, textual knowledge have also textual form, then frames, dependencies and rules, constructed from statements, contain structured texts as well, according to the goal of these considerations.

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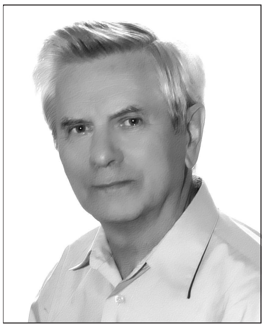

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Paper

Event mining based on observations of the system

Janusz Granat

Abstract—Event mining is becoming a challenging area of research. Events in system analysis is not a new concept. It has been used in Petri nets, stochastic modeling, etc. However, there are new opportunities that come from the lage amount of data that is stored in various databases. In this paper we will focus on formulating the event mining tasks that consider observations of the system as well as internal and external events.

Keywords— event mining, temporal data mining, telecommunications.

1. Introduction

Data mining have many industrial and scientific applications. However, the existing algorithms consider limited information about the events. Recently, an increased importance of events in modeling and understanding complex systems can be observed. D. Luckham [5] provides us with a framework for thinking about complex events and for designing systems that use such events (see also [6]). The event mining is new and challenging area of research and applications. Events are especially challenging for real-time analysis. Gartner Inc., a technology research and advisory firm, defines real-time as the complete compression of lag between the detection of an event, the reporting of that event, the decision-making, and the response. They further observe that the real-time enterprise (RTE) is an enterprise that competes by using up-to-date information to progressively remove delays to the management and execution of its critical business processes. Therefore, real-time computing might be the focal point of IT departments because it allows companies to provide on-line information for effective decision making.

A key to understanding events is knowing what caused them and having that knowledge at the time the events happen. Another issue is the knowledge about the consequences of events. The ability to track event and consequences is an essential step toward on-line decision support and an important challenge for new algorithms for event mining.

Many existing enterprise systems are distributed and event-driven. Events might be described by structured and unstructured information. The structured information is well recognized and is stored in databases. However, the organizations are working on improvement of the analysis of the external environment and influence of this environment on the performance of the organization. Environmental scanning is a new term and it means the acquisition and use of the information about events, trends, and relationships in an external environment. Therefore, the methods of dealing with unstructured information about events are especially important. The example of an event detection from online news documents is presented in [7].

Event mining might have various applications. On the business level it is the business activity monitoring (BAM). BAM is defined (by Gartner Inc.) as a concept that provides a real-time access to critical business performance indicators to improve the speed and effectiveness of business operations. BAM involves alerts, triggers, sensors, and agents that determine a transaction or event that is meaningful. Another group of applications are on the level of the network infrastructure of the company. Computer networks produce a large amount of event-based data that can be collected for network analysis. These data include alerts from firewalls and intrusion detection systems (IDS), log files of various software systems, routing information from the Internet and so on. An example of use of the concept of events for network analysis can be found in [4].

2. Basic definitions

We have a system which is influenced by internal and external events and the behavior of the system can be monitored (see Fig. 1).

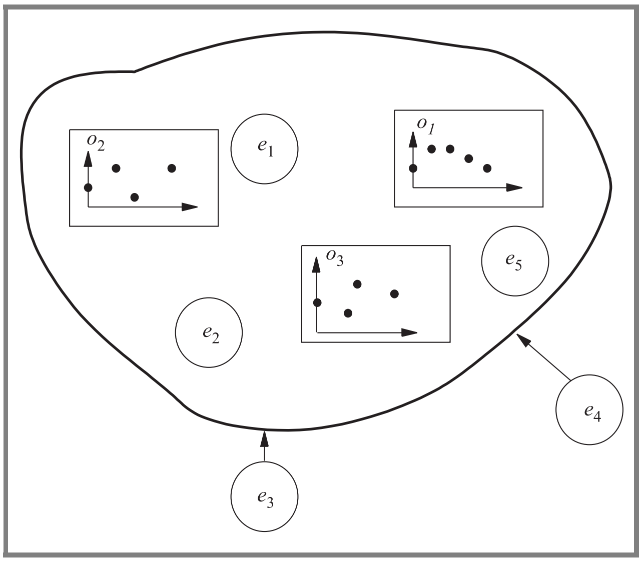

Fig. 1. The events and observations.

We define here the basic terms:

– an event,

– observations,

– observations pattern,

– event mining models.

2.1. An event

Definition 1: An event e_i is something that hapen in the system or its enviroment and can be described by a set of parameters.
Let us consider the finite set of events

$$\mathcal{E} = \{e_1, e_2, \ldots, e_n\}.$$

We can distinguish past and future events:
The set of past events:

$$\mathcal{E}_p = \{e_1, e_2, \ldots, e_e\}.$$

The set of future events:

$$\mathcal{E}_f = \{e_{e+1}, e_2, \ldots, e_n\},$$

$$\mathcal{E} = \mathcal{E}_p \cup \mathcal{E}_f.$$

A formal definition of the event.

Definition 2: Let $A_{e_i} = \{a_{1,e_i}, a_{2,e_i} \ldots, a_{m,e_i}\}$ be the set of attributes for an event e_i and $V_{a_{j,e_i}}$ be the domain of attribute $a_{j,e_i} \in A_{e_i}$. An event is defined as $(m+2)-$ tuple $(a_{1,e_i}, a_{2,e_i} \ldots, a_{m_{e_i},e_i}, t, \Delta t)$, where $a_{j,e_i} \in V_{a_{j,e_i}}$, t is the time of occurence of the event, Δt is the duration of the event.

If $\Delta t = 0$ then then we have a point event and for $\Delta t \neq 0$ we have an interval event. Each event might have different number of attributes and attributes of different types (numerical, textual, etc.) Attributes for interval event might change over time. We assume in this paper that attributes do not change at the time interval Δt.

2.2. Observations

Events might influence the behavior of the system.
An information system $\mathcal{O}$ of a set of observations can be defined as follows:

$$\mathcal{O} = (O, V, \rho, T, R), \quad (1)$$

where: T – is a nonempty set whose elements t are called moments of time, R – is an order on the set T (here we assume linear order), O – is finite and nonempty set of observations, $V = \bigcup_{o \in O} V_o$, V_o is the set of values of observation $o \in O$, called the domain of o, ρ – is an information function: $\rho : O \times T \rightarrow V$.

We assume that we will have the set of observations:

$$\mathbf{O} = (o_1, o_2, \ldots, o_l), \quad (2)$$

$$T = (t_1, t_2, \ldots, t_n).$$

For simplicity instead of $\rho(o_i, t)$ we will use $o_{i,t}$.

2.3. Observations pattern

Observations pattern p_i is a distinguishable sequence of observations:

$$(\hat{o}_1(t), \hat{o}_2(t), \ldots, \hat{o}_n(t)) \Rightarrow p_i \quad t \in \Delta t.$$

Pattern p_i might be described by a set of parameters, etc.
We can have set of patterns:

$$\mathcal{P} = (p_1, p_2, \ldots, p_p).$$

2.4. Event mining models

There are various tasks of modeling that consider events. In this paper we will focus on relations of events and changes of observations (Fig. 2).

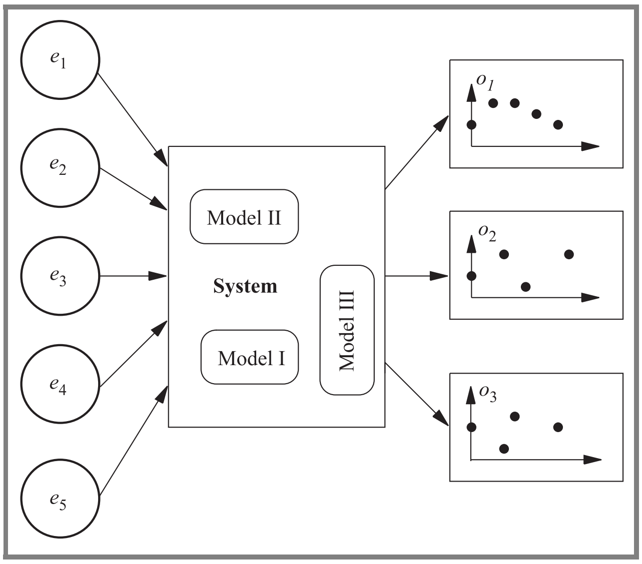

Fig. 2. Relations: system, models, events, observations.

The objectives of modeling:

1. For significant changes of observations find events that are the reasons of these changes

   if *change_detection_after_event*$(\mathbf{O}, w_s)$ **then** *the reasons are the events:* $e_1, e_2, \ldots, e_k \in \mathcal{E}_p$,

   w_s is an observation window after the event that occure at time t_i.

2. Prediction of future events by analysing the changes of observations

   if *change_before_event*$(\mathbf{O}, w_p)$ **then** *there is a probability of future events* $e_1, e_2, \ldots, e_k \in \mathcal{E}_f$,

   w_p is an observation window before the event that occure at time t_i.

3. Prediction of changes of observations after the event occurs

   if $e_1, e_2, \ldots, e_k \in \mathcal{E}_p$ **then** *there will occure pattern* p_i *of changes of observations* $\mathbf{O}$.

The Table 1 have a colum "events", where there is an information about a set of events E_{t_i} that occure at time t_i. Sometimes it is difficult to dettermine the exact time of event. In this paper we will not consider such cases.

Table 1
Event mining data

Time	Events	Observations			
t_1	$\emptyset$	o_{1,t_1}	o_{2,t_1}	$\ldots$	o_{n,t_1}
t_2	$\emptyset$	o_{1,t_2}	o_{2,t_2}	$\ldots$	o_{n,t_2}
t_3	E_{t_3}	o_{1,t_3}	o_{2,t_3}	$\ldots$	o_{n,t_3}
$\ldots$	$\ldots$	$\ldots$	$\ldots$	$\ldots$	$\ldots$
E_{t_i} – the set of events that occure at time t_i					

3. The faults of the network

In this section we will present a simple illustrative example. We are considering a small network with several ATM switches [2] (see Fig. 3). We have two observations of the network o_1 (Fig. 4) and o_2 (Fig. 5) and we know that two events (faults of one connection) occured at time 15:01:30 and 15:19:45. The data is shown in Table 2.

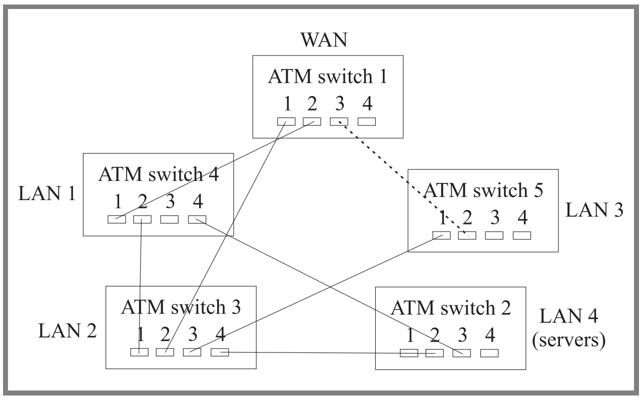

Fig. 3. The network.

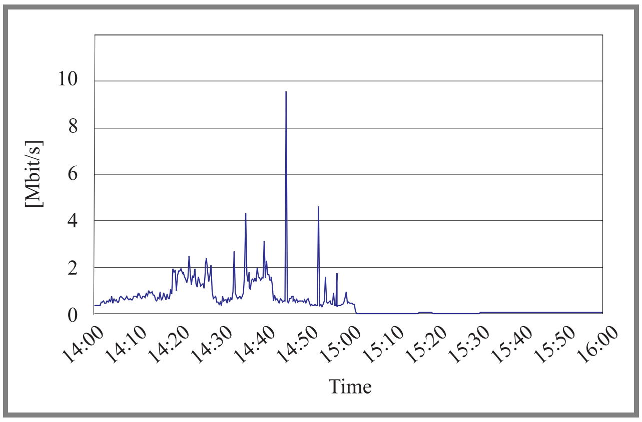

Fig. 4. The observation o_1.

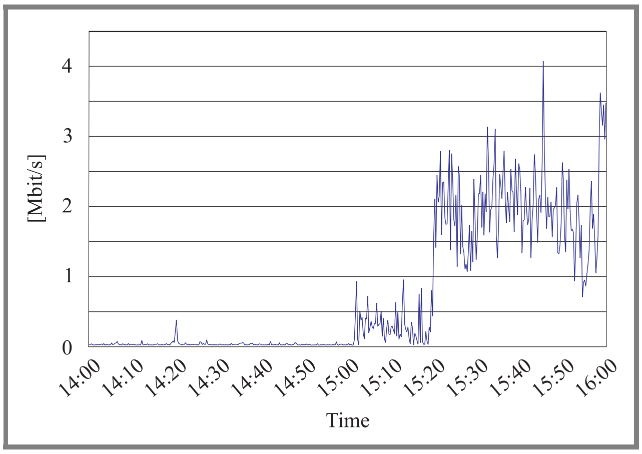

Fig. 5. The observation o_2.

Table 2
Event mining table (training data)

Time	Events	o_1	o_2
$\ldots$	$\ldots$	$\ldots$	$\ldots$
15:01:30	e_1	419 103	94 570
15:01:45	$\emptyset$	0	433 335
15:02:00	$\emptyset$	0	931 090
$\ldots$	$\ldots$	$\ldots$	$\ldots$
15:19:45	e_2	23 783	1 563 489
15:20:00	$\emptyset$	20 038	2 108 248
$\ldots$	$\ldots$	$\ldots$	$\ldots$

We have the following two apriori known events:

$e_1 = (t = 15{:}01{:}30,\ a =$ "fault of the connection of port 3 (switch 1) to port 2 (switch 5)"),

$e_2 = (t = 15{:}15{:}45,\ a =$ "fault of the connection of port 1 (switch 4) to port 2 (switch 1)").

We can observed significant changes of observations after the occurence of events. These changes can be easily detected by statistical algorithms (see [1, 3]). However, the correlation of patterns of changes of observations and events requires an representative training set of cases of faults and set of observation data that fully cover behavior of the system.

4. Conclusions

The observations of the system give an important information about the internal state of the system. However, finding the relations between observations of the system and internal or external events gives the possibility of finding the reasons of changing the behavior of the system. This paper shows mathematical formulation of event mining tasks basd on the set of observation of the system.

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Paper

Time series denoising with wavelet transform

Bartosz Kozłowski

Abstract—This paper concerns the possibilities of applying wavelet analysis to discovering and reducing distortions occurring in time series. Wavelet analysis basics are briefly reviewed. WaveShrink method including three most common shrinking variants (hard, soft, and non-negative garrote shrinkage functions) is described. Another wavelet-based filtering method, with parameters depending on the length of wavelets, is introduced. Sample results of filtering follow the descriptions of both methods. Additionally the results of the use of both filtering methods are compared. Examples in this paper deal only with the simplest "mother" wavelet function – Haar basic wavelet function.

Keywords— wavelet transform, WaveShrink, filtration, noise reduction, Haar basic wavelet function.

1. Introduction

Foundations of wavelet-based analysis methods were laid in the beginning of the 20th century. Back then, in the year 1909 Hungarian mathematician Alfred Haar introduced his two-state function in appendix to his doctoral thesis published later on [3]. Today a slightly modified version of this function is regarded as the first basic wavelet function. In the nineties of the 20th century a very swift developement of wavelet enforced methods began. Wavelets turned out to be very useful when applied to many problems, including analysis and synthesis of time series [9] (in acoustics, geology, filtration and forecasting [5, 11] in meteorology and economics), effective data storage, especially images [10] (computer graphics, image animation in movie industry). Lately a very fast development of wavelet-based data mining [6] techniques may be observed.

One of the tasks of knowledge discovery preprocessing is noise reduction. The goal of this sub process is to separate the noise from the signal and then to reduce or remove the noise. The definition of a noise stays imprecise because for some physical processes it is difficult to clearly define it. In this paper it is assumed that the goal of filtering is to change the input signal in such a manner that the values of a time series, which differ a lot from others, are changed but the characteristic of signal stays the way it was. Most commonly used methods consider statistical approach or involve Fourier transform. In this paper it will be shown why wavelet transform may be considered valuable for this task.

The paper is organized as follows. Section 2 presents absolute basics of wavelet analysis, including basic wavelet functions and discrete wavelet transform. Section 3 reviews WaveShrink methodology and presents results obtained by its application to the example data. In Section 4 the reason for searching for another method is clarified. A genuine method, also based on wavelets, is introduced. The results of application of the new methodology to the same example data are presented and compared with WaveShrink's results. Section 5 concludes the paper.

2. Wavelet theoretical background

Wavelets are functions which result in values different than zero in a relatively short interval. In this regard they differ from "normal", long waves, such as sinusoidal ones, which are determined on a whole time domain $(-\infty,\infty)$.

Let ψ be a real function of a real variable u which satisfies two conditions:

$$\int_{-\infty}^{\infty} \psi(u)du = 0 \quad (1)$$

and

$$\int_{-\infty}^{\infty} \psi^2(u)du = 1. \quad (2)$$

Condition (2) means that for any ε from an interval $(0,1)$ there is an interval $(-T,T)$ such that:

$$\int_{-T}^{T} \psi^2(u)du = 1-\varepsilon.$$

If ε is close to 0, it may be seen that only in an interval $(-T,T)$ corresponding to this ε values $\psi(u)$ are different than 0. Outside of this interval they must equal 0. Interval $(-T,T)$ is small compared to an interval $(-\infty,\infty)$, on which a whole function is determined. Condition (1) implies that if $\psi(u)$ has some positive values, it also has to have some negative ones (a function "waves"). Therefore Eqs. (1) and (2) introduce a concept of a small wave, or shorter, wavelet. If Haar function ϕ, which is a two-state function of real variable (Fig. 1a):

$$\phi(u) = \begin{cases} -1 & \text{for} \quad -1 < u \le 0 \\ 1 & \text{for} \quad 0 < u \le 1 \\ 0 & \text{for} \quad \text{other u} \end{cases}$$

would be transformed into:

$$\psi^{(H)}(u) = \begin{cases} -\frac{1}{\sqrt{2}} & \text{for} \quad -1 < u \le 0 \\ \frac{1}{\sqrt{2}} & \text{for} \quad 0 < u \le 1 \\ 0 & \text{for} \quad \text{other u} \end{cases}$$

then the resulting function $\psi^{(H)}$ satisfies conditions (1) and (2), and is called Haar basic wavelet function (Fig. 1b).

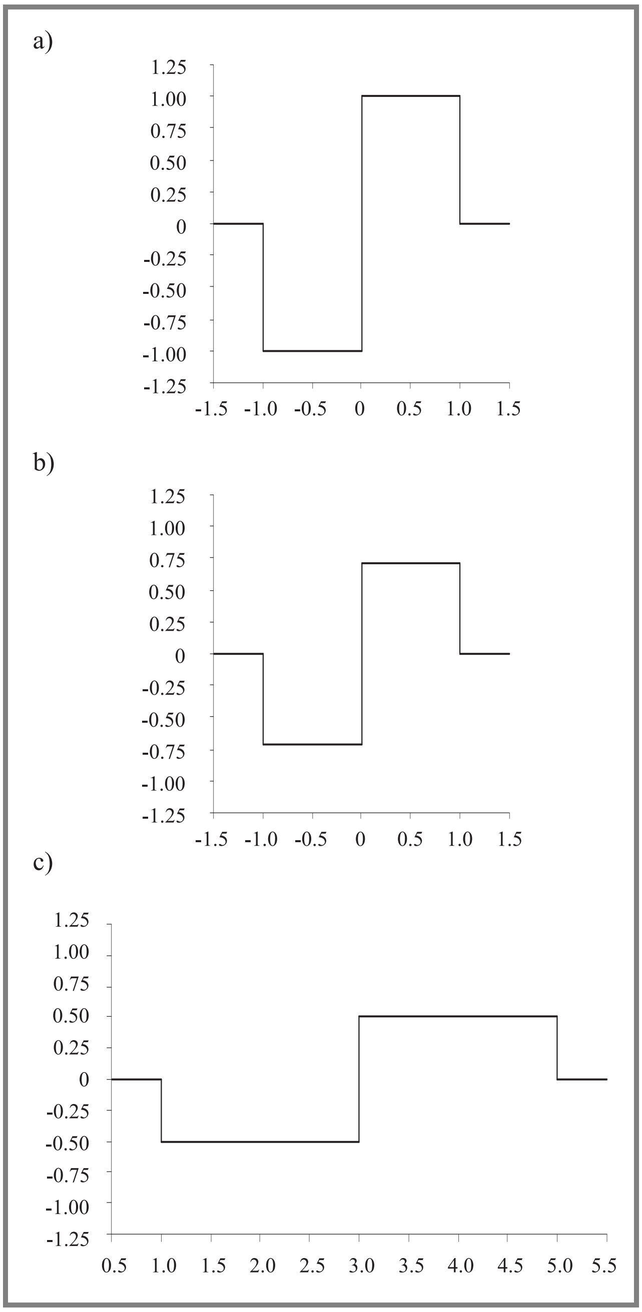

Fig. 1. Haar function (a), modified Haar function – the first basic wavelet function (b), and wavelet-based on Haar basic wavelet function with $\lambda = 2$ and $t = 3$ (c).

Introducing two parameters, namely scale (λ) and location (t) into the above definition of Haar basic wavelet function we get a family of scaled and transformed wavelets:

$$\psi^{(H)}_{\lambda,t}(u) = \begin{cases} -\frac{1}{\sqrt{2\lambda}} & \text{for} \quad t-\lambda < u \le t \\ \frac{1}{\sqrt{2\lambda}} & \text{for} \quad t < u \le t+\lambda \\ 0 & \text{for} \quad \text{other u} \end{cases}$$

as one on Fig. 1c, based on $\psi^{(H)}$. Figure 1b considers wavelets with scale λ equal to 1 ($\lambda = 1$) and is defined in point t equal to 0 ($t = 0$).

Wavelets $\psi^{(H)}_{\lambda,t}(u)$ may be established based on $\psi^{(H)}(u)$ according to the formula:

$$\psi^{(H)}_{\lambda,t}(u) \equiv \frac{1}{\sqrt{\lambda}}\psi^{(H)}(\frac{u-t}{\lambda}).$$

There are of course many other basic wavelet functions such as Mexican hat, Gauss wavelets, Morlet wavelet [8], family of Daubechies wavelets [1] just to name a few.
As a result of wavelet transform we obtain a set of wavelet coefficients calculated at different levels (scales) and in a wide range of observation points (locations). There are many ways of doing this. Two most often applied are (orthonormal) discrete wavelet transform (DWT) and its slightly modified version which preserves scales but calculates wavelets in more densely chosen observation points. For an DWT Mallat [7] proposed a very fast algorithm for calculating wavelets. With minor modifications it may be applied to other wavelet transforms. An important feature of DWT is that it may be reversed. Having wavelet coefficients it is possible to calculate original time series. This possibility is fundamental for a majority of current wavelet applications.

3. WaveShrink

3.1. WaveShrink method

One of the most explored signal smoothening or cutting method utilizing wavelets is Donoho's and Johnstone's [2] WaveShrink method.
The method is composed of three main steps. At the beginning observed time series is transformed into the wavelet space by DWT. In step two wavelet coefficients are modified, reduced according to the selected shrinkage function and a given threshold value. To accomplish this one of three shrinkage functions (presented below) may usually be used to establish how to modify wavelet time series coefficient. In the end inverse DWT is applied on wavelet coefficients and as a result smoothened original signal (with reduced noises) is derived.

3.2. Shrinkage functions

Shrinkage functions are formulas that define a correction coefficient $\delta_\lambda(x)$, which is subtracted from the corresponding wavelet coefficient. Calculated correction is relevant (different from 0) for those wavelet values, which exceed a given threshold parameter λ.

Hard shrinkage function

$$\delta^H_\lambda(x) = \begin{cases} 0 & |x| \le \lambda \\ x & |x| > \lambda \end{cases} . \tag{3}$$

Subtracting this correction reduces those wavelet coefficients of the wavelet time series, which exceed threshold value, to zero.

Soft shrinkage function

$$\delta_{\lambda}^{S}(x) = \begin{cases} 0 & |x| \leq \lambda \\ x - \lambda & x > \lambda \\ \lambda - x & x < -\lambda \end{cases} . \tag{4}$$

By subtracting δ_{λ}^{S} correction considered wavelet time series coefficients are reduced to λ for positive coefficients and to $-\lambda$ for negative ones.

Non-negative garrote shrinkage function

$$\delta_{\lambda}^{NN}(x) = \begin{cases} 0 & |x| \leq \lambda \\ x - \frac{\lambda^2}{x} & |x| > \lambda \end{cases} . \tag{5}$$

Results of δ_{λ}^{NN} function subtracted form wavelet time series coefficients modify them directing to zero in such a manner that the more a coefficient exceeds the threshold value the more it is reduced towards zero.

3.3. Estimating threshold value

The clue of WaveShrink method is to correctly estimate λ parameter. It may be achieved in various ways. Two of those are presented below.

Min-max approach

The first one is so called min-max threshold solution task. It is defined by Eq. (6).

$$\inf_{\lambda} \sup_{\theta} \left\{ \frac{R_{\lambda}(\theta)}{n^{-1} + \min(\theta^2, 1)} \right\} . \tag{6}$$

$R_{\lambda}(\theta)$ is defined as

$$R_{\lambda}(\theta) = E(\delta_{\lambda}(x) - \theta)^2 \tag{7}$$

and

$$x \sim N(\theta, 1).$$

What follows is it is assumed that the signal is of a normal distribution with expected value θ and a standard deviation equal to one. Solving this problem analytically requires, at least, altering signal so that it suits this assumed condition.

Standard deviation approach

In the following example another method was applied. Parameter λ is derived as a value of standard deviation's estimator (calculated for each wavelet level) multiplied by parameter n.

3.4. Illustrative example

Both this example and the example in next section are performed on data (shown in Fig. 2) collected from a network router. It is a traffic time series observed in one of the ports throughout the day.

In Fig. 3 there are the results of filtering using different values of a parameter n supplied for procedure of evaluating λ (one, two, and three) "within" the same shrinkage function (δ_{λ}^{H} – hard shrinkage function).

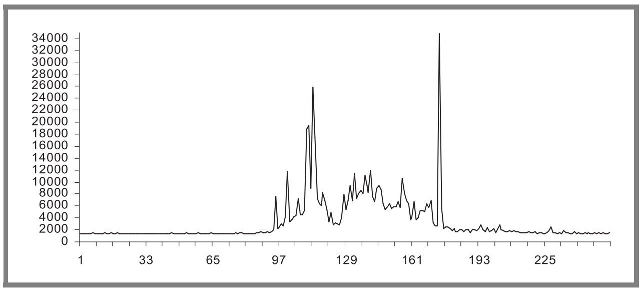

Fig. 2. Original time series.

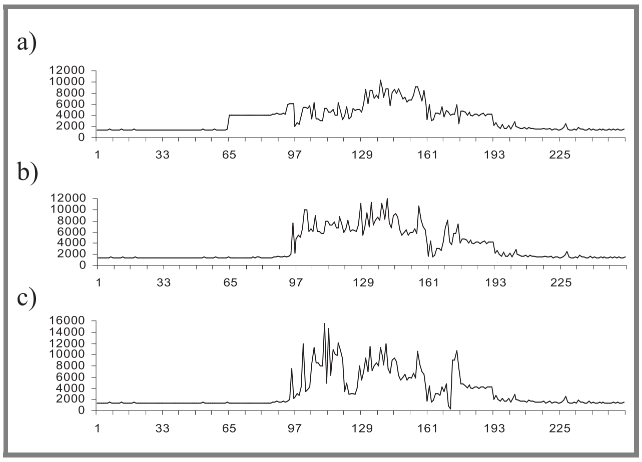

Fig. 3. The result of denoising by WaveShrink with $\delta_{\lambda} = \delta_{\lambda}^{H}$ and λ estimating parameter n equal to one (a), two (b), and three (c), respectively.

Figure 4 is organized in the same way but in this case $\delta_{\lambda} = \delta_{\lambda}^{S}$ – soft shrinkage function is used.

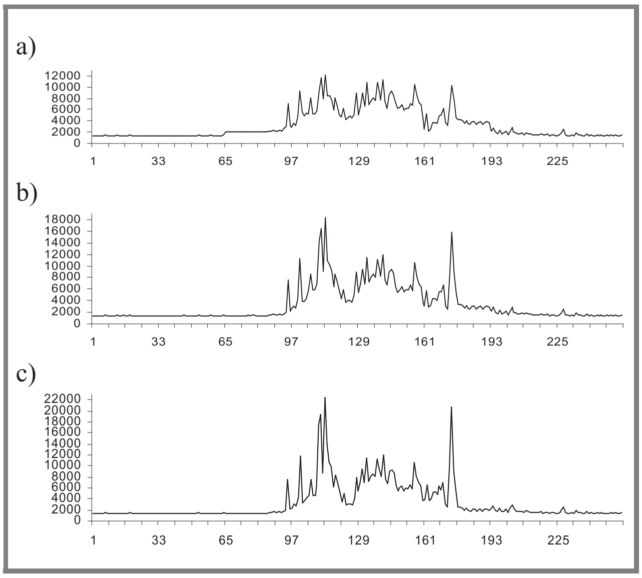

Fig. 4. The result of denoising by WaveShrink with $\delta_{\lambda} = \delta_{\lambda}^{S}$ and λ estimating parameter n equal to one (a), two (b), and three (c), respectively.

And finally Fig. 5 is obtained as a result of application of non-negative garrote shrinkage function δ_λ^{NN}.

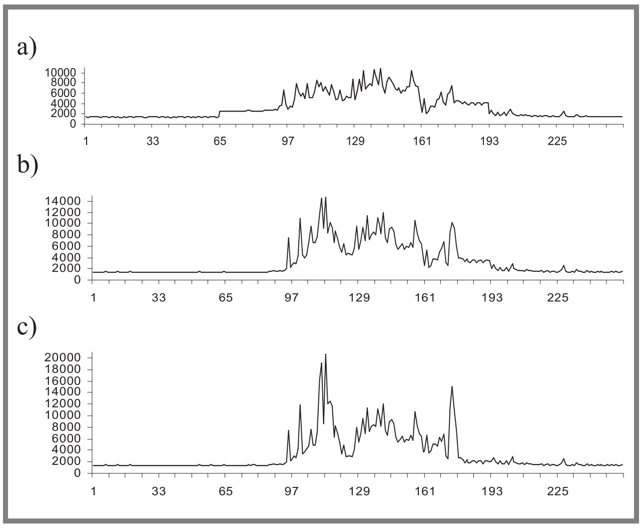

Fig. 5. The result of denoising by WaveShrink with $\delta_\lambda = \delta_\lambda^{NN}$ and λ estimating parameter n equal to one (a), two (b), and three (c), respectively.

One can easily see that for all shrinkage functions the "power of shrinkage" gets weaker as parameter n rises. On the other hand the fit of shrinkaged function to the original time series increases as parameter n rises.

In all cases this algorithm distorts the signal not only in noise spots but also in neighboring observation points. This is an obvious result of transforming altered wavelet coefficients into the original domain.

4. New method of denoising

4.1. Wavelet noise suspect approach

The final observation from the previous section is a direct reason for introducing another method of filtering. As wavelet domain offers great opportunities and we do not intend to drop the wavelet approach, it is necessary to design the methodology in a different way. The results of the wavelet transform are used as pointers of noise only. Wavelet coefficients are not altered and inverse transformation is not used at all. Wavelet coefficients may be considered as proportional to the difference between weighted (in case of Haar wavelet the weights are equal) sums of observations, in neighboring intervals of an equal length, and wavelet transform's multiscale provides multilevel analysis.

The new method also consists of three main steps. In the first one given time series is orthonormally converted into the wavelet domain. Then, for each wavelet level, wavelets qualified as noises are determined. Each wavelet coefficient, which is qualified as noise, is marked and considered as wavelet noise suspect (precisely a coefficient calculated of original time sub series, which includes suspected noise). Having all wavelet coefficients passed wavelet noise suspect marking procedure the second step may be processed. Every original time series point which "is covered" by the wavelet noise suspect is marked as the point noise suspect on the level which corresponds to the level of this wavelet noise suspect. Then for each observation point of original time series the depth of distortion is found. This depth D is the biggest level to which point is continuously (from level 0) marked as point noise suspect. In step three only these observation points, which have D greater than given threshold value $D_{\min}$ and are also marked as point noise suspects in time series space should be considered as noise (of strength D). Finally, each of these considered observation points is reduced by replacing original time series values with results of a function $b(D, D_{\min}, f[,...])$, where f is an original time series function.

This method leaves a lot of space for heuristics as it comes to designating methods of identifying distortion point and marking them, definition of function b, and method of determining D itself. One of the possible approaches is presented below.

4.2. Illustrative example

This example is performed on the same data set as the example in previous section. Function b is defined as

$$b(D, D_{\min}, f, x) = \begin{cases} f(x) & D \le D_{\min} \\ f(x)/D & D > D_{\min} \end{cases}, \qquad (8)$$

and $D_{\min}$ equals 1. Results are presented in Table 1 and Fig. 6.

Table 1
Parts of table used in the process of introduced method of denoising

		0	**1**	**2**	**3**	**4**	**5**	**...**
...	...	...	...	...	...	...	...	...
94	0	0	1	0	0	0	1	...
95	1	1	1	0	0	0	1	...
96	0	1	1	0	0	0	1	...
...	...	...	...	...	...	...	...	...
101	1	1	1	0	1	0	1	...
102	0	1	1	0	1	0	1	...
103	0	0	1	0	1	0	1	...
104	0	0	1	0	1	0	1	...
105	0	0	0	1	1	0	1	...
106	1	0	0	1	1	0	1	...
...	...	...	...	...	...	...	...	...
114	1	1	1	1	1	0	1	...
115	1	1	1	1	1	0	1	...
...	...	...	...	...	...	...	...	...
129	1	1	1	0	1	0	1	...
...	...	...	...	...	...	...	...	...
174	0	1	1	1	1	1	0	...
175	1	1	1	1	1	1	0	...
176	0	0	1	1	1	1	0	...
...	...	...	...	...	...	...	...	...

Table's rows correspond to the results of original time series' point identifications and columns correspond to original time series point number, original time series' noise identification mark, and wavelet coefficients series' noise identification mark respectively (1 in cell means that point was identified as noise suspect at column's level).
Numbers of rows 95, 101, 114, 115, 129, and 175 are numbers of observation points from original time series which are qualified as noises. In these cases D equals j. Rows 96, 102, 174 are not qualified as noises, despite the fact that they are qualified as noise within wavelet domain, because they were not qualified this way in original time series domain. Similarly row 106 is not qualified as noise (despite that it is qualified this way in the original time series space and in wavelet levels 2, 3, and 5) because there were qualification gaps (at wavelet levels 0 and 1).

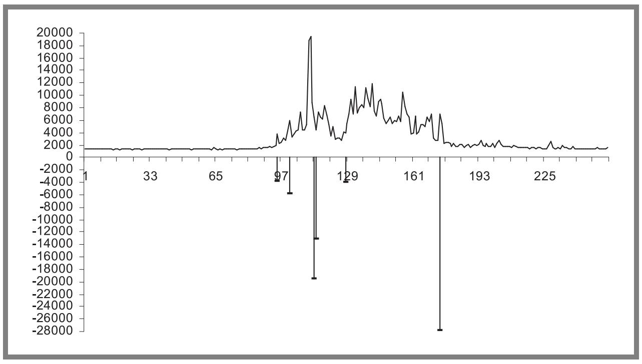

Fig. 6. Result of denoising by the new method.

Figure 6 shows the original time series transformed by filtration. Additionally it illustrates to what extent selected noise observation points were altered (the height of peek below horizontal axis equals δ). The first two peaks correspond to almost least reduced ($D = 2$) rows (observation points) 95 and 101 in Table 1. The last peak corresponds to most reduced ($D = 4$) observation point 175. In case of the two close, middle peeks (114 and 115) noise has propagated to the $D = 3$ wavelet level so observation points were reduced with power of something in between previous reductions. In case of point 129, D equals 1 and observation point 129 was the one, which was the least reduced.

5. Conclusions

Introduced method, together with WaveShrink, provide a complementary tandem of filtering based on wavelets. The main difference between introduced genuine method and WaveShrink method is that the reduction is performed on a signal, not on wavelet coefficients. These are only a tool for determining parameters of function reducing assumed noise. Therefore, the introduced method does not induct distortions into noise-neighboring observation points, what WaveShrink features.
WaveShrink suits better time series distorted in general, by malfunction for example, as it adjusts wide time series characteristics' intervals. The method introduced is better in cases where it is intended to remove "irregularities" from series as it does not change anything but the observation points qualified as noise.
Of course, this paper does not cover some other important issues. For example the order of complexity of calculations and reduction of their number was not discussed. Also the possibility of application of "more dense" wavelet transforms in case of the wavelet noise suspect method is not discussed here. Although this kind of transformations require a larger number of calculations, they seem to be an interesting direction of applications of wavelets to time series denoising.

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Regular paper

Influence of temperature and aging on polarization mode dispersion of tight-buffered optical fibers and cables

Krzysztof Borzycki

Abstract—This paper presents results of laboratory tests investigating influence of temperature on polarization mode dispersion (PMD) in variety of single mode optical fibers and cables. Research was focused primarily on tight-buffered fibers, where most pronounced effects resulting from buffer shrinkage or expansion were expected. The goal was to verify performance of optical fiber cable exposed to extreme temperatures and to compare behavior of different cables. Very strong temperature dependence of PMD was detected in standard single mode fibers with 0.9 mm tight buffer, commonly used in indoor cables, and in complete cable with such fiber. However, both nonzero dispersion-shifted fibers, spun during drawing and optical unit used in optical ground wire (OPGW), where 12 fibers are stranded showed good stability of PMD during thermal cycling. The same optical unit extracted from OPGW exhibited excellent PMD stability also during accelerated life test.

Keywords— single mode optical fiber, polarization mode dispersion, environmental testing, tight buffered fiber, optical fiber cable, optical ground wire, thermal cycling, accelerated life test.

1. Introduction

Polarization mode dispersion (PMD) is an important limit to performance of high speed optical transmission systems, in particular due to its random changes with time and wavelength and resulting difficulty of compensation, as opposed to chromatic dispersion. PMD of fiber is inherently sensitive to external mechanical disturbances, e.g., pressure applied by protective coating and buffer. PMD phenomenon and ways to control it have been extensively studied since around 1993, including monitoring of commercially installed cables [1, 2].

Research work, however, has been focused on PMD performance of fibers in thin (0.245 mm outer diameter) primary coating made of soft acrylate, as delivered (and tested) by fiber manufacturer or fibers used in loose tube and ribbon cables installed in long distance and metropolitan networks of most large operators. Fibers placed inside loose tube or (less commonly used) slotted core cables are suspended in viscous gel, minimizing hard contact with cable components, e.g., tube walls in all but extreme operating conditions. This in turn minimizes influence of external factors like temperature, cable bending, crush forces, etc., on operation of fibers inside. While providing good stability of transmission parameters and reliability, cables of "loose" designs are not suitable for several applications, particularly when small diameter, connectorization, installation in vertical shafts inside high rise buildings, fire safety or resistance to thermal shocks are required.

For such purposes, cables with so called "tight-buffered" fibers are used. The glass fiber with its primary coating is surrounded by one or more layers of solid plastics, providing mechanical strength and protection against humidity, abrasion and other harmful influences. While most products are intended for indoor operation in relatively benign conditions (except for bending), some manufacturers offer outdoor cables of this kind as well.

Another important product is optical ground wire (OPGW) – metallic aerial cable used by electric power utilities. While majority of manufacturers sell loose-tube OPGWs today, some older, but still available designs feature multi-fiber optical units with tight buffers. Over 7000 km of OPGW of such type has been installed in Poland between 1993 and 2004, constituting the core network of major operator.

Hard buffer made of polymers tends to shrink and stiffen at low temperatures, applying temperature-dependent mechanical pressure to glass fiber. This pressure can induce birefringence and significantly change fiber PMD. Aging of plastics can further modify their behavior, e.g., by shrinkage due to loss of plasticizer from poly-vinyl chloride (PVC) compounds, or softening due to decomposition.

Fiber networks are used to carry signals at ever increasing bit rates, currently up to 10 Gbit/s – both in local area networks (LANs) (gigabit Ethernet) and in long distance networks (STM-64), with link lengths in Poland up to 400 km. This makes the issue of PMD stability worthy detailed investigation.

Therefore, a research project was started by National Institute of Telecommunications in 2003 within EU research program COST Action 270 "Reliability of Optical Components and Devices in Communications Systems and Networks" to experimentally verify effects of temperature and accelerated aging on PMD in tight buffered single mode fibers and cables.

2. Tight buffered optical fiber – PMD issues

Indoor cables, commonly used in LANs and to make patchcords usually contain single fibers protected by buffer of 0.900 mm diameter, mechanically strippable and usually made of relatively hard, extruded thermoplastic materials.

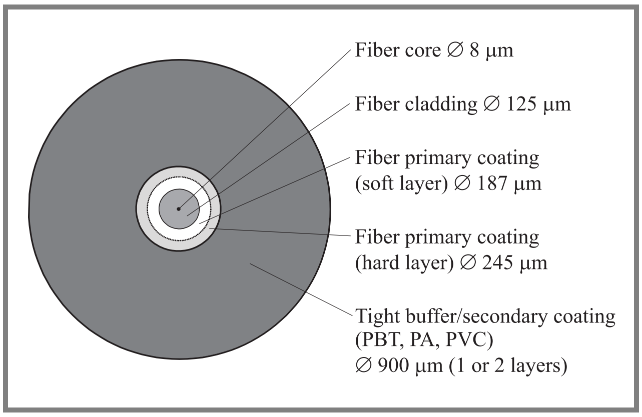

Fig. 1. Tight buffered single mode fiber for indoor cables.

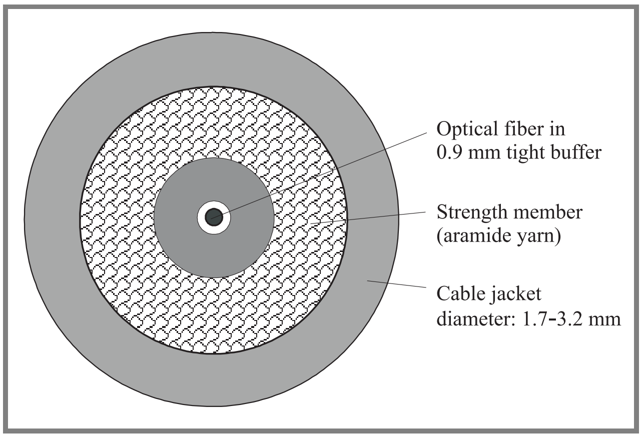

Fig. 2. Cross-section of simplex indoor cable.

The inner layer is sometimes made of soft material like silicone or even gel (in a so called "semi-tight buffer") to make buffer removal easier and faster. Such buffer is the industry standard when fitting optical connectors on the fiber or complete cable is required (Figs. 1 and 2).

Details of OPGW with 12-fiber tight-buffered unit are shown on Figs. 3 and 4. This design is representative of products installed by Polish power companies during the 1990s.

Tight-buffered fiber is surrounded by one or more layers of relatively rigid polymeric materials having much higher thermal expansion coefficient than silica glass (Table 1).

Operation at extreme low temperatures results in considerable mechanical pressure applied in radial direction towards the fiber, while high temperatures cause the buffer to expand and soften, reducing pressure. As long as the pressure remains isotropic, it shall not induce birefringence in glass and increase fiber PMD. When the symmetry is lost, e.g., due to noncircular buffer shape, pressure in one direction becomes dominant and birefringence appears. Soft, inner layers of buffer and primary coating redistribute mechanical forces more evenly (like hydrostatic pressure), mitigating this problem. Efficiency of this mechanism is dependent on materials used and particulars of buffer design [3, 4].

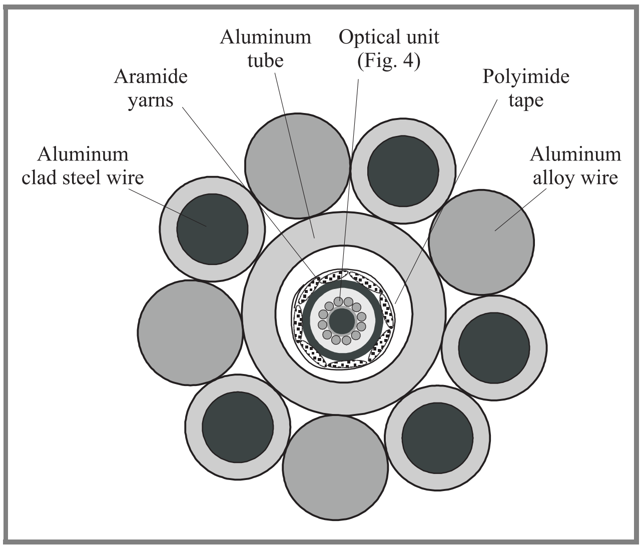

Fig. 3. OPGW tested: AlumaCore OPT-GW 30/38 mm^2/496 (Alcoa Fujikura Ltd.). External diameter: 12.7 mm.

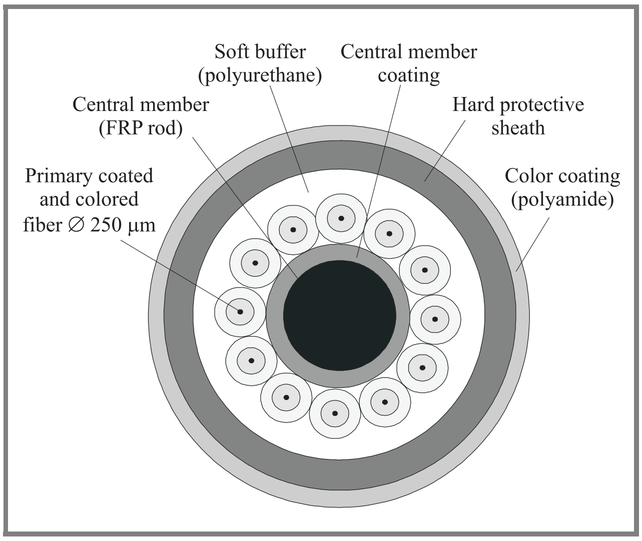

Fig. 4. Optical unit for OPGW with central strength member and 12 fibers. External diameter: 2.5 mm.

So far, temperature (and aging) dependence of PMD in tight-buffered fibers has not been a subject to either a theoretical analysis or experiments in any paper known to the author. The manufacturer has published results of tests and inspection made on similar OPGW after 17 years of field operation [5, 6]. Excellent stability of fiber attenuation was demonstrated, but no PMD measurements were made.

Table 1

Thermal expansion coefficient (TEC) of materials used to make optical fiber cables. Typical values shown; commercial products differ due to variations in manufacturing process

Material	Typical application(s)	TEC (linear)
Fused silica (SiO_2)	Basic fiber material	$+5.5 \cdot 10^{-7}$/K
Fiber reinforced plastic (FRP): glass/epoxy	Strength member – rigid	$+5.5 \cdot 10^{-6}$/K
Aramide yarn (Twaron)	Strength member – fibrous	$-3.5 \cdot 10^{-6}$/K
Fiber reinforced plastic (FRP): aramide/epoxy	Strength member – rigid	$-2.0 \cdot 10^{-6}$/K
Polyamide (nylon 6)	Tight buffer, loose tube	$+8.3 \cdot 10^{-5}$/K
Poly-buthyl-terephtalate (PBT)	Tight buffer, loose tube	$+7.4 \cdot 10^{-5}$/K
Polycarbonate (PC)	Loose tube	$+6.8 \cdot 10^{-5}$/K
High density polyethylene (HDPE)	Sheath – outdoor cables	$+5.9 \cdot 10^{-5}$/K

Table 2

Fiber and cable samples tested for temperature dependence of PMD

Sample	Description of fiber/cable; manufacturer's designation	Manufacturer	Fiber count	Fiber type	Sample length [m]	Year of manufacture	Remarks
A	Fiber in 0.245 mm primary coating, uncolored (SM-02R)	Optical Fibres Ltd.	1	G.652	6409	1997	Wound on plastic spool
B	Fiber in 0.245 mm primary coating, uncolored (SM-02R)	Optical Fibres Ltd.	1	G.652	6458	1997	Wound on plastic spool
C	Fiber in 0.9 mm tight buffer; two lengths spliced (J2B)	TP SA OTO Lublin	1	G.652	4555 1645	2001 2004	UV cured buffer
D	Fiber in 0.9 mm tight buffer; two lengths spliced (J5A)	TP SA OTO Lublin	1	G.655	5034	2005	UV cured buffer – new design
E	Indoor cable – 2 mm diameter, LSZH jacket (W-NOTKSd 1J5A)	TP SA OTO Lublin	1	G.655	4083	2005	UV cured buffer – new design
F	Indoor cable – 2 mm diameter, LSZH jacket (W-NOTKSd J-2,0)	TeleFonika KFK	1	G.652	4040	2004	Extruded buffer
G	OPGW with tight-buffered unit (OPT-GW 14/37 mm^2/443)	Alcoa Fujikura Ltd.	12	G.652	570	1994 (approx.)	Old cable after service (ca 8 yrs.)
H	OPGW with tight-buffered unit (OPT-GW 30/38 mm^2/496)	Alcoa Fujikura Ltd.	12	G.652	460	1994 (approx.)	Old cable after service (ca 8 yrs.)
I	Optical unit from OPT-GW 14/37 mm^2/443; 2 lengths spliced	Alcoa Fujikura Ltd.	12	G.652	170 230	1994 (approx.)	Extracted from old cable

3. Test program

Experiments conducted in 2004 and first half on 2005 included:

- **Temperature cycling of variety of fiber or cable samples with monitoring of fiber PMD and (in most cases) its attenuation**.

  In order to establish a PMD-temperature characteristics a single cycle with multiple, equally spaced temperature values was used. Exposure (dwell) time at each temperature was typically 12 h or 24 h, enough to stabilize temperature inside spool or drum under test. Samples contained standard single mode fibers conforming to ITU-T Rec. G.652 [7] and nonzero dispersion-shifted fibers (NZ-DSF) conforming to ITU-T Rec. G.655 – category A [8]. Following tests, all samples were inspected for signs of deterioration: cracks, discoloration, shrinkage, etc.

- **Accelerated aging test at +85°C, executed on a single sample**.

  This was a simplified, preparatory experiment to verify the principle accelerated life test and equipment used. Nevertheless, results were very useful.

Previous activities carried out in 2003 are described in report [9] and papers [10, 11].
Characteristics of samples are summarized in Table 2.

4. Test equipment

Figures 5 and 6 show general arrangement of test equipment used.

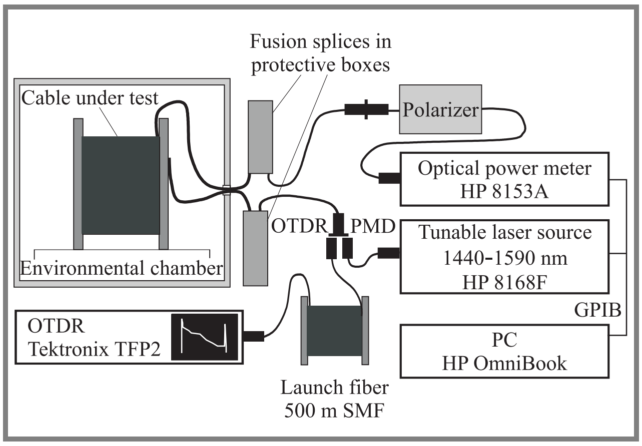

Fig. 5. Test setup for investigation of environmental effects on PMD and attenuation of optical fiber cables.

Fig. 6. Equipment used, including environmental chamber (right).

Polarization mode dispersion measurements were based on the fixed analyzer method defined by ITU-T Rec. G.650.2 [12]. Spectral scan of ratio of optical power transmitted through the fiber only (during loss calibration) and through the fiber and in-line polarizer (during actual PMD measurement), produced a curve with multiple peaks and valleys (Fig. 7), known as "extrema", counted either manually or by program running on PC. For the 150 nm spectral range used, fiber PMD was calculated according to formula:

$$\mathrm{PMD} = n_{EX} \cdot 0.0216 \text{ ps},$$

where n_{EX} is the extrema count.

The lowest value of PMD corresponding to single peak or valley was 0.0216 ps. In order to detect a 25% change in PMD, its initial value had to be greater than 0.1 ps. This required fiber lengths of 4–10 km; unfortunately not all samples available conformed to such requirements. Several relatively old fibers and cables were tested (see Table 2), but their PMD did not significantly differ from products currently available.

In case of multi-fiber cables or optical units, all fibers have been fusion spliced to maximize fiber length, making PMD measurements more accurate and replicating real network conditions, where several cable lengths are spliced. Fibers under test were carefully fixed in order to minimize uncontrolled polarization changes; all splices were located outside environmental chamber.

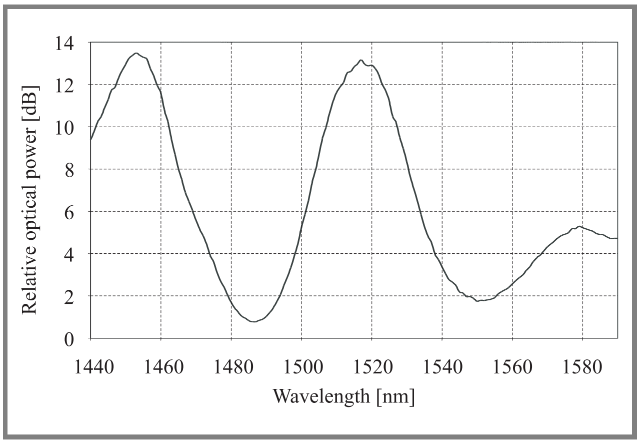

Fig. 7. Example of spectral scan made during PMD measurement. Extrema count: 5. Sample: C.

Spectral scanning was made in 1 nm increments; single measurement took approximately 5 min. During most tests PMD measurements were made repeatedly at fixed intervals ranging from 10 to 120 min.

The purpose of this project was to establish behavior of installed cables, where heat transfer between cable and its environment is relatively fast – unlike testing of coiled cable on drums or spools. Therefore, transient effects were not analyzed, except for establishing PMD stabilization time needed to design test schedule.

A standard test schedule involved the following sequence for each temperature:

– attenuation measurements at 1310 nm and 1550 nm,

– beginning of periodic PMD measurements, e.g., every 15 min for 22 h,

– change of temperature setting to a new value after the first PMD measurement.

Test data representing transients were discarded, and those belonging to steady-state periods were averaged in order to increase accuracy and remove artifacts resulting from vibration, minor fiber movements, etc. PMD values obtained in this way were normalized to initial value measured at +20°C, assumed to be 100%. The final result was a PMD versus temperature plot. For most samples, attenuation changes were negligible.

Temperature ranges were generally in accordance with Polish standards. Ranges for indoor cable and 0.9 mm buffered fiber were extended to investigate performance during possible outdoor use.

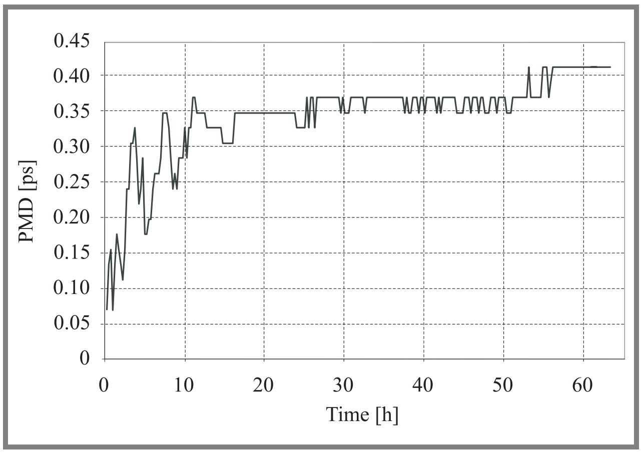

Fig. 8. Changes of PMD in temperature cycled indoor cable. Sample F on plywood drum. Temperature setting was changed from +20°C to −20°C after the first measurement. Due to slow heat transfer between cable layers its PMD has stabilized after 20 h.

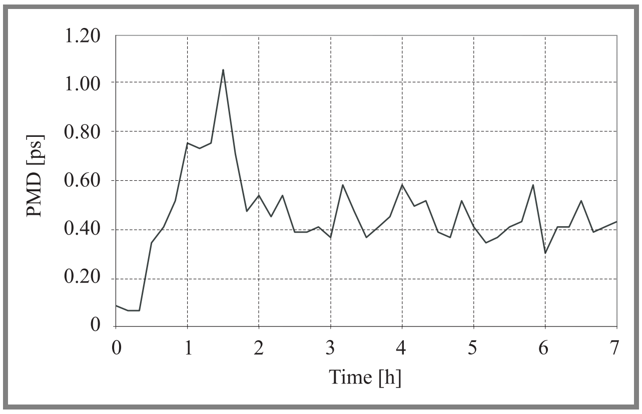

Fig. 9. Changes of PMD in tight buffered fiber during thermal cycling. Sample C on plastic spool. Temperature profile: +20°C (0–0.3 h), −55°C (0.3-1.5 h), −40°C (1.5–7 h).

Reaching a stable PMD at each new temperature required considerable time, ranging from 1 h for 3 kg spool with fiber to about 20 h in case of 15 kg plywood drum holding 4 km of indoor cable (Fig. 8) and 300 kg wooden drum with 570 m of OPGW. Forced air circulation in the chamber was always used. It is apparent (Fig. 9) that response to decreasing temperature is generally slower that to a rising one.

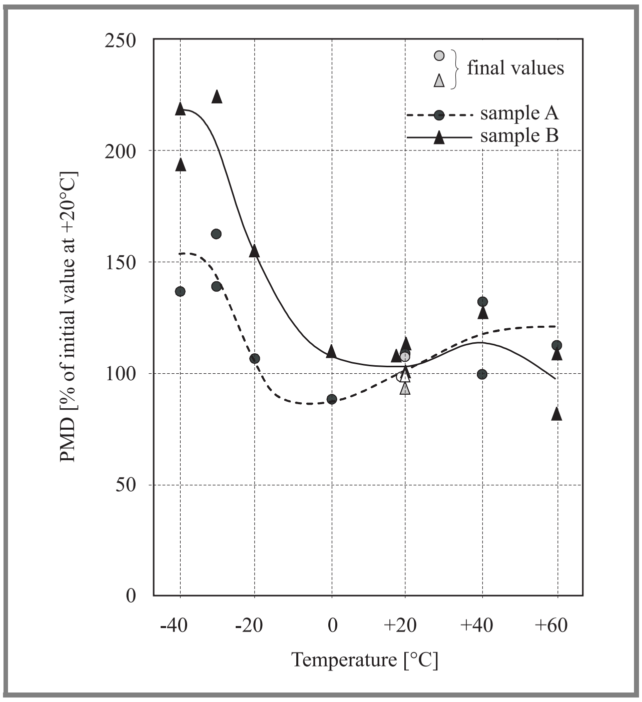

Fig. 10. PMD-temperature characteristics of two lengths of G.652 fiber in primary coating. Samples A and B tested on plastic spools.

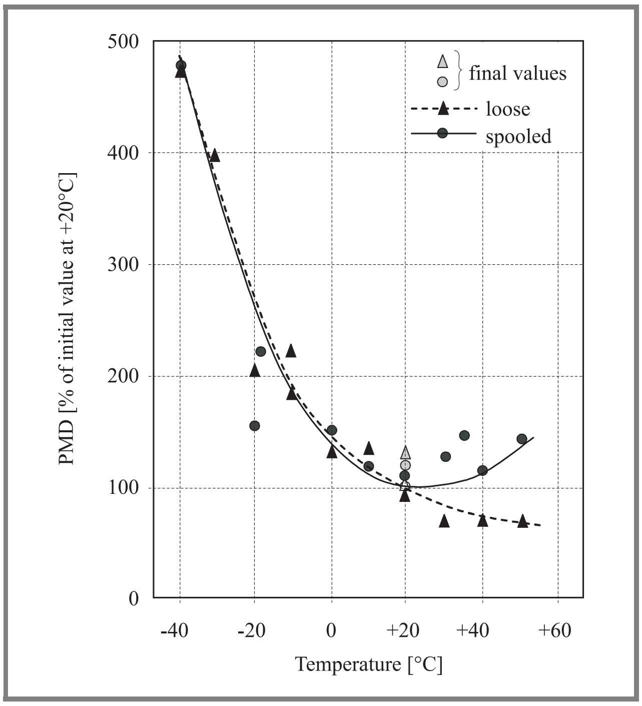

Fig. 11. PMD-temperature characteristics of G.652 fiber in 0.9 mm UV-cured tight buffer. Sample C tested twice: on spool and loose.

5. Test results

Selected characteristics of PMD changes with temperature for different fiber and cable samples are presented on Figs. 10–16. All results shown are steady-state characteristics.

The first experiment was performed on primary coated standard single mode fibers, conforming to ITU-T Rec. 652 [7]. Such fibers are used primarily in loose tube cables.

Behavior of two samples (Fig. 10) is very similar. As expected, shrinkage and stiffening of acrylate coating at low temperatures results in increased pressure on glass fiber and higher PMD. However, PMD growth looks to saturate below −30°C, despite continued increase of pressure. The most likely explanation is increased mixing of polarization modes. Changes of PMD were fully reversible, considering measurement uncertainty of about 10%.

Adding harder and thicker tight buffer changes the situation dramatically (Fig. 11 – solid line):

1. Increase of PMD at low temperatures is fast: almost 5-fold at −40°C and 10-fold at −55°C, as predicted.

2. PMD of tight buffered fiber started rise also at elevated temperatures, beginning from about +40°C. This is contrary to expectations, as expanding buffer is expected to exert less pressure on the fiber inside.

3. Despite exposure to very wide range of temperatures, including a brief −55°C transient due to malfunction of environmental chamber (Fig. 9), changes of PMD are reasonably reversible.

4. Fiber attenuation was very stable: 0.191 dB/km before test, 0.196 dB/km at −40°C, 0.192 dB/km at +60°C and 0.193 dB/km after the test; all measurements being done at 1550 nm wavelength.

Effect (2) was surprising, as continued reduction of PMD with rising temperature was expected. To eliminate crush forces between fibers wound on spool, generated when fiber buffer expands, the test was repeated on the fiber removed from spool and freely suspended on 160 mm HDPE tube. Continuous reduction of PMD with temperature was observed, the ratio of extreme values being as high as 7:1 (Fig. 11).

This shows that all tests in elevated temperatures (higher than temperature at which the fiber was originally wound) shall be performed on samples in loose condition. Figure 10 reveals the same problem, too. Cables in which fibers are protected by jacket and strength members against crush forces should not be affected.

Next tests were executed on lightweight indoor cables with single tight buffered fiber and aramide yarn strength member (see Fig. 2), used in LANs and to make patchcords for optical cross-connects. Variants of this design are also used as field-deployable outdoor cables for military use and temporary optical networks.

Two cables of identical design and size, but with different fibers and purchased from two suppliers behave very differently (Figs. 12 and 13).

Sample F had G.652 fiber encased in buffer made of PBT by hot extrusion. The PMD – temperature characteristics (Fig. 12) exhibits minimum at room temperature and dramatic increase at low temperatures. This cable was tested on lightweight plywood drum, as supplied by the manufacturer. Increase of PMD at temperatures starting from +30°C resembles behavior of fiber samples tested on spools (Figs. 10 and 11), so the pressure between layers of cable is probably responsible. Permanent increase of PMD after single thermal cycle is worth further investigation, as this may indicate shrinkage of buffer material or movement of fiber inside cable. Visual inspection of the cable did not indicate any problem.

Fiber attenuation at 1550 nm was stable: 0.183 dB/km at +20°C before the test, 0.206 dB/km at −20°C,

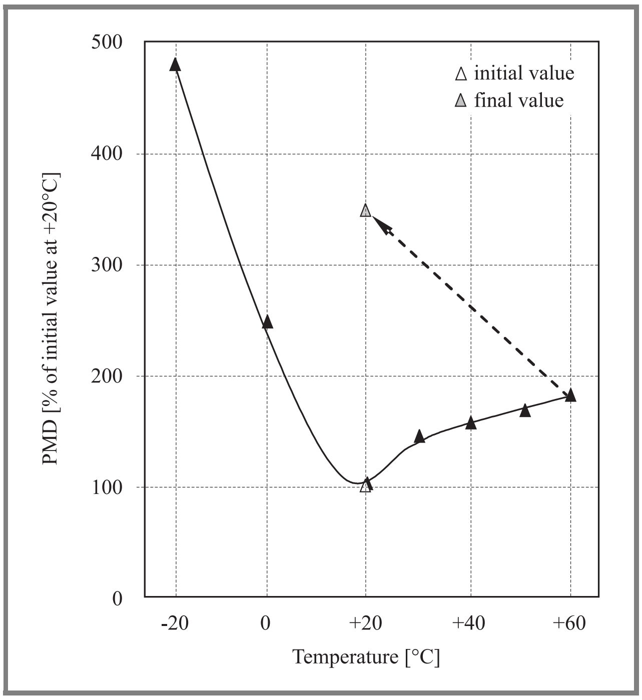

Fig. 12. PMD-temperature curve of 2 mm indoor cable with G.652 fiber. Sample F, wound on plywood drum.

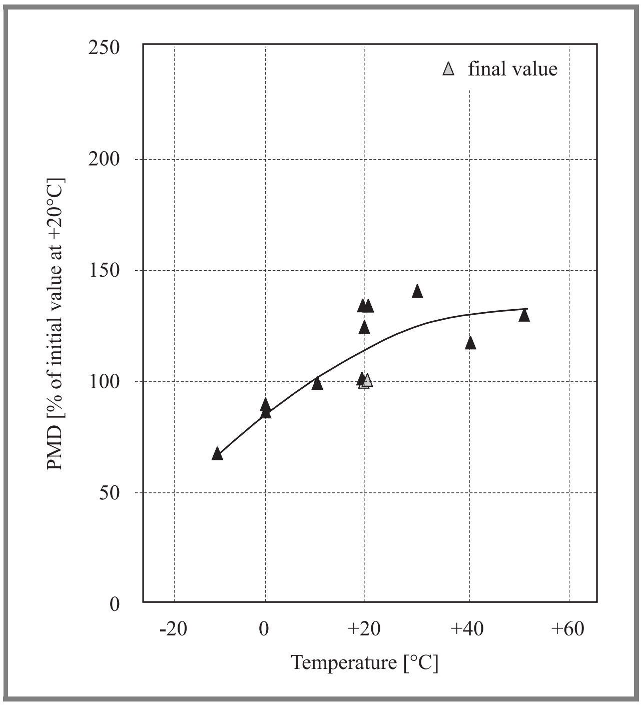

Fig. 13. PMD-temperature curve of 2 mm indoor cable with G.655.A fiber. Sample E, hanged loose on 110 mm tube.

and 0.187 dB/km at +20°C after test, despite large variations of PMD.

Sample E had a G.655 fiber (OFS TrueWave) buffered with acrylate material UV-cured at room temperature. The cable was loosely suspended on plastic tube to eliminate crush forces between layers. Buffer design was the same as in sample C, but comparison of Figs. 11 (dashed line), 12 and 13 reveals dramatic difference: PMD in G.655 fiber **decreased** at low temperatures and remained stable above +20°C.

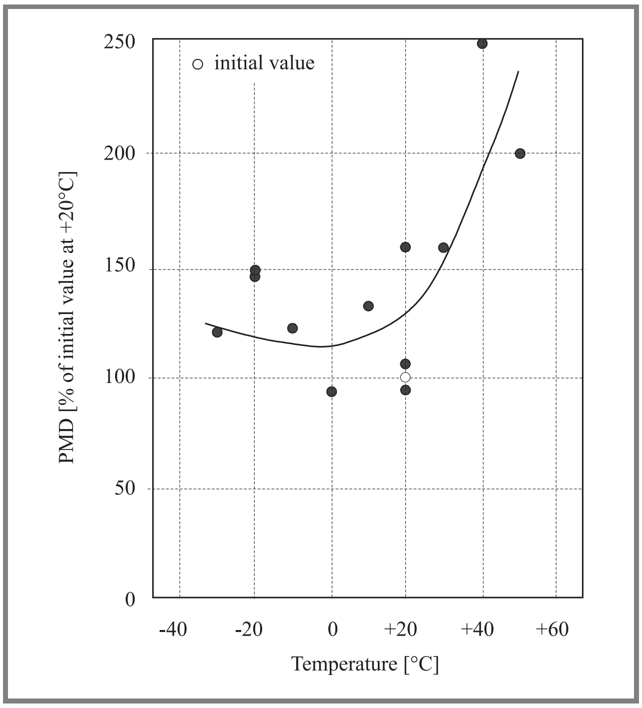

Fig. 14. PMD-temperature characteristics of G.655 fiber in 0.9 mm UV-cured tight buffer. Sample D tested on plastic spool.

Experiment on un-cabled G.655 fiber in tight buffer supplied by the same company (Fig. 14) revealed only small increase of PMD (20–45%) down to −30°C. Rise of PMD at elevated temperatures likely results from testing fiber on spool. Unfortunately, buffer cracked during test at −40°C, this prevented tests on loose fiber.

What caused this difference?

The only tangible explanation is difference in fiber manufacturing process. G.655 fibers are usually spun during drawing from the preform to reduce PMD, because higher concentration of GeO_2 dopant, larger difference of refractive index and increased mechanical strain in core area are all conductive to increased birefringence. Research papers [13, 14] indicate that advanced multi-frequency or frequency-modulated spin reduces PMD to less than 10% of value found in identical, but un-spun fiber. This mechanism is similarly effective at reducing PMD induced by external crush forces.

On the other hand, G.652 fibers are usually drawn without spinning, as their “raw” PMD has been reduced to satisfactory level by improved control of geometry and dopant deposition, so they lack effective defense against external factors increasing PMD.

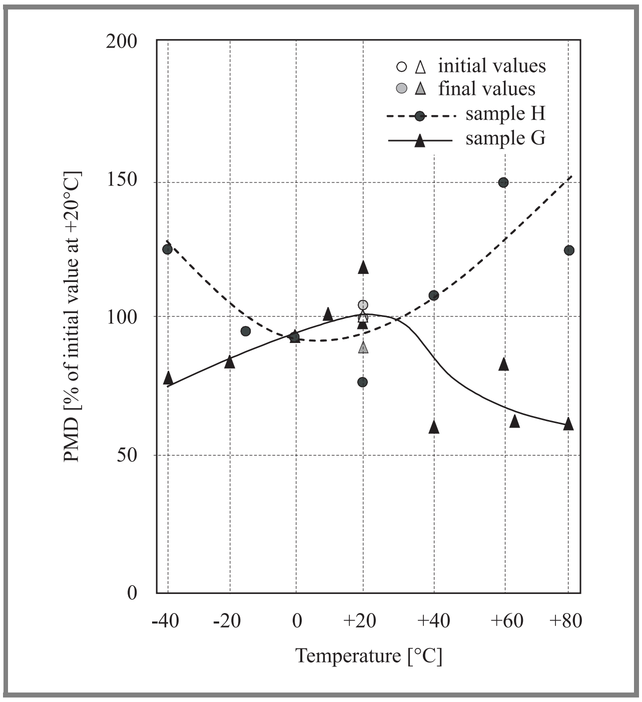

Fig. 15. PMD-temperature characteristics of OPGW cable (see Fig. 3). Samples G and H on wooden drums.

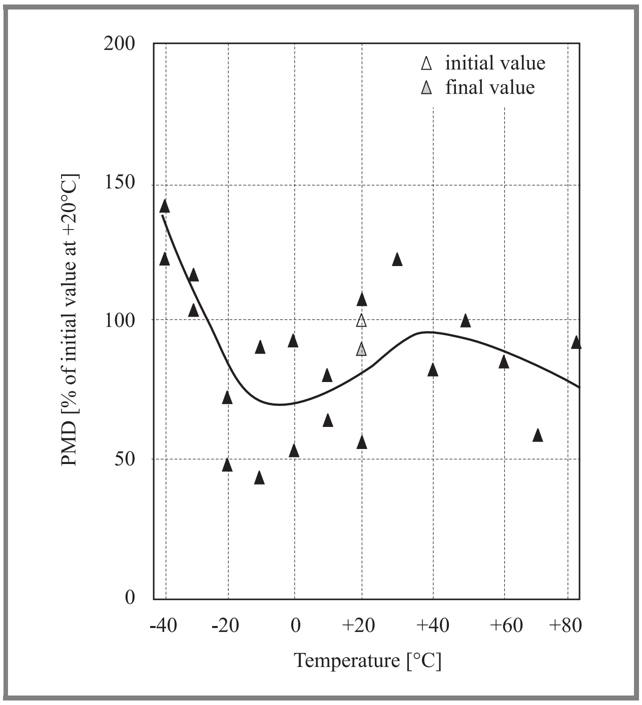

Fig. 16. PMD-temperature characteristics of 12-fiber unit extracted from OPGW (Fig. 4). Sample I, coiled in loose loops.

Optical ground wires are used on aerial power lines in harsh conditions, and have been tested in wide temperature range. Large PMD deviations were expected, especially

Table 3

Results of PMD and attenuation measurements in optical unit extracted from OPGW (sample I) during accelerated life test. Optical loop included 4400 m of fiber and 24 fusion splices. Two fibers from one length were broken and not spliced

Temperature [°C]	Time of measurement	PMD [ps]	Relative value of PMD [%]	Attenuation at 1550 nm [dB]
+20	Before ALT 1	0.076	100	3.69
+85	After 24 h exposure	0.086	114	3.74
+85	After 336 h exposure	0.076	100	3.72
+20	After ALT 1	0.051	71	3.71
+20	Before ALT 2	0.051	71	3.69
+85	After 24 h exposure	0.086	114	3.74
+85	After 336 h exposure	0.065	86	3.78
+20	After ALT 2	0.065	86	3.76

Notes:
1. **Changes of PMD remained within uncertainty of measurement** (±0.022 ps).
2. **A small increase of attenuation:** 0.016 dB/km was detected only in Test 2.
3. Inspection of sample after tests revealed no signs of physical degradation.

as the samples have been degraded by long service and exposure of fiber unit to moisture during post-removal storage. A complete OPGW and the optical unit of the same size removed from OPGW were tested, the results (Figs. 15 and 16) being surprising:

- PMD was remarkably stable between −40°C and +85°C, despite differences between two samples. Deviations from room temperature value are within ±50%. Unfortunately, resolution of PMD measurements was hardly adequate for rather short fiber samples.
- Optical unit extracted from OPGW exhibited 50% increase of PMD and increase of attenuation by 0.02–0.04 dB/km (1550 nm) at temperatures between −30°C and −40°C.
- Changes of PMD were fully reversible.

Following thermal cycling, an accelerated life test (ALT) was conducted on the same sample (I) of 12 fiber optical unit extracted from OPGW. The sample was heat-aged at +85°C for a 336 h (14 days) twice, with 14 days of storage at room temperature between tests. Telcordia standard GR-20-CORE specifies 7 days (168 h) of heat aging at +85°C to simulate 40 years of cable life.

Results of ALT proved excellent stability of transmission parameters (see Table 3).

Good stability of PMD despite the fact that cable manufacturer used un-spun G.652 fibers (Corning SMF-28) is explained by spiral stranding of fibers around strength member during forming of multi-fiber unit. Apparently, this has an effect similar to spinning of fiber during drawing.

6. Conclusions

Results of experiments on variety of fibers and cables are often contrary to assumptions made when the COST-270 research project was initiated in December 2002.

Performance of old type OPGW with tight buffered optical unit is much better than anticipated:

- Stability of PMD across range of temperatures specified in Polish standard (−40°C...+85°C) is excellent.
- There was no change of PMD and negligible increase of attenuation during extended accelerated life test.
- Such properties were shown after several years of operation and temporary penetration of moisture.

Fibers in 0.9 mm tight buffers and indoor cables with such fibers can be sensitive to temperature:

- PMD of un-spun G.652 fiber in tight buffer or cable with such fiber starts to rise quickly when temperature drops below approximately 0°C. If the cable must operate in winter conditions, substantial PMD margin must be included during system design.
- Optical fibers, which were spun during drawing are very resistant to induction of PMD by external forces. Use of such fibers in cables intended for low operating temperatures is strongly recommended. Spinning is used mostly during manufacture of fibers conforming to ITU-T Recs. G.655 and G.656.
- PMD in fiber or cable may increase by factor of 5 or more, while attenuation remains very stable.

As only a limited range of products were tested, the results may not be universally applicable.

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Regular paper

Evolutionary paths in wireless communication systems

Wojciech Michalski

Abstract—The paper contains a review and analysis of evolutionary paths of seven most important, from network development strategy point of view, wireless communication systems, especially the WLAN operating according to the IEEE 802.11 standard. With respect to WLAN, trends related to evolution toward mobile network and self organizing network as well as toward integration of WLAN and GSM using GPRS are presented. Concerning WLAN architecture development, evolution paths toward distributed controlled network as well as centrally coordinated and controlled network are described. Moreover, analysis includes cellular mobile radio systems, wireless personal networks, wide area wireless packet data systems, satellite-based mobile systems, paging/messaging systems and cordless telephones. With respect to these systems, general evolution process and trends associated with this process are described.

Keywords—* *cellular mobile radio systems, wireless local-area networks, wireless personal networks, wide area wireless packet data systems, satellite-based mobile systems, paging/messaging systems, cordless telephones, access points, switches, GPRS, mobile IP, WLAN cellular integration.

1. Introduction

Wireless communications today is not a single technology, not a single system, and not a single service, but comprises many technologies, systems and services optimized for different applications. Wireless technologies are evolving as technology advances and in the evolution process some trends are becoming clear. The most significant of them are: mobility in communications and transformation from physical connections to communications networks. This article presents different trends observed in evolution of wireless communication systems, some factors stimulating this process as well as perspective and real chances of development of particular groups of wireless communications in the future.

2. Communications in wireless networks

Technologies and systems that are currently providing wireless communications services (or will provide them in near future) can be grouped into seven following groups:

- cellular mobile radio systems,
- wireless local-area networks,
- wireless personal networks,
- wide area wireless packet data systems,
- satellite-based mobile systems,
- paging/messaging systems,
- cordless telephones.

Systems mentioned above are treated as the most important from network development strategy point of view. The grouping is generally done with respect to degree of mobility and communications applications or modes. Particular groups and evolutionary trends concerning these groups will be presented below. Although evolution path is individual for each system, the purpose of evolution process is common – to be competitive and to satisfy needs of particular user groups.

Wireless communications may be considered from different points of view. The most important of them are user location and degree of mobility. User location can be determined as either indoors or outdoors as well as on an airplane, train or car. Degree of mobility can be defined by speed, e.g., vehicular, pedestrian, or stationary, as well as by size of area in which communications are provided. Mobility and portability may be implemented on following scales:

- within a house or building (cordless telephones, wireless personal networks, wireless local area networks),
- within a campus, a town, or a city (cellular mobile radio systems, wireless local area networks, wireless personal networks, wide area wireless packet data systems, paging/messaging systems, extended cordless telephones),
- in a region area (cellular mobile radio systems, wireless local area networks, wireless personal networks, wide area wireless packet data systems, satellite-based mobile systems, paging/messaging systems),
- country- or continent-wide (cellular mobile radio systems, wireless local area networks, wireless personal networks, paging/messaging systems),
- worldwide (cellular mobile radio systems, wireless local area networks, wireless personal networks, satellite-based mobile systems).

Systems working in both fixed and mobile networks can use two modes of communications: messaging and real time two way communications. The first one is for message transmission, storage and retrieval. It is used where the communications is in not real time. This mode is typical for voice mail, fax and e-mail. The second one is represented by the telephone, cellular mobile radio telephone, interactive text and graphics exchange over data networks as well as video phone.

Wireless communications is based on different applications called agents. They are new high level software applications or entities being incorporated into some computer networks. When introduced into data network they are used to find information by some title or characteristic, and to return the information to the point from which the agent was initiated.

3. Cellular mobile radio systems

Cellular mobile radio systems may be defined as a solution providing high mobility (refers to vehicular speed), widespread coverage and wide ranging voice communications.

Cellular mobile radio systems have been evolving for many years. One evolutionary path was development from analog to digital technology. Systems working at 800 MHz have evolved to digital radio technologies operating in accordance with the following standards:

– global system for mobile communications (GSM) in Europe,

– time division multiple access (TDMA) digital cellular known as IS-54 and in the form of the code division multiple access (CDMA) digital cellular known as IS-95 in USA,

– Japanese or personal digital cellular (JDC or PDC) in Japan.

The most significant problem for digital cellular systems, considered in the design stage, is the cost of base stations. For this reason, process of digital system design is leaded toward maximizing users per MHz and per base station. Generally, regions between cities have a low population density, so a relatively high transmitter power to provide maximum range from high antenna locations is needed to cover highways running through such regions.

Digital cellular mobile radio systems have been evolving for over a decade and still evolve in different directions, e.g., toward solutions for small coverage areas or microcells. Thanks to these solutions, it is possible to increase the capacity in high user density areas and to improve coverage of shadowed areas. The microcells base stations installed inside (e.g., in conference center lobbies and similar places of high user concentrations) allow to reduce transmitter power. Microcells base stations are less expensive than conventional cell sites. Thus one can say that micro cell base stations increase system capacity and reduce cost per radio channel.

Another evolutionary path of digital cellular mobile radio systems is the development of GSM toward universal mobile telecommunication system (UMTS). There are two ways: directly according to third group partnership project (3GPP) or indirectly, with evolutionary path divided into following stages:

– GSM stage (solution providing 9600 bit/s data transmission in circuit-mode),

– general packet radio service (GPRS) stage (solution providing 160 kbit/s data transmission in packet-mode),

– enhanced data rates for GSM evolution (EDGE) stage treated as a type of GPRS (solution providing 384 kbit/s data transmission in packet-mode),

– GSM EDGE radio access network (GERAN) stage being second phase of EDGE (solution providing 1920 kbit/s data transmission in packet-mode),

– UMTS release R99 (phase 1) stage (solution providing 2 Mbit/s data transmission in packet-mode),

– UMTS release R4 and R5 stage (related to successive phases of UMTS development).

Till now, only GSM and GPRS technologies have a practical meaning. Although cellular mobile data systems have existed for over 10 years, but only recently their speeds become faster due to introduction of GPRS on GSM.

4. Wireless local-area networks

Wireless local-area networks (WLANs) may be a solution providing network connectivity in areas difficult or impossible to wire and allowing mobile applications to work with traditional wired local-area network (LAN) applications. They provide simultaneous mobility and connectivity as well as high flexibility for moves, adds and changes. They do not replace wired solutions, but only complement them. They are treated as a LAN solution for true mobile devices. This technology has been in contention whether it is a competitor or a complement to 3G. Now, it is generally accepted to be a complement.

Wireless LANs is a technology that provides access to mobile Internet as opposed to Internet over a cable. Internet access may be achieved through asymmetrical digital subscriber line (ADSL), cable television (CATV) combined with WLANs at home or in office, and hot spots belonging to WLANs, GPRS or UMTS. Wireless LANs give coverage everywhere and high speed coverage in hot spot areas. They provide roaming between hot spots and mobile areas. This technology may be used mainly (but not only) by mobile operators because they have access to mass market and technologies to mitigate problems arising from billing and roaming.

Wireless LANs have existed for many years, but only recently, when 802.11a standard [7] solution was built, they are treated as feasible high speed wireless solution. With speeds up to 54 Mbit/s and cheap access points (APs), WLANs can provide communication anywhere, anytime and with expanded range of services, using several APs. In many countries of the word, fixed and mobile network operators as well as Internet service providers have deployed APs in places such as hotels, airports, coffee shops, etc. These areas known as hot spots are the places where people are concentrated and like to use wireless services.

Wireless LANs are evolving in several directions. The first one is evolution from corporate to public networks (based on hot spots). The second one is toward extending mobility (wireless communication anytime and anywhere provided by "mobile" hot spots). The last one is toward self-organizing (*ad hoc* networking).

Another evolutionary paths of WLAN are integration of WLAN and GSM/UMTS and development of WLAN architecture toward distributed controlled network (intelligent peripheral devices) as well as toward centrally coordinated and controlled network (intelligent central nodes).

4.1. Development of WLAN toward extending mobility

According to the needs of users, WLANs as well as Internet protocol (IP)-based networks are evolving toward mobile networks. The most important thing in this process was implementation of appropriate protocols, thanks to which there are micro- and macro-mobility in WLANs. Micro-mobility exists when mobile node (MN) moves between different base stations (BSs) within the same region. Macro-mobility appears when MN goes to BS to the other region. Micro-mobility may be supported by, e.g., cellular IP (CIP) protocol which provides host-based routing schemes as well as hierarchical tunneling schemes. Macro-mobility is based on mobile IP (MIP) protocol, which can provide means to roam among networks without changing IP address. Generally, one can say that MIP and CIP play a key role in integration of different link layer technologies with the promise of enabling transparent mobility through use of a unified network layer.

Extended mobility in WLANs is supported mainly by roaming and handoff mechanisms. Thanks to roaming, it is possible to move between different networks (WLANs, GSM) and different operators. When mobile nodes roam among networks, they are handled by two agents: home and foreign agent. Home agent (HA), located in the home network, provides central mapping of home address to care-of address. Foreign agent (FA) provides possibilities for tunneling endpoint in case when MN is using foreign agent care-of address. Both home address and care-of address are associated with MN at all times when MN is away from the home network and they support the routing procedures. The first one is an address of the MN on its home network. The second one is temporary address on a foreign network. Handoff provides means needed to move from one area to another without interruption of active session. Handoff procedure may be performed by hardware and software in three steps. In the first one, an active MN sends a route update packet when it moves to the new BS. In the second one, when the MN changes to the new, BS packets from the old BS are lost (by hardware). In the last one, packets from both BS are received during overlapping (by software).

4.2. Development of WLAN toward self-organizing network

Wireless LANs as well as wireless personal networks distinguish *ad hoc* nature of their compositions (e.g., interconnecting structures and local foreign as well as remote personal and foreign devices). Moreover, their compositions characterize the heterogeneity of technologies, applications and devices. Access to infrastructure may not always be available or may be incidental (*ad hoc*). On the other hand, they have rapidly changing parameters, e.g., caused by terminal mobility and link failures. For these reasons, as typical mobile *ad hoc* networks, WLANs are currently developing toward improvement of *ad hoc* networking solution. In this solution, currently some concepts concerning resources recognition (which resources are around and available either locally or remotely), techniques for resources recognition (proactive and reactive strategies, advertising, soliciting, etc.), architecture for resource recognition, context recognition (which units constitutes the context for WLAN and its parts) as well as security and privacy are being deployed.

It is very important, that in self organizing network, based on *ad hoc* networking solution, every terminal has the same function, and network is formed dynamically (*ad hoc*) by communications exchange among terminals.

4.3. Integration of WLAN and GSM

This direction of WLAN evolution results mainly from need to broadband access services by some group of mobile subscribers. Development of UMTS is limited by necessity to build access networks (almost always from the beginning) and low interest in the UMTS services by large group of potential users. In this situation WLAN associated with GPRS is treated as a good solution for mobile networks.

There are three methods of integrating WLAN and mobile networks using GPRS:

– tight coupling,

– loose coupling,

– open coupling.

Tight coupling is the case when WLAN is directly connected to GPRS/GSM core network. It means integration on a radio level. Controllers can handle both base stations of WLAN as well as base stations belonging to GPRS/GSM. In this model all traffic generated in WLAN goes through the GPRS/GSM core network. It gives the service provider the ability to exclusively own and operate the WLAN network. The benefit of this solution is using (by WLAN) the cellular data core infrastructure, authentication and billing

systems as well as providing access to core service such as short message service (SMS) and multimedia messaging service (MMS).
In case of loose coupling, WLAN is treated as an access network complementary to the cellular network. It means that WLAN is not directly connected to GPRS/GSM core network but it only adopts some mechanisms used in GSM network, especially authentication and billing functions. In this case, networks are integrated on the link layer (it concerns link connecting authorization, authentication, accounting (AAA) server (located in WLAN) and home location register (HLR) (located in GSM)). This solution enables easy roaming between different WLANs and cellular networks, which is important requirement as many service providers exist.
In case of open coupling, interworking WLAN and GPRS/GSM is on billing level only. This solution has problems, eg., concerning authentication that are overcome by providing WLAN direct access to external networks and having common location for this.
The best solution is tight coupling, because it provides common mechanisms concerning security and handover procedure in both networks. But it requires many changes in access network infrastructure. Open coupling is the simplest solution, but it does not give possibilities offered by tight coupling. The optimum solution is loose coupling, because it ensures low cost and provides modern network with GSM security mechanisms, especially authorization and authentication procedures used in GSM network.

4.4. WLAN architecture development toward distributed controlled network

Wireless LANs development toward distributed controlled network means developing intelligence of peripheral devices. This evolutionary path is related to increasing WLAN functionality in access points exclusively. This concept is preferred to develop the most typical WLANs based on the hot spots acting independently.
Architecture of the networks based on intelligent peripheral devices consists of segment of WLAN radio access, switch and router. Segment of WLAN radio access is composed of access points working in accordance to a given standard, e.g., IEEE 802.11b [7]. In this segment the mechanisms concerning access control and billing are implemented. APs are connected with router by switch. The role of access points is connecting WLAN to structured wired networks and transmitting data between wired and wireless networks. APs provide access to others networks, but don't participate in internal connections (peer-to-peer). Users may roam between APs without interrupting active connections (same as in cellular networks). The main role of router is handling dynamic host configuration protocol (DHCP) as well as WAN connecting hot spots and backbone network. Backbone network as a main segment of WLAN concentrates traffic generating in particular hot spots.
Wireless LANs based on intelligent access points have decentralized control functions. They may support wide scope of functions and services implemented in their software. Due to large number of APs, installation costs are higher than in case of intelligent switch solution, especially related to multisystem devices working in both IEEE 802.11a and b standards.

4.5. WLAN architecture development toward centrally coordinated and controlled network

This direction of WLAN development is related to deploying software functions of the switch treated as main control device. It concerns all the functions including control, network management, handling of protocol layers (layer 2, 3, 4), filtering and data packets management as well as functions concerning security and privacy in WLAN. Thanks to filtering, administrator may assign priority to delay-sensitive packets. It concerns especially packets carrying voice data related to voice over WLAN (VoWLAN) (the service being a new application of WLAN). Moreover, in accordance with this idea, the switch is responsible for handling quality of service (QoS) functions, load balancing and access to telecommunication services. Some intelligent switches can handle simple as well as intelligent APs.
Intelligent switch has implemented typical application used to transferring packets between wired and wireless networks as well as traffic observation and measurement functions. Moreover, VoWLAN is possible thanks to implementation of QoS mechanisms. In this solution access points perform functions of network adapters only. They are used for simple transfer of data packets through radio link to the subscriber mobile nodes. They haven't any control and management functions or authentication and encryption mechanisms, because all these functions are implemented also in the switch. Switch acts in centralized mode and performs supervision functions too (in particular functions concerning reconfiguration of APs). Thanks to concentration of all the main functions at central point of the network, access points may be simple and cheap. For this reason, network construction and maintenance costs may be lower.
Solution based on intelligent central nodes is related to large WLAN, composed of large number of APs. It is dedicated to operators having hot spots in such locations as airports, hospitals, schools or large shopping centres. Due to location of all functionality in one place, two switches are required for security, especially for WLAN which must work without interruptions (in hospitals and critical public places). This model enables easy creation and upgrade of wireless networks.

5. Wireless personal networks

Wireless personal networks may be defined as systems working in the intelligent spaces, using radio signal for transferring information exchanged between different electronic devices. These networks make possible enhancement of our personal living environment and work by dynamically networking computers, PDAs, phones, headsets, viewers, appliances, sensors and actuators. They are confined to the vicinity of the person, or personal operating space and have generally an *ad hoc* character.

Wireless personal networks are centered around a person and his/her needs. Resources and partners are not necessarily in the close vicinity of the person. These networks are distinguished by their dynamics. First, dynamics of personal networks results from large variety of heterogeneous nodes connected in an *ad hoc* network and in a dynamic fashion. Second, state of the nodes may change from active to stand by and disconnected during the operation of the network. Moreover, network topology, hierarchy and constituent nodes may change continuously and access to the infrastructure network may not always be available or may be incidental (*ad hoc*). Dynamics of composition, configuration and connectivity depends on time and place. Core consisting of a PAN may be extended on demand with personal resources or resources belonging to others.

Wireless personal networks may handle internal calls as well as incoming and outgoing calls (from/to fixed, mobile, IP and others personal networks). External calls initiated by mobile phones are routed through GSM interface, calls initiated by information devices are routed through Bluetooth interface. Bluetooth is a new technology developed by consortium of Ericsson, Intel, IBM, Nokia, Toshiba and others, providing connection of peripheral devices, bridging of networks and supporting of *ad hoc* networking (e.g., GPRS via mobile phone – Bluetooth – laptop).

Wireless personal networks, especially based on Bluetooth technology, are treated as scatternets. They are built in piconets topology. Each piconet has one master and up to 7 slaves. Master determines hopping sequence and slaves have to synchronize. Participation in piconet means synchronization to hopping sequence. Communication between piconets means that devices jump back and forth between the piconets. Unit located in all piconets is the main point connecting two or more piconets. Each Bluetooth device may work as slave in multiple piconets and as master in one piconet only. Thus, the following types of Bluetooth devices may operate in scatternets:

- unit treated as a master,
- unit treated as a slave,
- unit treated as a master in one and as a slave in another piconet,
- unit treated as a slave in two or more piconets,
- unit treated as a master in one and as a slave in two or more piconets.

Wireless personal networks also have been evolving in several different directions. First, they have changed from systems using IrDA and UWB technologies to Bluetooth and IEEE.802.15 standards. Second, they have evolved from personal networks based on private branch exchange (PBX) and Centrex applications to the edge networks providing different applications, e.g., telepresence sessions, walking through a smart building, health monitoring and business environment extended to the car. Examples of edge networks are:

- personal area networks (PAN),
- body area networks (BAN),
- personal networks (PN),
- smart environments,
- sensor networks,
- home networks,
- vehicle networks,
- inter-vehicle networks.

Another evolutionary path is the *ad hoc* networking. It is optimized for use in personal environments, because WLANs have typically *ad hoc* nature. For this reason, security and privacy are very important for these networks. In more cases, current wireless infrastructure based on self-organization cannot furnish the necessary bandwidth and protocols to provide services to users traveling at highway speeds, although there are several security and handoff protocols providing required protecting mechanisms and QoS.

Mobile personal networks based on GSM, UMTS as well as public WLANs are built. Fixed personal networks are built using private WLANs and Bluetooth technology. Mobile users and vehicles may use multimode terminals working in GSM and UMTS networks and WLANs. Currently, some operators offer different applications for wireless personal communication dedicated fixed, mobile and wireless communication in vehicles (virtual home or virtual office in vehicles).

6. Wide area wireless packet data systems

Wide area wireless packet data systems may be described as a solution providing high mobility, wide ranging, low rate digital data communications to both vehicles and pedestrians. These systems have been deployed for several years and have established a customer base in many countries. They haven't experienced such a spectacular, rapid growth like voice technologies although they have existed for a long time.

Wide area wireless packet data systems have been evolving in different directions, especially toward improving base station capacity, reducing total cost, and improving scope and attractiveness of services. Finally, the cellular digital packet data (CDPD) was developed, serving as overlay to cellular radio network. CDPD shares 30 kHz spaced 800 MHz voice channels; CDPD base stations share cell sites with a voice cellular radio system.

Thanks to this, the new wide area packet data network allows to reduce the cost of packet data service and to provide 19.2 kbit/s data rate.
Another evolutionary path leads toward smaller coverage areas as well as microcells. By using very small and inexpensive base stations the service costs may be reduced. Small base stations may be located inside buildings, and may be widely distributed throughout a region. To reduce cost of the interconnecting data network, sometimes the base station to base station wireless links are used. But if the same radio channels are seized that are used to provide service, the overall capacity to serve users is reduced. Capacity may be increased by adding base stations connected to fixed distribution network. It may be increased also by using other dedicated radio channels to interconnect base stations. Generally, the microcell data networks are dedicated to stationary and low speed mobile users.

7. Satellite-based mobile systems

Satellite-based systems may be defined as solution providing two- or one-way limited quality voice transmission, and/or very limited messaging and data transmission, to very wide ranging vehicles and fixed locations. They provide both large mobility and very wide coverage better than very expensive base station systems. The best known system is Motorola's Iridium, but there are also other satellite-based systems like Globalstar, Odyssey and Teledesic.
The purpose of satellite-based systems is to provide large area (often global) coverage to users. Providing the small coverage cells (e.g., inside buildings as well as in locations shadowed by buildings, trees and mountains) from earth orbit is very difficult. Moreover, the wide overall coverage, low capacity and high cost of orbital base stations makes telecommunication services offered by these systems very expensive. For this reason, satellite systems cannot compete with terrestial systems operating in populated areas. They may be treated as complement of terrestial cellular systems dedicated to low population density areas only.
Satellite-based systems include low earth orbit systems (LEOS), intermediate or medium height systems (MEOS) and geostationary or geosynchronous orbit systems (GEOS). LEOS is a system having tens to hundreds of satellites. It consists of more, but less expensive, satellites covering the earth. LEOS satellites can more easily cover smaller coverage areas and provide higher capacity within a given bandwidth. Moreover, transmission delay in LEOS is significantly smaller and quality of voice transmission is significantly better than in other systems. GEOS consists of few satellites only (perhaps only three). The satellites are more expensive. GEOS provide lower capacity within a given bandwidth and transmission delay is significant (about 0.5 s). MEOS is located between LEOS and GEOS in both technical as well as economic aspects.
One can say, there are not and there will (in the near future) not enough users in low population density regions of the world, having enough money, to make satellite-based systems economically viable. Possible development of satellite systems is limited to being a complement to GSM system only.

8. Paging/messaging systems

Paging generally may be defined as one-way message delivering solution operating over wide area. One-way radio link is adopted to the asymmetry of transmission. This system is distinguished by high antennas and high power transmitter located at the fixed base station as well as very low power consumption pocket paging receivers. Thanks to this, it provides long usage time from small batteries. Paging was deployed and has experienced rapid growth for many years. This technology began many years ago as a one bit messaging system. The one bit can only inform that someone wants to communicate with called subscriber.
Paging has evolved in several directions. First, it has changed from analog tone coding technology enabling user identification to digitally encoded messages technology. Second, it has evolved from one bit messages to multi-bit messages. In this evolutionary path it has evolved from one bit information meaning "someone wants to communicate", through the calling party's telephone number to short e-mail text messages, offered at the end of evolution process.
Another evolutionary path was two way paging. However, in practice this concept was unrealizable, because two-way communication disturbs the asymmetrical transmission associated with paging. Moreover, two-way paging technology requires a transmitter in the user set, and involves all the problems of two-way radio system that must be developed. For this reason, one can say that two way-paging is not an appropriate solution for this purpose.

9. Cordless telephones

Cordless telephones may be described as a technology providing voice communications with low mobility with reference to the range and user speed.
First cordless telephones based on analog radio technologies were introduced in late 1970s. In this period they experienced the greatest growth. Analog cordless telephones have evolved toward digital radio technologies in different forms. One of them was 2nd generation cordless telephone (CT-2) technology. The second one was digital European cordless telephone (DECT) technology and several different solutions of industrial scientific medical (ISM) band technologies developed in the USA.
In Europe, CT-2 technologies have been evolving for few years in the direction of extending their domain of use besides residences. This evolutionary path comprises development of CT-2 toward telepoint solution as well as phone point service. CT-2 phone point service has grown rapidly mainly in Asian countries (e.g., Singapore and Hong Kong).

Among European countries it was implemented in UK, but it was introduced twice before it became attractive enough for customers. Customers of CT-2 using handsets registered with the point provider can initiate and receive calls in area handled by their telepoint. However, they can not use their handsets if they move besides area to which the call was initiated, because CT-2 technology does not support transferring active wireless calls from one phone point to another. Limited range of capabilities of handoff is provided by CT-2+ technology, which is being deployed in Canada. As in case of other wireless communications, CT-2 base stations were located in places of big concentration of people (along city streets, in the shopping malls, train stations, etc.).

Another evolutionary path of European cordless telephones is DECT. This technology provides handoff capabilities when users move between different base stations. DECT base stations are handled by controllers connected to public exchanges or private branch exchanges.

Originally, the fundamental task of cordless telephones was providing economical voice communications inside residences. For this purpose, the wire located between telephone base unit and its handset was replaced by a short wireless link. The benefits obtained by developing this technology include minimizing total cost and maximizing the talk time.

10. Conclusion

Wireless communication systems presented in this paper include seven groups of technologies and systems providing voice and data services and having different degrees of mobility. All of them are evolving to better meet the demands of various groups of users. Different trends and several different development paths may be observed in development of wireless communications. The highest growth distinguishes three groups of technologies: cellular mobile radio systems, wireless local area networks and wireless personal networks. It is not yet clear in which direction the remaining groups (i.e., wide area wireless packet data systems, satellite-based mobile system, paging, and cordless telephones) will go, and whether they will be completely merged with one of above mentioned groups. It is quite clear that part of them will be included in functionality of 3rd generation networks. One can say that cellular mobile radio systems will evolve toward packets data systems based on IP technologies, WLANs toward high speed technologies and wireless personal network toward edge networks operating in intelligent spaces. WLANs will be integrated with existing GSM cellular networks, although integration of these networks is not as simple as it may seem. Many issues exist for which solutions are required or must be improved, e.g., session continuity, inter-carrier roaming, developing a dual band devices (WLAN and cellular system) as well as authentication and billing.

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Regular paper

The Kummer confluent hypergeometric function and some of its applications in the theory of azimuthally magnetized circular ferrite waveguides

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Abstract— Examples of the application of the confluent hypergeometric functions in miscellaneous areas of the theoretical physics are presented. It is suggested these functions to be utilized as a universal means for solution of a large number of problems, leading to: cylindrical, incomplete gamma, Coulomb wave, Airy, Kelvin, Bateman, Weber's parabolic cylinder, logarithmic-integral and exponential integral functions, generalized Laguerre, Poisson-Charlier and Hermit polynomials, integral sine and cosine, Fresnel and probability integrals, etc. (whose complete list is given), which are their special cases. The employment of such an approach would permit to develop general methods for integration of these tasks, to generalize results of different directions of physics and to find the common features of various phenomena, governed by equations, pertaining to the same family. Emphasis is placed here on the use of the Kummer function in the field of microwaves: the cases of normal and slow rotationally symmetric *TE* modes propagation in the azimuthally magnetized circular ferrite waveguide are considered. Lemmas on the properties of the argument, real and imaginary parts, and positive purely imaginary (real) zeros of the function mentioned in the complex (real) domain, of importance in the solution of boundary-value problem stated for normal (slow) waves, are substantiated analytically or numerically. A theorem for the identity of positive purely imaginary and real zeros of the complex respectively real Kummer function for certain parameters, is proved numerically. Tables and graphs support the results established. The terms for wave transmission are obtained as four bilaterally open intervals of variation of the quantities, specifying the fields. It turns out that the normal (slow) modes may exist in one (two) region(s). The theoretically predicted phase curves for the first waves of the two *TE* sets examined show that the structure explored is suitable for ferrite control components design.

Keywords— microwave propagation in anisotropic media, microwave guides and components, ferrite phase shifters, switches and isolators, eigenvalue problems, function-theoretic and computational methods in electromagnetic theory, theoretical and numerical analysis of special functions.

1. Introduction

The Kummer confluent hypergeometric function (CHF) belongs to an important class of special functions of the mathematical physics [1–19] with a large number of applications in different branches of the quantum (wave) mechanics [2, 5–7, 9, 10, 12, 17, 20, 21], atomic physics [2, 5, 22, 23], quantum theory [23], nuclear physics [23], quantum electronics [24, 25], elasticity theory [2, 5, 7, 9, 26], acoustics [5, 10, 27, 28], theory of oscillating strings [2, 5, 29], hydrodynamics [5, 10, 30], random walk theory [2, 7], optics [31], wave theory [2, 7], fiber optics [32–34], electromagnetic field theory [5, 7, 35, 36], plasma physics [37–39], the theory of probability and the mathematical statistics [5, 7, 10, 13, 40], the pure [5, 41–43] and applied mathematics [44]. In the microwave physics and in particular in the theory of waveguides, such examples are the problems for rotationally symmetric wave propagation in closed and opened circular guiding structures, containing: radially inhomogeneous isotropic dielectric [45–48] or azimuthally magnetized radially stratified anisotropic media (e.g., ferrite or semiconductor) [48–74]. The possibility to obtain signal phase shifting at microwaves makes the geometries of the second type of filling attractive for the development of nonreciprocal devices for this frequency band and is the reason for their extensive study [48–88].

In this paper some properties of the complex and real Kummer CHF and its positive purely imaginary, respectively real zeros are investigated, which are employed in the analysis of normal and slow rotationally symmetric TE modes in the simplest canonical structure of the aforesaid family of anisotropic transmission lines: the circular waveguide, entirely filled with ferrite. Obtained are the propagation conditions and phase characteristics in both cases, too. It is found that there is one (there are two) area(s) of normal (slow) wave transmission, available for both signs (only for the negative sign) of magnetization. The potentialities of the configuration as phaser, switch or isolator are discussed. Symbols with (without) hats "^" stand for quantities, relevant to the slow (normal) TE modes, respectively to the real (complex) Kummer functions.

Besides, the idea is also expressed to replace in the applications the special cases of the CHFs (that are enumerated) by the functions themselves (to replace the multitudinous schemes, utilized at present by a more universal technique) as much as possible. In this way lots of the common traits of different processes which usually remain hidden, owing to the usage of a rather diverse mathematics, would come into sight.

2. Confluent hypergeometric functions

2.1. Basic concepts

Confluent hypergeometric are called four functions: the Kummer and the connected with it Tricomi function $\Phi(a,c;x)$ and $\Psi(a,c;x)$, respectively, and the Whittaker first, and second ones $M_{\kappa,\mu}(x)$ and $W_{\kappa,\mu}(x)$ [10]. The functions $\Phi(a,c;x)$ and $\Psi(a,c;x)$ are solutions of the confluent hypergeometric equation (CHE), written in the standard form of Kummer [1–14, 16–19, 44, 54, 55, 57–59, 61, 69, 72], whereas $M_{\kappa,\mu}(x)$ and $W_{\kappa,\mu}(x)$ – of the same equation, presented in its modified form, suggested by Whittaker [3, 5, 8, 10, 11, 13, 15–17, 19, 44, 55]. The quantities a and c (κ and μ) are called parameters and x – variable [3]. The CHFs except the Kummer one are multiple-valued for which the zero is a branch point. Their main branch is taken in the complex x – plane with a cut along the negative real axis. Both $\Phi(a,c;x)$ and $M_{\kappa,\mu}(x)$ are regular at zero, whereas $\Psi(a,c;x)$ and $W_{\kappa,\mu}(x)$ tend to infinity for $x \to 0$ [1–19, 55, 57–59, 61, 69, 72]. The greater symmetry with respect to the parameters observed in the formulae, involving Whittaker functions [5, 15], as well as the symmetry in the functions themselves (in their values) [55], is the reason for discussing them in parallel with the Kummer and Tricomi ones. In our opinion however, though not symmetrical, the couple $\Phi(a,c;x)$ – $\Psi(a,c;x)$ is to be preferred in the applications in view of the simpler character of power series, determining them. In addition to above definition, due to L. J. Slater [10], worth mentioning also is the one, given by Tricomi who ascertains that CHF is called any solution of CHE, considered in whichever of its forms [3]. Accordingly, such are for example the $\Phi^*(a,c;x)$, $\mathscr{M}_{\kappa,\mu}(x)$ and $N_{\kappa,\mu}(x)$ functions, too, introduced by Tricomi [2, 3, 7, 9, 61], Buchholz [5] and Erdélyi [10], respectively. Beside the notations, accepted here following F. G. Tricomi [2–4, 7, 9] and our previous works [54, 55, 57–61, 63, 64, 66–74], the symbols $M(a,b,x)$, ${}_1F_1[a;b;x]$, $\overset{\infty}{u}(a,b,x)$, and $F(\alpha,\beta,x)$ are employed also in literature instead of $\Phi(a,c;x)$, the symbols $U(a,b,x)$, $\overset{\infty}{v}(a,b,x)$ and $G(a,b,x)$ – instead of $\Psi(a,c;x)$, and the ones $\sqrt{2x/\pi}\, m_\kappa^{(2\rho)}(x)$ and $\sqrt{2x/\pi}\, w_\kappa^{(2\rho)}(x)$ – instead of $M_{\kappa,\mu}(x)$ and $W_{\kappa,\mu}(x)$, respectively [1, 5, 10, 12, 13]. The term "confluent" in the name of the functions is used, since the Kummer one might be deduced from the Gauss hypergeometric function ${}_2F_1(a,b;c;x)$ through a limiting process, leading to a confluence of two of its three regular singularities (1 and ∞) into an irregular one (the point ∞) [3, 5, 10]. (The regular singularity 0 remains unchanged.) The word "hypergeometric" is applied, as the expressions for the functions can be obtained by adding factors to the terms of the infinite geometric progression [10].

2.2. Special cases

A lot of special functions can be regarded as special cases of CHFs, or combinations of them:

- the ordinary and modified cylindrical and spherical Bessel functions: $J_\nu(x)$, $I_\nu(x)$, $\sqrt{\pi/(2x)}J_{n+1/2}(x)$ or $\sqrt{\pi/(2x)}J_{-n-1/2}(x)$ and $\sqrt{\pi/(2x)}I_{n+1/2}(x)$, respectively [1–3, 7, 9, 10, 12, 13, 15, 16];
- the Hankel functions $H_\nu^{(1)}(x)$ and $H_\nu^{(2)}(x)$ [1, 2, 7, 12, 13, 16];
- the Neumann function $N_\nu(x)$ [3, 7];
- the cylindrical and spherical McDonald functions $K_\nu(x)$ and $\sqrt{\pi/(2x)}K_{n+1/2}(x)$ [7, 13, 15, 16];
- the Coulomb wave functions: the two pairs $P_L(a,x)$ and $Q_L(a,x)$, and $U_L(a,x)$ and $V_L(a,x)$, considered by Curtis [17], the couples $G_L(\sigma)$ and $H_L(\sigma)$, defined by Hartree [17], and $U(\alpha,\gamma,Z)$ and $V(\alpha,\gamma,Z)$, introduced by Jeffreys and Jeffreys [17] and the most preferable in the applications standard pair $F_L(\eta,\rho)$ and $G_L(\eta,\rho)$, discussed by Abramowitz and Stegun [5, 10, 13, 17];
- the function $H(m,n,x)$, named Coulomb wave function and function of the paraboloid of revolution by Tricomi [2, 7, 9] or confluent hypergeometric function by Miller [13];
- the Laguerre functions $L_\nu^{(\mu)}(x)$ and $U_\nu^{(\mu)}(x)$ [3, 5, 10, 16], denoted also as $S_\nu^\mu(x)$ and $V_\nu^\mu(x)$ by Mirimanov [5, 10, 35];
- the Airy functions $Ai(x)$ [13, 16, 44] and $Bi(x)$ [13, 16];
- the incomplete $\gamma(a,x)$, the complementary $\Gamma(a,x)$, the modified $\gamma^*(a,x)$ and the fourth incomplete $\gamma_1(a,x)$ gamma functions [1–3, 7, 9, 10, 13, 15, 16, 44], as well as the derivative of them $g(a,x)$, $g_1(a,x)$, $G(a,x)$ and $k(a,x)$ ones, treated by Tricomi [2, 7, 9];
- the Kelvin (Thomson) functions $bei_n(x)$, $ber_n(x)$, $kei_n(x)$, $ker_n(x)$ [13, 16], $hei_n(x)$ and $her_n(x)$ [3, 11], met also like $bei_n x$, $ber_n x$, etc. [13];
- the Bateman function $k_\nu(x)$ [3, 5, 7, 10, 13, 16];
- the Weber's parabolic cylinder functions $D_\nu(x)$ in the Whittaker's notation [2, 3, 5, 7, 9, 10, 13, 15, 16, 44], $E_\nu^{(0)}(x)$ and $E_\nu^{(1)}(x)$ in the Buchholz's one [5, 7, 10, 13, 16], $D_\nu^+(x)$ and $D_\nu^-(x)$, proposed by Tricomi [7], $U(a,x)$, $V(a,x)$ and $W(a,x)$ in the Miller form [13], or $\delta(\xi,\nu)$ and $\rho(\xi,\nu)$ in the symbols by Magnus [5, 10] and $\varphi_n(x)$ and $\Psi_n(x)$, suggested by Janke, Emde and Lösch [11];
- the Cunningham function $\omega_{m,n}(x)$ [5, 10, 13], known as Pearson-Cunningham function, too [15];
- the Heatly Toronto function $T(m,n,r)$ [3, 5, 10, 13, 16];
- the Meixner's functions $F_1(\alpha,\beta,x)$ [3, 5, 10, 16] and $F_2(\alpha,\beta,x)$ [10];

– the MacRobert's function $E(\alpha, \beta :: x)$ [3, 5, 16];
– the Erdélyi function ${}_2F_0(\alpha, \beta; x)$ [3, 10, 16];
– the Poiseuille functions $pe(r, w)$ and $qe(r, w)$ [10];
– the Krupp functions ${}_1R(v, l; x)$ and ${}_2R(v, l; x)$ [5, 10];
– the Schlömilch function $S(v, x)$ [5, 15];
– the Chappell function $C(x, k)$ [5, 10];
– the logarithmic-integral function $li(x)$ or lix [1, 3, 7, 9, 10, 12, 13, 15, 16];
– the exponential integral functions Eix or $Ei(x)$ and $E_1(x)$ [3, 7, 9, 10, 13, 16], the generalized exponential integral function $E_n(x)$ [13, 16], marked also as $\mathscr{E}_n(x)$ [16] and the modified exponential integral one $Ein(x)$, used by Tricomi [2, 7, 13];
– the error $erfx$ or $Erf(x)$ and $Erfi(x)$, and complementary error $erfcx$ or $Erfc(x)$ functions (the error and probability integrals) [2, 3, 7, 9, 10, 13, 15, 44], as well as the ones $\Phi(x)$ and $F(x)$ [1, 5, 11–13], $\phi(x)$ and $L(x)$ [10], $\Theta(x)$, $H(x)$ and $\alpha(x)$ [3, 11], connected with them, the multiple probability integral $i^n\ erfcx$ [13] and the Hh – probability function $Hh_n(x)$ [13];
– the normal (Gauss) $P(x)$ and $Z(x)$ [13], and the χ^2-distribution $P(\chi^2|v)$ and $Q(\chi^2|v)$ functions [40], and the F-distribution $P(F|v_1, v_2)$ one [13];
– the Lagrange-Abel function $\phi_m(x)$ [15];
– some elementary (exponential e^x, power x^n, circular $\sin x$ and hyperbolic shx) functions [1, 3, 7, 13, 16];
– the reduced to $n+1$th degree exponential series $e_n(x)$ [7, 9];
– the Laguerre and generalized Laguerre polynomials $L_n(x)$ and $L_n^{(\alpha)}(x)$ [1–3, 7, 9, 10, 12, 13, 16, 44];
– the Sonine polynomials $T_\mu^{(n)}(x)$ [5, 15];
– the Poisson-Charlier polynomials $\rho_n(v, x)$ [10, 13] or $p_n(x)$ in the Tricomi's notation [3];
– the Hermit and modified Hermit polynomials $He_n(x)$ and $H_n(x)$ [1–3, 7, 10, 12, 13, 16, 44];
– some polynomials (in general incomplete) in $1/x$ of nth degree [7];
– the integral sine $Si(x)$ and cosine $Ci(x)$ [1–3, 5, 10, 12, 13, 15, 16]; and the modified cosine $Cin(x)$, employed by Tricomi [2, 7, 13];
– the Fresnel integrals $C(x)$ and $S(x)$ [3, 7, 10, 12, 13, 16], the related to them $C^*(x)$ and $S^*(x)$, and the generalized Fresnel ones $C(\alpha, x)$ and $S(\alpha, x)$ [2, 7].

2.3. Examples of application

The CHFs play an exceptional role in many branches of physics and mathematics. Several examples of their applications are:

– the solution of Schrödinger equation for charged particle motion (e.g., electron motion) in Coulombian field in the quantum mechanics, atomic physics and quantum theory [2, 5–7, 9, 10, 12, 17, 20–23];
– the energy spectrum specification of the isotropic (spherically symmetric) harmonic oscillator in nuclear physics and other related areas [5, 12, 23];
– the quantum mechanical treatment of the operation of the masers and lasers [24, 25];
– the elasticity problem for the flexion of circular or annular plates of lenticular form (resembling to a concave or convex lens), resting on, or rabbeted along its contour, subjected to a normal load whose value at certain point depends on its radial elongation from the center of the plate [2, 5, 7, 9, 26];
– the theory of the reflection of sound waves by a paraboloid [5, 10, 27];
– the consideration of sound waves propagation in parabolic horn, excited by a point source in its focus, and in the space between two co-focal paraboloids of revolution and the construction of the three-dimensional Green function for the homogeneous boundary-value problem of the first kind (Dirichlet problem) and of the second one (Neumann problem) for the wave equation in both cases [5, 28];
– the inquiry of the natural oscillations of a tight stretched string whose mass is distributed symmetrically with respect to its middle, following a parabolic law [5, 29];
– the investigation of a heat generation in a laminary Poiseuille flow through (in a viscous incompressible liquid, flowing through) a thin cylindrical capillary tube of circular cross-section [5, 10, 30];
– the determination of the length of the resultant of a large number of accidentally directed vectors (a special case, connected with the problems of random walk) [2, 7];
– the task for cylindrical-parabolic mirrors [31];
– the description of sea waves motion against a sheer coast [2, 7];
– the analysis of guided modes along a cladded optical fiber of parabolic-index core and homogeneous cladding [32–34];
– the portrayal of electromagnetic waves transmission in parabolic pipes [5];
– the study of the reflection of electromagnetic waves by a parabolic cylinder [2, 5, 7];
– the solution of the diffraction problem for a plane and a spherical electromagnetic wave in a paraboloid of revolution of infinite dinemsions [5, 35];

- the exploration of radiation electromagnetic field in a hollow paraboloid of revolution, launched by an axially oriented electric or magnetic dipole, placed at or before its focus, and between two co-focal paraboloids [5];
- the electrodynamic characterization of the field in an excited by a loop cavity resonator, consisting of two co-focal caps of the form of paraboloids of revolution [5];
- the finding of the normal (Gauss), the χ^2- and the F-distribution for arbitrary quantities in the theory of probability and mathematical statistics [13, 40];
- the development of a mathematical model of the electrical oscillations in a free ending wire [5];
- the assessment of the noise voltages transfer over a linear rectifier [5];
- the explanation of radiation of magnetized dipole in a stratified medium of spherical symmetry (in a globular layered atmosphere) [5];
- the case of electromagnetic waves in plasma with electron density changing linearly along one of the co-ordinate axes, if an infinitely large constant magnetic field is applied along the latter [37];
- the problem for electromagnetic waves in an inhomogeneous plasma whose collision frequency is a constant and the electron density varies in one direction only as a second-degree polynomial of the last-mentioned (or following a parabolic profile) [38];
- the examination of the radiation field from a uniform magnetic ring current around a cylindrical body of infinite length covered by a plasma sheath in the presence of a uniform azimuthal static magnetic field which is of practical application to improve radio communications during the blackout period in the re-entry of a conical space vehicle in the earth's atmosphere at hypersonic speed [39];
- the Tricomi euristic approximate evaluation of the distribution of the positive integers which can be presented as sums of two *k*th powers of possible value in the theory of probability [7];
- the finding of the normal (Gauss) and χ^2 – and F – distribution for arbitrary quantities in the theory of probability and mathematical statistics [13, 40];
- the series expansion of an arbitrary function in terms of eigenfunctions, of significance in the theory of hydrogen atom to describe the point (discrete) and continuous energy spectrum [5, 41];
- some continued fractions expressions of analytic functions in the complex plane, employable in the computational methods [13, 42];
- the realization of irreducible (simple) representations of a group of third order triangular matrixes, in which integral operators whose kernels are written through Whittaker functions, correspond to certain of its elements [43];
- the inspection of TE_{0n} and TM_{0n} modes, sustained in radially inhomogeneous circular dielectric waveguides (plasma columns or optical fibers) whose permittivity alters in radial direction following certain profiles [45–48];
- the theory of normal and slow surface TM_{0n} waves in the azimuthally magnetized millimeter-wave semiconductor (solid-plasma) coaxial waveguides, using *n*-type InSb and GaAs cooled to 77K as a plasma material [49, 53, 56, 62, 65];
- the problems for normal and slow TE_{0n} modes in the azimuthally-magnetized ferrite and ferrite-dielectric circular and co-axial waveguides and for slow waves, propagating along cylindrical helices, closely wound around (or surrounded by) an azimuthally magnetized ferrite rod (toroid) [49–52, 54, 55, 57–61, 63, 64, 66–74];
- the study of microwave radiation from a magnetic dipole in an azimuthally magnetized ferrite cylinder [89] which may also be explored by means of the functions considered.

3. The confluent hypergeometric functions – a universal means for solution of problems of mathematical physics

The above analysis shows that: a lot of tasks from different areas of mathematical physics lead to various representations of CHFs and a large number of functions are special cases of the latter and can be expressed in terms of them. In view of this one might expect to meet the CHFs throughout the literature. In fact, as Lauwerier wrote, "they are only sparingly used" [30]. Even one of the problems from the class examined was categorized as "not a particularly fortunate one" in the words of Suhl and Walker [49]. An attempt to substantiate these inferences is the following assertion (standing nowadays in plenty of fields): "The reason may be that these functions are still too little known, and are therefore evaded as much as possible." [30].

Indeed, the CHFs are more complicated than many other special functions, since they possess two parameters and an independent variable. The lack of numerical tables, or the insufficient tabulation of the functional values and their zeros were a grave obstacle in their applications [30, 49, 75, 80]. Serious computational predicaments arise, if the parameters and variable get large and especially, provided they are complex. The relations between these three quantities also influence the speed of convergence of power series, determining the functions. Due to this, coming upon them, some authors gave only formal analytical results [2, 5, 7, 9, 12, 21, 23, 24, 27, 30, 36–39, 49, 51], whereas others tried to avoid them through:

– reducing the CHFs to their special cases (if possible) [5, 10, 12];

– defining new functions which replace them [75, 79, 80] or harnessing such ones [83, 89];

– elaborating various numerical methods [48, 76, 82, 86, 87].

In our opinion the usage of so many very diverse artificially devised approaches hampers tracing out the connections among the different phenomena explored (which obviously exist, since the latter could be described by the same mathematical language), and impedes the establishment of their common characteristics. It is our conviction that in spite of the drawbacks pointed out, or the difficulties, appearing as a result of their complexity, the CHFs have indisputable advantages: generality and well developed theory together with valuable properties, such as for example symmetry in case of Whittaker functions. Therefore, a way out of this complicated situation, is to find means to overcome the computational challenges, instead of inventing contrivances to obviate the CHFs.

In essence the employment of the special cases, debated in Subsection 2.2, has a similar effect on the process of investigation of the phenomena and their properties, as the just discussed one, when the CHFs have been excluded from the solutions. Utilizing such a great number of functions entails as well a fragmentation of the analysis methods of corresponding tasks. However, unlike before, this state of affairs has sprung up in a natural way, when different problems have been attacked by different schemes.

As a set-off to that, it is suggested to replace the functions in question (the special cases) everywhere, where they attend by the having more universal character CHFs. To this end, the following statement is formulated:

***Statement for universality*: The confluent hypergeometric functions, considered in any of their forms, could be used as a universal means instead of any of the functions, being their special cases and the related to them, such as: the cylindrical, incomplete gamma, Coulomb wave, Weber's parabolic cylinder functions, etc. (whose complete list is given above), in the tasks in which they are met.**

Corrolary: Moving from a fragmentation to a generalization would permit:

– to solve enormous number of problems by the same universal mathematical technique;

– to develop general methods for their solution;

– to generalize results of different branches of physics;

– to find common features in different phenomena, governed by equations from the same family.

An undoubted benefit could be derived even from the partial realization of the programme proposed (when the computational hardships are surmountable).

4. Kummer confluent hypergeometric function

4.1. Definition

The Kummer CHF is defined by the absolutely convergent infinite power series [1–14, 16–19, 54, 55, 57–59, 61, 69, 72]:

$$\Phi(a, c; x) = \sum_{0}^{\infty} \frac{(a)_\nu}{(c)_\nu} \frac{x^\nu}{\nu!} . \tag{1}$$

It is analytic, regular at zero entire single-valued transcendental function of all a, c, x, (real or complex) except $c = 0, -1, -2, -3, \ldots$, for which it has simple poles. $\Phi(a, c; x)$ is a notation, introduced by Humbert, $(\lambda)_\nu = \lambda(\lambda+1)(\lambda+2)\ldots(\lambda+\nu-1) = \Gamma(\lambda+\nu)/\Gamma(\lambda)$, $(\lambda)_0 = 1$, $(1)_\nu = \nu!$, where λ stands for any number (real or complex) and ν for any positive integer or zero, is the Pochhammer's symbol and $\Gamma(\lambda)$ is the Euler gamma function. The series (1) is a solution of the Kummer CHE that is a second order ordinary differential equation [1–14, 16–19, 54, 55, 57–59, 61, 69, 72]:

$$x\frac{d^2y}{dx^2} + (c-x)\frac{dy}{dx} - ay = 0 , \tag{2}$$

having regular and irregular singularities at 0 and at ∞, respectively.

4.2. Asymptotic expansion

The asymptotic expansion of $\Phi(a, c; x)$ for large values of variable $x = |x|e^{j\varphi}$, $0 < \varphi < \pi$, is [6, 54, 57–59]:

$$\Phi(a, c; x) \approx \frac{\Gamma(c)}{\Gamma(a)}|x|^{a-c}e^{j(a-c)\varphi}e^{|x|e^{j\varphi}} + \frac{\Gamma(c)}{\Gamma(c-a)}|x|^{-a}e^{ja(\pi-\varphi)} . \tag{3}$$

If $x = jz$ ($z = |x|$ – real, positive), i.e., $\varphi = \pi/2$, both terms in the expression are approximately equally large and should be taken into account. Provided x is real, positive ($\varphi = 0$), the first term in formula (3) is considered only, since the second one becomes less than the unavoidable error, inherent to the asymptotic expansions. When x is real, negative ($\varphi = \pi$), the second term is used solely for the same reason [6, 54, 57–59].

5. Some properties of the complex Kummer function

5.1. Properties due to the analytical study

The case a – complex ($a = \mathrm{Re}\,a + \mathrm{jIm}\,a$), $c = 2\mathrm{Re}\,a$ – positive integer, $\mathrm{Im}\,a = -k$, k – real $[a = c/2 - \mathrm{j}k,\ k = \mathrm{j}(a - c/2)]$, $x = \mathrm{j}z$ – positive purely imaginary ($x = \mathrm{Re}\,x + \mathrm{jIm}\,x$, $\mathrm{Re}\,x = 0$, $\mathrm{Im}\,x = z$, $|x| = z$, z – real, positive, $\varphi = \arg x = \mathrm{Im}\,x/\mathrm{Re}\,x$, $\varphi = \pi/2$), is discussed. Under these assumptions an application of the first Kummer theorem [1–3, 5–7, 9–13, 16] facilitates to prove the statement [57–59]:

Lemma 1: If $c = 2\mathrm{Re}\,a$, $\mathrm{Re}\,x = 0$ ($x = \mathrm{j}z$ – purely imaginary), then

$$\arg\Phi(a, 2\mathrm{Re}\,a; \mathrm{j}z) = z/2\,, \tag{4}$$

where $\arg\Phi$ stands for the argument of the Kummer function.

In addition, a new modulus-argument representation of the asymptotic expansion (3) is obtained [57, 58]:

$$\Phi(a, 2\mathrm{Re}\,a; \mathrm{j}z) \approx 2F(\cos v)\mathrm{e}^{\mathrm{j}(z/2)} = 2F|\cos v|\mathrm{e}^{\mathrm{j}(z/2+n\pi)}, \tag{5}$$

where $F = [\Gamma(2\mathrm{Re}\,a)/|\Gamma(a)|]\mathrm{e}^{-(\pi/2)\mathrm{Im}\,a} z^{-\mathrm{Re}\,a}$, $v = (z/2) + \mathrm{Im}\,a \ln z - \arg\Gamma(a) - \mathrm{Re}\,a(\pi/2)$ and $n = 1, 2, 3\ldots$ denotes the number of corresponding zero of cosine, $\arg\Gamma(a)$ is the argument of gamma function. An inspection of expression (5) permits to formulate further to Lemma 1.

Lemma 2: If $c = 2\mathrm{Re}\,a$, $\mathrm{Re}\,x = 0$ ($x = \mathrm{j}z$ – positive purely imaginary), the function $\Phi(a, 2\mathrm{Re}\,a; \mathrm{j}z)$ has an infinite number of simple zeros $\zeta_{k,n}^{(c)}$ in z both for $k > 0$ and $k < 0$ ($k = -\mathrm{Im}\,a$, $n = 1, 2, 3\ldots$), at which $\mathrm{Re}\,\Phi = \mathrm{Im}\,\Phi = |\Phi| = 0$ [57, 58, 69].

Lemma 3: If $c = 2\mathrm{Re}\,a$, $\mathrm{Re}\,x = 0$ ($x = \mathrm{j}z$ – positive purely imaginary) and z exceeds the nth zero $\zeta_{k,n}^{(c)}$ of Kummer CHF $\Phi(a, 2\mathrm{Re}\,a; \mathrm{j}z)$ in z ($k = -\mathrm{Im}\,a$, $n = 1, 2, 3\ldots$) then its argument

$$\arg\Phi(a, 2\mathrm{Re}\,a; \mathrm{j}z) = (z/2) + n\pi \tag{6}$$

is a linear function of z with finite increase by π at each consecutive zero of the function [57, 58, 69].

Lemma 4: If $c = 2\mathrm{Re}\,a$, $\mathrm{Re}\,x = 0$, ($x = \mathrm{j}z$ – positive purely imaginary), then for the real and imaginary parts of Kummer CHF it holds $\mathrm{Re}\,\Phi(a, 2\mathrm{Re}\,a; \mathrm{j}z) = 0$ for $z = (2m+1)\pi$, whereas $\mathrm{Im}\,\Phi(a, 2\mathrm{Re}\,a; \mathrm{j}z) = 0$ for $z = 2m\pi$, $m = 0, 1, 2, 3, \ldots$, irrespective of the value of $\mathrm{Im}\,a$, (k) [57, 58].

Corollary: An infinite decreasing (if $\mathrm{Re}\,a > 0$) or increasing (if $\mathrm{Re}\,a < 0$ and $\mathrm{Re}\,a \neq t/2$, $t = 0, -1, -2, -3, \ldots$) sequence of maxima of $|\Phi(a, 2\mathrm{Re}\,a; \mathrm{j}z)|$ and a sequence of its zeros alternate with each other when z grows in case $c = 2\mathrm{Re}\,a$, $\mathrm{Re}\,x = 0$ ($x = \mathrm{j}z$ – positive purely imaginary) [57].

5.2. Properties due to the numerical study

The statements of Lemmas 1–4 are confirmed by the numerical evaluation of the function $\Phi(1.5 - \mathrm{j}k, 3; \mathrm{j}z)$ made, using series (1). Figure 1 is a plot of the loci curves of Φ in the complex plane for $k = +0.5$, 0 and -0.5 (solid, dotted and dashed lines, respectively), Fig. 2 visualises

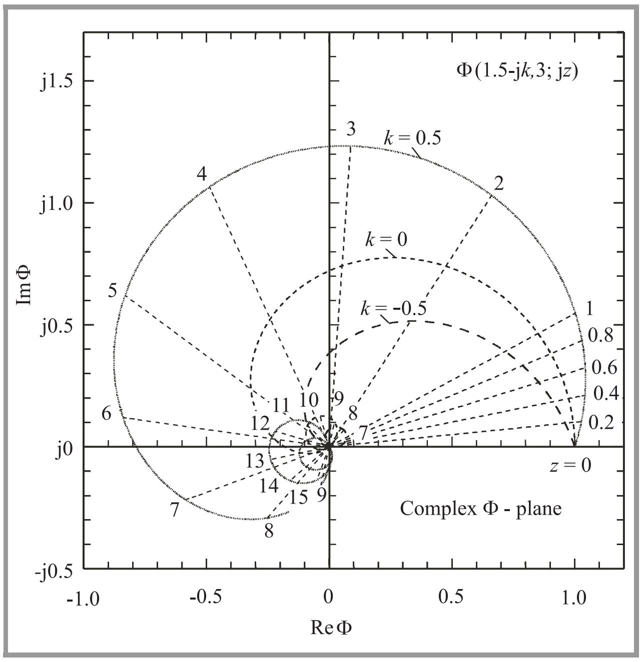

Fig. 1. Loci curves of $\Phi(1.5 - \mathrm{j}k, 3; \mathrm{j}z)$ in the complex plane for $k = +0.5$, 0 and -0.5.

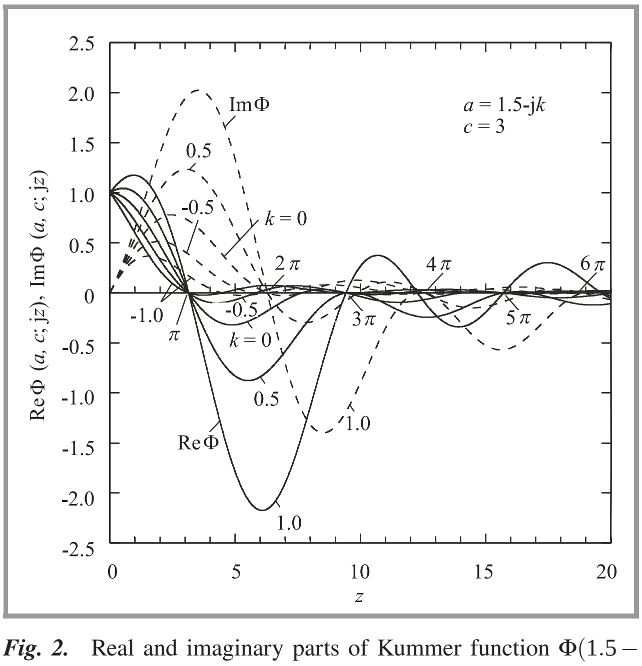

Fig. 2. Real and imaginary parts of Kummer function $\Phi(1.5 - \mathrm{j}k, 3; \mathrm{j}z)$ against z for $k = 0$, ± 0.5 and ± 1.0.

the variation of $\mathrm{Re}\,\Phi$ (solid lines) and $\mathrm{Im}\,\Phi$ (dashed lines) versus z for $k = 0, \pm 0.5, \pm 1.0$ and Fig. 3 gives the dependence of modulus and argument of Φ on z for $k = +0.5$, 0 and -0.5 (solid, dotted and dashed lines, re-

Table 1
First six positive purely imaginary zeros $\zeta_{k,n}^{(3)}$ of $\Phi(1.5-\mathrm{j}k,3;\mathrm{j}z)$ for $k=-1.0\,(0.2)+1.0$

k	$\zeta_{k,1}^{(3)}$	$\zeta_{k,2}^{(3)}$	$\zeta_{k,3}^{(3)}$	$\zeta_{k,4}^{(3)}$	$\zeta_{k,5}^{(3)}$	$\zeta_{k,6}^{(3)}$
−1.0	4.4750 5671	9.5777 9569	15.0744 6601	20.7758 5770	26.6000 3381	32.5053 0790
−0.8	4.9618 8564	10.3259 3914	15.9980 9339	21.8286 7627	27.7540 5190	33.7420 5957
−0.6	5.5218 6556	11.1477 3249	16.9911 7329	22.9469 7930	28.9703 0361	35.0384 2135
−0.4	6.1595 3442	12.0428 8636	18.0516 0729	24.1278 4699	30.2454 3063	36.3907 7149
−0.2	6.8751 0735	13.0069 8966	19.1734 8573	25.3647 3201	31.5725 5798	37.7920 4131
0.0	7.6634 1194	14.0311 7334	20.3469 3627	26.6473 8388	32.9412 6801	39.2317 1702
0.2	8.5142 1018	15.1029 6417	21.5590 7859	27.9628 4223	34.3385 7601	40.6968 5232
0.4	9.4140 5779	16.2082 5362	22.7959 6241	29.2973 5379	35.7509 1422	42.1739 8392
0.6	10.3489 2135	17.3336 0506	24.0447 2652	30.6384 5569	37.1660 9203	43.6511 3385
0.8	11.3063 8822	18.4679 5058	25.2951 0103	31.9763 7998	38.5746 8212	45.1191 1960
1.0	12.2767 8251	19.6032 3531	26.5398 8420	33.3044 4623	39.9703 5445	46.5718 6228

Table 2
First positive purely imaginary zeros $\zeta_{k,1}^{(3)}$ of $\Phi(1.5-\mathrm{j}k,3;\mathrm{j}z)$ and products $|k|\zeta_{k,1}^{(3)}$ and $|a|\zeta_{k,1}^{(3)}$ for large negative k

k	$\zeta_{k,1}^{(3)}$	$\lvert k\rvert\zeta_{k,1}^{(3)}$	$\lvert a\rvert$	$\lvert a\rvert\zeta_{k,1}^{(3)}$
−10000	0.00065 93654 06232	**6.59365 4**0623	10 000.00011 25000	**6.59365 41**365
−20000	0.00032 96827 04784	**6.59365 4**0956	20 000.00005 62500	**6.59365 41**142
−40000	0.00016 48413 52600	**6.59365 410**40	40 000.00002 81250	**6.59365 410**86
−60000	0.00010 98942 35093	**6.59365 410**55	60 000.00001 87500	**6.59365 410**76
−80000	0.00008 24206 76327	**6.59365 4106**1	80 000.00001 40625	**6.59365 410**72
−100000	0.00006 59365 41063	**6.59365 4106**2	100 000.00001 12500	**6.59365 410**70

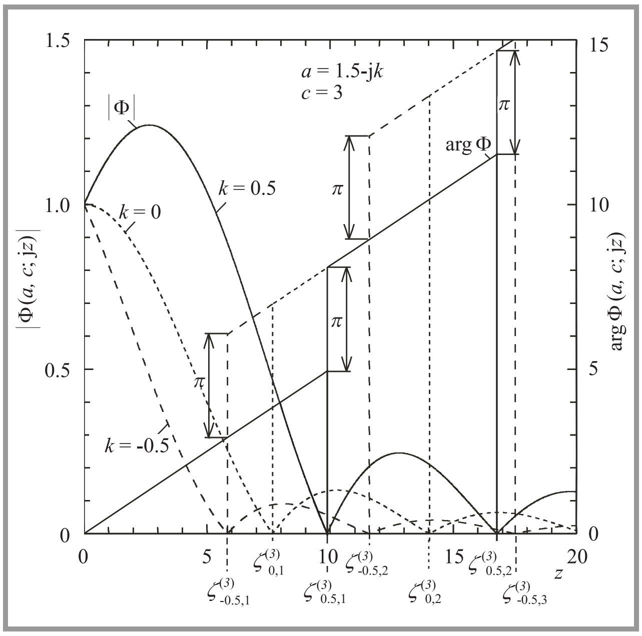

Fig. 3. Modulus and argument of Kummer function $\Phi(1.5-\mathrm{j}k,3;\mathrm{j}z)$ versus z for $k=+0.5$, 0 and -0.5.

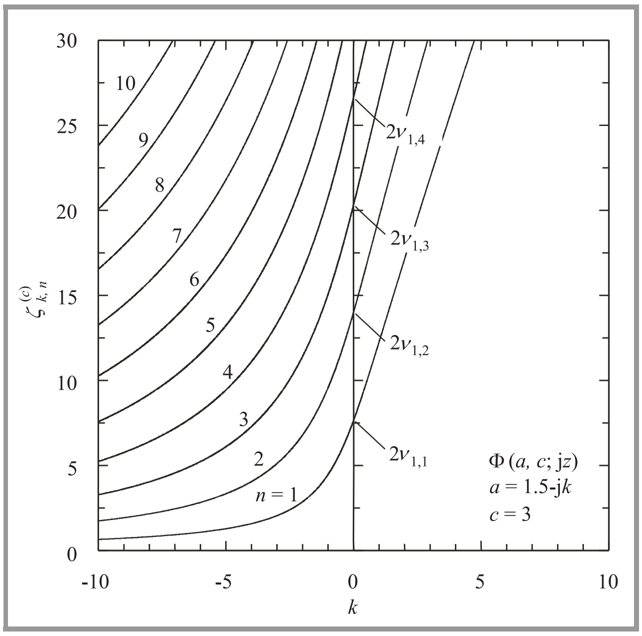

Fig. 4. Distribution of the first ten positive purely imaginary zeros of Kummer CHF $\Phi(1.5-\mathrm{j}k,3;\mathrm{j}z)$ with k.

spectively). The distribution of the first ten zeros of Φ with k is plotted in Fig. 4. The curves intersect the ordinate axis ($k=0$) at points $\zeta_{0,n}^{(3)}=2\nu_{1,n}$ [$\nu_{1,n}$ is the nth zero of Bessel function $J_1(x)$] which could be proved, using the second Kummer theorem [1–4, 7, 9, 10, 13, 16]. Values of $\zeta_{k,n}^{(c)}$ for small and large $|k|$ are listed in Tables 1 and 2. The analysis shows that it is true: $\lim_{k\to-\infty}\zeta_{k,n}^{(c)}=0$ and $\lim_{k\to+\infty}\zeta_{k,n}^{(c)}=+\infty$. The products $|k|\zeta_{k,n}^{(c)}$ and $|a|\zeta_{k,n}^{(c)}$ are of special interest, if k gets very large negative (see Table 2). It is valid [72]:

Lemma 5: If $\zeta_{k,n}^{(c)}$ is the nth positive purely imaginary zero of Kummer function $\Phi(a,c;x)$ in x ($n=1,2,3\ldots$) provided $a=c/2-\mathrm{j}k$ – complex, $c=2\,\mathrm{Re}\,a$ – restricted positive integer, $x=\mathrm{j}z$ – positive purely imaginary, z – real, positive, $k=\mathrm{j}(a-c/2)$ – real, then the infinite sequences of positive real numbers $\{\zeta_{k,n}^{(c)}\}$, $\{|k|\zeta_{k,n}^{(c)}\}$ and $\{|a|\zeta_{k,n}^{(c)}\}$ are convergent for $k\to-\infty$ (c, n – fixed). The limit of the first sequence is zero and the limit of the second and third ones is the same. It equals the finite positive real number L, where $L=L(c,n)$. It holds:

$$\lim_{k\to-\infty}|k|\zeta_{k,n}^{(c)}=L(c,n)\,, \tag{7}$$

$$\lim_{k\to-\infty}|a|\zeta_{k,n}^{(c)}=L(c,n)\,. \tag{8}$$

For any $|k|$ and relevant $|a|$ it is true $|k|\zeta_{k,n}^{(c)}<L(c,n)<|a|\zeta_{k,n}^{(c)}$. In case $k\to+\infty$, $\{\zeta_{k,n}^{(c)}\}$, $\{|k|\zeta_{k,n}^{(c)}\}$, and $\{|a|\zeta_{k,n}^{(c)}\}$ also tend to $+\infty$. Results for complex Φ – function can be found in [55, 57–59, 72], too.

6. Some properties of the real Kummer function

6.1. Properties due to the analytical study by F. G. Tricomi

Tricomi has proved that if $\hat a,\hat c,\hat x$ are real, $\hat x>0$ and $\hat c>0$:

- the Kummer CHF $\Phi(\hat a,\hat c,\hat x)$ has real positive zeros only if $\hat a<0$;
- the number of zeros $\hat l=abs[\hat a]$ is finite, $[\hat a]$ is the largest integer less or equal to $\hat a$, i.e., $[\hat a]\le\hat a$;
- at the point $\hat a=[\hat a]=-\hat n$ ($\hat n\le\hat l$ – a positive integer, $\hat n=1,2,\ldots,\hat l$) a new zero appears [1–3, 7, 9, 10, 44].

6.2. Properties due to the numerical study

The case $\hat a=\hat c/2+\hat k$ – real, $\hat c$ – positive integer, $\hat k$ – real ($\hat k=\hat a-\hat c/2$), $\hat x$ – real, positive, is treated. Computations of the function $\Phi(1.5+\hat k,3;\hat x)$ have been performed, making use of series (1). Figures 5 and 6 represent Φ versus $\hat x$ for $\hat k>0$ (solid lines), $\hat k=0$ (dotted curve) and $\hat k<0$ (dashed lines). The monotonous (oscillating) character of curves for $\hat k>-1.5$ ($\hat k<-1.5$) is in agreement with above analytical results. Values of the real zeros $\hat\zeta_{\hat k,\hat n}^{(\hat c)}$ of the same function are given in Tables 3–5 for different intervals of variation of $\hat k$. The distribution of the first eight zeros of Φ against $\hat k$ is drawn in Fig. 7. The numerical analysis indicates

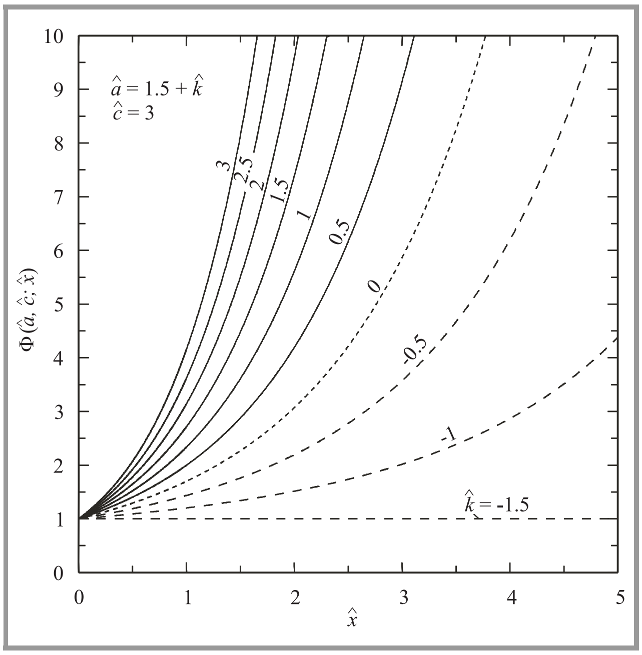

Fig. 5. Kummer function $\Phi(1.5+\hat k,\,3;\,\hat x)$ against $\hat x$ for $\hat k=-1.5(0.5)3$.

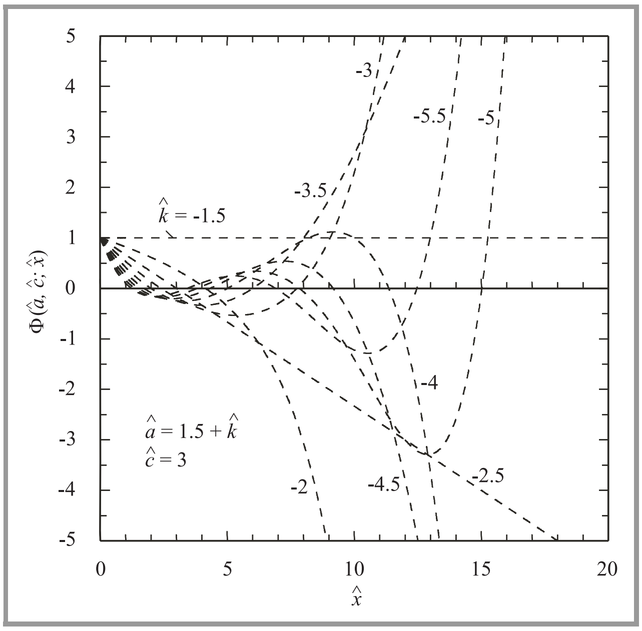

Fig. 6. Kummer function $\Phi(1.5+\hat k,\,3;\,\hat x)$ versus $\hat x$ for $\hat k=-5(0.5)-1.5$.

that it holds: $\lim_{\hat k\to-\infty}\hat\zeta_{\hat k,\hat n}^{(\hat c)}=0$ and $\lim_{\substack{\hat k\to-(\hat n-1)-\hat c/2\\ \hat k<-(\hat n-1)-\hat c/2}}\hat\zeta_{\hat k,\hat n}^{(\hat c)}=+\infty$.

The products $|\hat k|\hat\zeta_{\hat k,\hat n}^{(\hat c)}$ and $|\hat a|\hat\zeta_{\hat k,\hat n}^{(\hat c)}$ are of interest, if $\hat k$ is very large negative (Table 5). It is true:

Lemma 6: If $\hat\zeta_{\hat k,\hat n}^{(\hat c)}$ is the $\hat n$th positive real zero of Kummer function $\Phi(\hat a,\hat c;\hat x)$ in $\hat x$ ($\hat n=1,2,\ldots,\hat l$, $\hat l=abs[\hat a]$) provided $\hat a,\hat c,\hat x$ are real, $\hat c$ – restricted positive integer and $\hat k=\hat a-\hat c/2$ – real ($\hat a=\hat c/2+\hat k$), then

Table 3
First six positive real zeros $\hat{\zeta}^{(3)}_{\hat{k},\hat{n}}$ of $\Phi(1.5+\hat{k}, 3; \hat{x})$ for $\hat{k} = -\left[(2\hat{n}+1)/2 + 1.10^{-\hat{s}}\right]$ and $\hat{s} = 2(1)10$

$\hat{s}$	$\hat{\zeta}^{(3)}_{\hat{k}(\hat{s}),1}$	$\hat{\zeta}^{(3)}_{\hat{k}(\hat{s}),2}$	$\hat{\zeta}^{(3)}_{\hat{k}(\hat{s}),3}$	$\hat{\zeta}^{(3)}_{\hat{k}(\hat{s}),4}$	$\hat{\zeta}^{(3)}_{\hat{k}(\hat{s}),5}$	$\hat{\zeta}^{(3)}_{\hat{k}(\hat{s}),6}$
10	32.6943 6952	39.2832 5273	45.2869 3680	50.9779 8646	56.4636 0593	61.8003 1150
9	30.1381 7435	36.6025 7840	42.4984 2791	48.0931 9869	53.4911 8502	58.7470 5165
8	27.5553 2227	33.8846 6661	39.6641 8688	45.1555 0085	50.4595 9370	55.6290 4210
7	24.9390 3482	31.1204 5777	36.7733 9084	42.1526 1734	47.3553 1077	52.4316 6489
6	22.2793 6643	28.2967 9449	33.8104 1524	39.0668 9583	44.1589 4739	49.1340 0339
5	19.5607 1308	25.3932 7731	30.7511 9691	35.8712 6945	40.8408 6789	45.7041 6336
4	16.7561 8418	22.3751 7209	27.5550 4993	32.5201 8558	37.3513 6214	42.0887 8082
3	13.8134 2126	19.1750 2405	24.1432 3161	28.9257 1322	33.5946 6329	38.1852 0648
2	10.6181 4852	15.6405 7545	20.3351 4451	24.8844 5526	29.3480 2383	33.7537 3550

Table 4
First six positive real zeros $\hat{\zeta}^{(3)}_{\hat{k},\hat{n}}$ of $\Phi(1.5+\hat{k}, 3; \hat{x})$ for $\hat{k} = -2(-1)-10$

$\hat{k}$	$\hat{\zeta}^{(3)}_{\hat{k},1}$	$\hat{\zeta}^{(3)}_{\hat{k},2}$	$\hat{\zeta}^{(3)}_{\hat{k},3}$	$\hat{\zeta}^{(3)}_{\hat{k},4}$	$\hat{\zeta}^{(3)}_{\hat{k},5}$	$\hat{\zeta}^{(3)}_{\hat{k},6}$
−2	4.1525 7778					
−3	2.3908 7384	7.7342 0261				
−4	1.7240 3430	4.9963 8913	11.3550 3906			
−5	1.3562 4234	3.8054 2722	7.8425 2881	15.0185 8200		
−6	1.1202 9295	3.0969 9425	6.1880 6299	10.8491 1987	18.7168 8187	
−7	0.9552 6444	2.6191 1978	5.1554 0981	8.7786 0273	13.9709 0761	22.4429 7395
−8	0.8330 6998	2.2725 0326	4.4346 8551	7.4417 8723	11.5221 5873	17.1799 4235
−9	0.7388 2652	2.0086 2555	3.8982 3676	6.4852 2265	9.9005 3270	14.3837 0603
−10	0.6638 7020	1.8005 8410	3.4814 0975	5.7592 2215	8.7176 8903	12.4948 1718

Table 5
First positive real zeros $\hat{\zeta}^{(3)}_{\hat{k},1}$ of $\Phi(1.5+\hat{k}, 3; \hat{x})$ and products $|\hat{k}|\hat{\zeta}^{(3)}_{\hat{k},1}$ and $|\hat{a}|\hat{\zeta}^{(3)}_{\hat{k},1}$ for large negative $\hat{k}$

$\hat{k}$	$\hat{\zeta}^{(3)}_{\hat{k},1}$	$\lvert\hat{k}\rvert\hat{\zeta}^{(3)}_{\hat{k},1}$	$\hat{a}$	$\lvert\hat{a}\rvert\hat{\zeta}^{(3)}_{\hat{k},1}$
−10000	0.00065 93654 15127	**6.59365 41**512	−9998.5	**6.59**266 51031
−20000	0.00032 96827 05895	**6.59365 41**179	−19998.5	**6.593**15 95938
−40000	0.00016 48413 52739	**6.59365 410**95	−39998.5	**6.593**40 68475
−60000	0.00010 98942 35134	**6.59365 410**80	−59998.5	**6.593**48 92666
−80000	0.00008 24206 76343	**6.59365 410**74	−79998.5	**6.593**53 04764
−100000	0.00006 59365 41072	**6.59365 410**72	−99998.5	**6.593**55 52023

the infinite sequences of positive real numbers $\{\hat{\zeta}^{(\hat{c})}_{\hat{k},\hat{n}}\}$, $\{|\hat{k}|\hat{\zeta}^{(\hat{c})}_{\hat{k},\hat{n}}\}$ and $\{|\hat{a}|\hat{\zeta}^{(\hat{c})}_{\hat{k},\hat{n}}\}$ are convergent for $\hat{k} \to -\infty$ ($\hat{c}, \hat{n}$ – fixed). The limit of the first sequence is zero and the limit of the second and third ones is the same. It equals the finite positive real number $\hat{L}$, where $\hat{L} = \hat{L}(\hat{c}, \hat{n})$. It is valid:

$$\lim_{\hat{k}\to-\infty} |\hat{k}|\hat{\zeta}^{(\hat{c})}_{\hat{k},\hat{n}} = \hat{L}(\hat{c}, \hat{n}), \tag{9}$$

$$\lim_{\hat{k}\to-\infty} |\hat{a}|\hat{\zeta}^{(\hat{c})}_{\hat{k},\hat{n}} = \hat{L}(\hat{c}, \hat{n}). \tag{10}$$

For any $|\hat{k}|$ and corresponding $|\hat{a}|$ it holds $|\hat{a}|\hat{\zeta}^{(\hat{c})}_{\hat{k},\hat{n}} < \hat{L}(\hat{c}, \hat{n}) < |\hat{k}|\hat{\zeta}^{(\hat{c})}_{\hat{k},\hat{n}}$. If $\hat{k} \to +\infty$, $\{\hat{\zeta}^{(\hat{c})}_{\hat{k},\hat{n}}\}$, $\{|\hat{k}|\hat{\zeta}^{(\hat{c})}_{\hat{k},\hat{n}}\}$ and $\{|\hat{a}|\hat{\zeta}^{(\hat{c})}_{\hat{k},\hat{n}}\}$ go to $+\infty$, too.

Lemma 7: Let $\hat{\zeta}^{(\hat{c})}_{\hat{k},\hat{n}}$ is the $\hat{n}$th positive real zero of Kummer function $\Phi(\hat{a}, \hat{c}; \hat{x})$ in $\hat{x}$ ($\hat{n} = 1, 2, \ldots, \hat{l}$, $\hat{l} = abs[\hat{a}]$) provided $\hat{a}, \hat{c}, \hat{x}$ are real, $\hat{c}$ – restricted positive integer and $\hat{k} = \hat{a} - \hat{c}/2$ – real ($\hat{a} = \hat{c}/2 + \hat{k}$). If $\hat{k} = -\left[(2\hat{n}+1)/2 + 1.10^{-\hat{s}}\right]$, and $\hat{s} = 1, 2, 3, \ldots$ is a positive integer, then the dif-

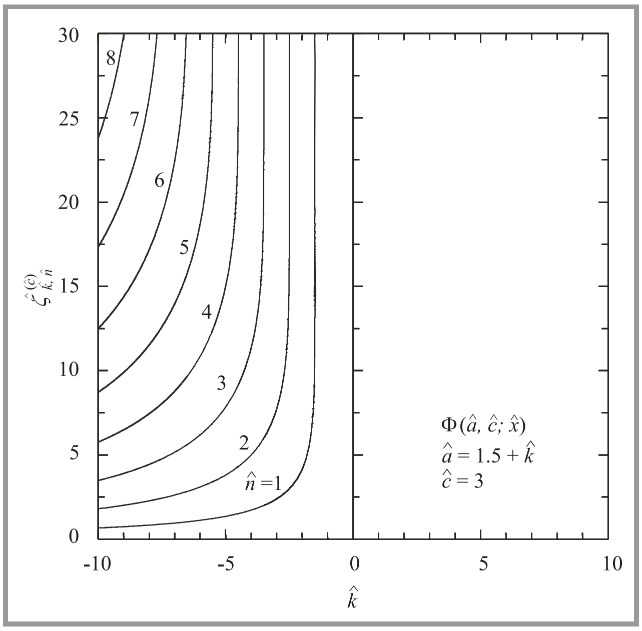

Fig. 7. Distribution of the first eight positive real zeros of Kummer function $\Phi(1.5+\hat{k}, 3; \hat{x})$ with $\hat{k}$.

ferences $\hat{\Delta}_{\hat{s}+1,\hat{s},\hat{n}} = \hat{\zeta}^{(\hat{c})}_{\hat{k}(\hat{s}+1),\hat{n}} - \hat{\zeta}^{(\hat{c})}_{\hat{k}(\hat{s}),\hat{n}}$ and $\hat{\Delta}^2_{\hat{s}+2,\hat{s}+1,\hat{s},\hat{n}} = \hat{\Delta}_{\hat{s}+1,\hat{s},\hat{n}} - \hat{\Delta}_{\hat{s}+2,\hat{s}+1,\hat{n}}$ where $\hat{k}(\hat{s}+1)$ and $\hat{k}(\hat{s})$ are any two neighbouring parameters for certain $\hat{n}$, tend to a finite real positive number and zero, respectively, especially if $\hat{s}$ gets large and $\hat{n}$ is small. Accordingly, the zeros $\hat{\zeta}^{(\hat{c})}_{\hat{k},\hat{n}}$, situated close to the points $\hat{k} = -(2\hat{n}+1)/2$ can be computed from the approximate formula:

$$\hat{\zeta}^{(\hat{c})}_{\hat{k}(\hat{s}+2),\hat{n}} \approx \hat{\zeta}^{(\hat{c})}_{\hat{k}(\hat{s}+1),\hat{n}} + \hat{\Delta}_{\hat{s}+1,\hat{s},\hat{n}} = 2\hat{\zeta}^{(\hat{c})}_{\hat{k}(\hat{s}+1),\hat{n}} - \hat{\zeta}^{(\hat{c})}_{\hat{k}(\hat{s}),\hat{n}}. \quad (11)$$

This relation permits to obtain the subsequent zero, if the values of the preceding two are known (Table 4). Results for real Φ – function are available also in [10, 11, 13, 14].

7. A theorem for the identity of zeros of certain Kummer functions

Theorem 1: If $\zeta^{(c)}_{k,n}$ is the nth positive purely imaginary zero of complex Kummer function $\Phi(a, c; x)$ in x ($n = 1, 2, 3, \ldots$) provided $a = c/2 - \mathrm{j}k$ – complex, $c = 2\mathrm{Re}\,a$ – restricted positive integer, $x = \mathrm{j}z$ – positive purely imaginary, z – real, positive, $k = \mathrm{j}(a - c/2)$ – real, and if $\hat{\zeta}^{(\hat{c})}_{\hat{k},\hat{n}}$ is the $\hat{n}$th positive real zero of real Kummer function $\Phi(\hat{a}, \hat{c}; \hat{x})$ in $\hat{x}$ ($\hat{n} = 1, 2, \ldots, \hat{l}$, $\hat{l} = abs[\hat{a}]$) provided $\hat{a}, \hat{c}, \hat{x}$ are real, $\hat{c}$ – restricted positive integer and $\hat{k} = \hat{a} - \hat{c}/2$ – real ($\hat{a} = \hat{c}/2 + \hat{k}$), then the infinite sequences of positive real numbers $\{\zeta^{(c)}_{k,n}\}$, $\{|k|\zeta^{(c)}_{k,n}\}$ and $\{|a|\zeta^{(c)}_{k,n}\}$ are convergent for $k \to -\infty$ (c, n – fixed), and the infinite sequences of positive real numbers $\{\hat{\zeta}^{(\hat{c})}_{\hat{k},\hat{n}}\}$, $\{|\hat{k}|\hat{\zeta}^{(\hat{c})}_{\hat{k},\hat{n}}\}$ and $\{|\hat{a}|\hat{\zeta}^{(\hat{c})}_{\hat{k},\hat{n}}\}$ are convergent for $\hat{k} \to -\infty$ ($\hat{c}, \hat{n}$ – fixed). The limits of $\{\zeta^{(c)}_{k,n}\}$ and $\{\hat{\zeta}^{(\hat{c})}_{\hat{k},\hat{n}}\}$ equal zero. The limits of $\{|k|\zeta^{(c)}_{k,n}\}$ and $\{|a|\zeta^{(c)}_{k,n}\}$ coincide. They equal the positive real number L, where $L = L(c, n)$. The same is fulfilled for the limits of $\{|\hat{k}|\hat{\zeta}^{(\hat{c})}_{\hat{k},\hat{n}}\}$ and $\{|\hat{a}|\hat{\zeta}^{(\hat{c})}_{\hat{k},\hat{n}}\}$ which equal the positive real number $\hat{L}$, where $\hat{L} = \hat{L}(\hat{c}, \hat{n})$. On condition that $c = \hat{c}$ and $n = \hat{n}$, it is correct:

$$L(c, n) = \hat{L}(\hat{c}, \hat{n}). \quad (12)$$

In addition, in case $k = \hat{k}$ – large negative, it is true:

$$\zeta^{(c)}_{k,n} \approx \hat{\zeta}^{(\hat{c})}_{\hat{k},\hat{n}}. \quad (13)$$

It holds $\zeta^{(c)}_{k,n} < \hat{\zeta}^{(\hat{c})}_{\hat{k},\hat{n}}$ and $|\hat{a}|\hat{\zeta}^{(\hat{c})}_{\hat{k},\hat{n}} < |k|\zeta^{(c)}_{k,n} < L(c, n) < |a|\zeta^{(c)}_{k,n} < |\hat{k}|\hat{\zeta}^{(\hat{c})}_{\hat{k},\hat{n}}$ for any $c = \hat{c}$, $n = \hat{n}$, $|k| = |\hat{k}|$ and $|a| \approx |\hat{a}|$, ($|\hat{a}| < |a|$). The rate of convergence decreases as follows $\{|k|\zeta^{(c)}_{k,n}\}$, $\{|a|\zeta^{(c)}_{k,n}\}$, $\{|\hat{k}|\hat{\zeta}^{(\hat{c})}_{\hat{k},\hat{n}}\}$ and $\{|\hat{a}|\hat{\zeta}^{(\hat{c})}_{\hat{k},\hat{n}}\}$. When $c << |k|$ ($\hat{c} << |\hat{k}|$), the sequences $\{|\hat{a}|\hat{\zeta}^{(\hat{c})}_{\hat{k},\hat{n}}\}$ and $\{|a|\zeta^{(c)}_{k,n}\}$ converge faster. For $c = 3$ and $n = 1, 2, \ldots, 10$, it is valid $L(c, n) = \hat{L}(\hat{c}, \hat{n})$ =6.593654107, 17.71249973, 33.75517722, 54.73004731, 80.6387791, 111.48189218, 147.25958974, 187.9719664, 233.61907045, 284.20092871. Assuming that $k \to +\infty$ ($\hat{k} \to +\infty$), $\{\zeta^{(c)}_{k,n}\}$, $\{|k|\zeta^{(c)}_{k,n}\}$ and $\{|a|\zeta^{(c)}_{k,n}\}$, $(\{\hat{\zeta}^{(\hat{c})}_{\hat{k},\hat{n}}\}$, $\{|\hat{k}|\hat{\zeta}^{(\hat{c})}_{\hat{k},\hat{n}}\}$ and $\{|\hat{a}|\hat{\zeta}^{(\hat{c})}_{\hat{k},\hat{n}}\})$ tend to $+\infty$. The proof of Theorem 1 is based on the numerical study of the zeros of Kummer CHF, respectively on Lemmas 5 and 6 (Tables 2 and 5).

8. Azimuthally magnetized circular ferrite waveguide

An infinitely long, homogeneous, perfectly conducting circular waveguide of radius r_0, entirely filled with lossless ferrite, magnetized in azimuthal direction to remanence by an infinitely thin switching wire, is considered. The anisotropic load has a scalar permittivity $\varepsilon = \varepsilon_0 \varepsilon_r$ and a Polder permeability tensor $\overleftrightarrow{\mu} = \mu_0 [\mu_{ij}]$, $i, j = 1, 2, 3$ of nonzero components $\mu_{ii} = 1$ and $\mu_{13} = -\mu_{31} = -\mathrm{j}\alpha$, ($\alpha = \gamma M_r/\omega, \gamma$ – gyromagnetic ratio, M_r – remanent magnetization, ω – angular frequency of the wave). The propagation of normal and slow rotationally symmetric TE modes in the structure is examined. The following quantities are used in the study of the fields of first type: β – phase constant of the wave in the guide, $\beta_f = \beta_1\sqrt{\mu_{eff}}$, $\beta_1 = \beta_0\sqrt{\varepsilon_r}$, $\beta_0 = \omega\sqrt{\varepsilon_0\mu_0}$ – natural propagation constants of the unbounded azimuthally magnetized ferrite and dielectric media of relative permittivity ε_r and of free space, respectively, $\mu_{eff} = 1 - \alpha^2$ – effective relative permeability and $\beta_2 = (\beta_f^2 - \beta^2)^{1/2}$ – transverse distribution coefficient. The expressions: $\overline{\beta} = \beta/(\beta_0\sqrt{\varepsilon_r})$, $\overline{\beta}_f = \beta_f/(\beta_0\sqrt{\varepsilon_r})$, $\overline{\beta}_2 = \beta_2/(\beta_0\sqrt{\varepsilon_r})$ and $\overline{r}_0 = \beta_0 r_0\sqrt{\varepsilon_r}$ provide universality of the results.

9. A microwave application of the complex Kummer function

9.1. Propagation problem for normal TE_{0n} modes in an azimuthally magnetized circular ferrite waveguide

The guided TE_{0n} waves in configuration described are normal, if $\bar{\beta}_2 = \left(\bar{\beta}_f^2 - \bar{\beta}^2\right)^{1/2}$ is real $\left(\bar{\beta}_f = \sqrt{\mu_{eff}}\right)$, i.e., $\bar{\beta} < \bar{\beta}_f$, $\left(\bar{\beta} > 0,\ \bar{\beta}_f > 0\right)$. They are governed by the following characteristic equation [54, 55, 57, 59–61, 63, 66, 69, 70, 72–74]:

$$\Phi\left(a, c; x_0\right) = 0\,, \tag{14}$$

where $a = 1.5 - \mathrm{j}k$, $c = 3$, $x_0 = \mathrm{j}z_0$, $k = \alpha\bar{\beta}/\left(2\bar{\beta}_2\right)$, $z_0 = 2\bar{\beta}_2\bar{r}_0$. It holds, provided $\bar{\beta}_2 = \zeta_{k,n}^{(c)}/\left(2\bar{r}_0\right)$ which defines the eigenvalue spectrum of the fields examined.

9.2. Phase characteristics

Using the roots $\zeta_{k,1}^{(3)}$ of Eq. (14) and the relations between barred quantities, the dependence of $\bar{\beta}$ on $\bar{r}_0$ with α as parameter for the normal TE_{01} mode in the ferrite waveguide is computed and plotted in Fig. 8. The solid (dashed) lines, corresponding to positive (negative) magnetization are of infinite (finite) length. Hence, transmission is possible for $\alpha_+ > 0$ $\left(\alpha_- < 0\right)$ in an unlimited from above (restricted from both sides) frequency band. The common starting point of the curves for the same $|\alpha|$ at the horizontal axis depicts the pertinent cutoff frequency $\bar{r}_{0cr} = \left[\zeta_{0,1}^{(3)}/2\right]/\left(1-\alpha^2\right)^{1/2}$. The ends of characteristics for $M_r < 0$ of co-ordinates ($\bar{r}_{0en-}$, $\bar{\beta}_{en-}$) form an envelope (dotted line), labelled with En_{1-}, limiting from the right the area of propagation for negative magnetization. The curve, marked with $\alpha = 0$ (the ferrite degenerates into isotropic dielectric) is infinitely long (transmission takes place in an unlimited from above frequency range). The characteristics for $\alpha_+ > 0$ ($\alpha_- < 0$) are single-valued (double-valued below cut-off, with an inversion point of abscissa $\bar{r}_{0i-}$). The envelope En_{1-} possesses a minimum $\min \bar{r}_{0right-} = 2L(3,1)$ at $\alpha_{-,\min} = -1\sqrt{2}$ and $\bar{\beta}_{right-,\min} = 1/\sqrt{2}$.

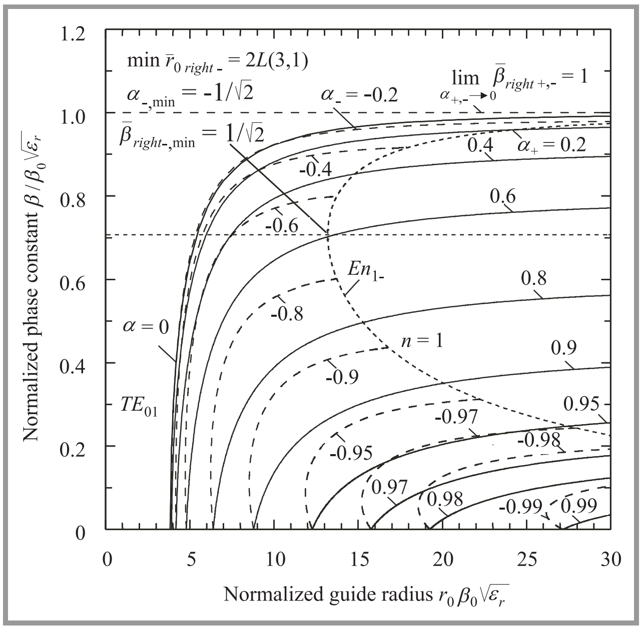

Fig. 8. Phase curves $\bar{\beta}(\bar{r}_0)$ of the normal TE_{01} mode in the circular ferrite waveguide.

9.3. Propagation conditions

Integrating the results of analysis of complex Kummer CHFs and of the problem studied, it turns out that the normal TE_{0n} waves propagate in one region whose boundaries for $M_r > 0$ and $M_r < 0$ are determined by the terms: $\alpha_{left+,-} < \alpha_{+,-} < \alpha_{right+,-}$, $k_{left+,-} < k_{+,-} < k_{right+,-}$, $\bar{\beta}_{left+,-} < \bar{\beta}_{+,-} < \bar{\beta}_{right+,-}$, $\bar{r}_{0left+,-} < \bar{r}_{0+,-} < \bar{r}_{0right+,-}$, where $\alpha_{left-} = -1$, $\alpha_{right-} = 0$, $\alpha_{left+} = 0$, $\alpha_{right+} = 1$, $k_{left-} = -\infty$, $k_{right-} = 0$, $k_{left+} = 0$, $k_{right+} = +\infty$, $\bar{\beta}_{left+,-} = 0$, $\bar{\beta}_{right+,-} = \left(1 - \alpha_{+,-}^2\right)^{1/2}$, $\bar{r}_{0left+} = \bar{r}_{0cr}$, $\bar{r}_{0right+} = +\infty$, $\bar{r}_{0left-} = \bar{r}_{0i-}$, $\bar{r}_{0right-} = \bar{r}_{0en-} = L(c,n)/\left[|\alpha_-|\left(1-\alpha_-^2\right)^{1/2}\right]$. Moreover $\bar{\beta}_{right} = \bar{\beta}_{en-}$. The subscripts "*left*", "*right*" designate the limits of domain in which certain quantity varies and the ones "+", "–" show the sign of magnetization to which the latter is relevant.

9.4. Phaser operation

The waveguide may provide differential phase shift $\Delta\bar{\beta} = \bar{\beta}_- - \bar{\beta}_+$ for TE_{01} mode when latching M_r in the area of partial overlapping $\Delta = \bar{r}_{0right-} - \bar{r}_{0left+} = \bar{r}_{0en-} - \bar{r}_{0cr}$ of the intervals $\Delta_- = \left(\bar{r}_{0left-},\ \bar{r}_{0right-}\right)$, and $\Delta_+ = \left(\bar{r}_{0left+},\ \bar{r}_{0right+}\right)$, pertinent to $\bar{\beta}_-(\bar{r}_{0-})$ and $\bar{\beta}_+(\bar{r}_{0+})$ curves for the same $|\alpha|$ ($\Delta = \Delta_- \cap \Delta_+$, Fig. 8 and Fig. 1 [74]). Hence, the condition for the geometry to operate as phaser at fixed $|\alpha|$ (the working point $\bar{r}_0$ to be part of Δ), is $\bar{r}_{0cr} < \bar{r}_0 < \bar{r}_{0en-}$, or [69]:

$$\zeta_{0,1}^{(3)}|\alpha| < 2\bar{r}_0|\alpha|\sqrt{1-\alpha^2} < 2L(3,1)\,. \tag{15}$$

Save from the graphs, $\Delta\bar{\beta}$ could be computed also directly from structure parameters, using the formulae $\Delta\bar{\beta} = A|\alpha|$, $\Delta\bar{\beta} = B/\bar{r}_0$, $\Delta\bar{\beta} = (C/\bar{r}_0)|\alpha|$ [66, 74]. The values of factors A, B, C are tabulated in [66, 74]. If $\bar{r}_0 > \bar{r}_{0en-}$, the configuration has potentialities as current controlled switch or isolator.

10. A microwave application of the real Kummer function

10.1. Propagation problem for slow $\widehat{TE}_{0\hat{n}}$ modes in an azimuthally magnetized circular ferrite waveguide

The guided $\widehat{TE}_{0\bar{n}}$ waves examined are slow, if $\hat{\bar{\beta}}_2 = \left(\hat{\beta}^2 - \hat{\beta}_f^2\right)^{1/2}$ is real ($\hat{\beta}_f^2 = \hat{\mu}_{eff}$, $\hat{\mu}_{eff} = 1 - \hat{\alpha}^2$), i.e., provided $\bar{\hat{\beta}}^2 > \bar{\hat{\beta}}_f^2$, ($\bar{\hat{\beta}} > 0$, $\bar{\hat{\beta}}_f^2 > 0$, $\bar{\hat{\beta}}_f^2 < 0$ or $\bar{\hat{\beta}}_f^2 = 0$). The solution of Maxwell equations subject to boundary condition at the wall $\bar{r} = \bar{r}_0$ yields the corresponding characteristic equation [69]:

$$\Phi(\hat{a}, \hat{c}; \hat{x}_0) = 0 \qquad (16)$$

with $\hat{a} = 1.5 + \hat{k}$, $\hat{c} = 3$, $\hat{x}_0 = 2\bar{\hat{\beta}}_2 \bar{r}_0$, $\hat{k} = \hat{\alpha}\bar{\hat{\beta}}/(2\bar{\hat{\beta}}_2)$. It is valid in case $\bar{\hat{\beta}}_2 = \hat{\zeta}^{(\hat{c})}_{\hat{k},\hat{n}}/(2\bar{r}_0)$, giving the eigenvalue spectrum looked for. (Equation (16) could be obtained from (14) putting $k = \mathrm{j}\hat{k}$ and $\bar{\beta}_2 = -\mathrm{j}\bar{\hat{\beta}}_2$).

10.2. Propagation conditions

Combining the outcomes of the study of real Kummer CHFs and of the problem regarded, it is found that the slow $\widehat{TE}_{0\hat{n}}$ modes could be guided for $\hat{M}_r < 0$ solely in two areas, set by the criteria: $\hat{\alpha}^{(1),(2)}_{left-} < \hat{\alpha}^{(1),(2)}_{-} < \hat{\alpha}^{(1),(2)}_{right-}$, $\hat{k}^{(1),(2)}_{left-} < \hat{k}^{(1),(2)}_{-} < \hat{k}^{(1),(2)}_{right-}$, $\bar{\hat{\beta}}^{(1)}_{left-} < \bar{\hat{\beta}}^{(1)}_{-} < \bar{\hat{\beta}}^{(1)}_{right-}$, $\bar{\hat{\beta}}^{(2)}_{left-} > \bar{\hat{\beta}}^{(2)}_{-} > \bar{\hat{\beta}}^{(2)}_{right-}$, $\bar{r}^{(1),(2)}_{left-} < \bar{r}^{(1),(2)}_{0-} < \bar{r}^{(1),(2)}_{0right-}$, with $\hat{\alpha}^{(1)}_{left-} = -1$, $\hat{\alpha}^{(1)}_{right-} = 0$, $\hat{\alpha}^{(2)}_{left-} = -\infty$, $\hat{\alpha}^{(2)}_{right-} = -(2\hat{n}+1)$, $\hat{k}^{(1)}_{left-} = -\infty$, $\hat{k}^{(1)}_{right-} = -(2\hat{n}+1)/2$, $\hat{k}^{(2)}_{left-} = \hat{\alpha}^{(2)}_{-}/2$, $\hat{k}^{(2)}_{right-} = -(2\hat{n}+1)/2$, $\bar{\hat{\beta}}^{(1)}_{left-} = \left[1 - \left(\hat{\alpha}^{(1)}_{-}\right)^2\right]^{1/2}$, $\bar{\hat{\beta}}^{(1)}_{right-} = \left\{\left[1-\left(\hat{\alpha}^{(1)}_{-}\right)^2\right] / \left[1 - \left(\hat{\alpha}^{(1)}_{-}/(2\hat{n}+1)\right)^2\right]\right\}^{1/2}$, $\bar{\hat{\beta}}^{(2)}_{right-} = \left\{\left[\left(\hat{\alpha}^{(2)}_{-}\right)^2 - 1\right] / \left[\left(\hat{\alpha}^{(2)}_{-}/(2\hat{n}+1)\right)^2 - 1\right]\right\}^{1/2}$, $\bar{\hat{\beta}}^{(2)}_{left-} = +\infty$, $\bar{r}^{(1)}_{0left-} = \hat{L}(\hat{c}, \hat{n}) / \left\{\left|\hat{\alpha}^{(1)}_{-}\right| \left[1 - \left(\hat{\alpha}^{(1)}_{-}\right)^2\right]^{1/2}\right\}$, $\bar{r}^{(1)}_{0right-} = +\infty$, $\bar{r}^{(2)}_{0left-} = 0$, $\bar{r}^{(2)}_{0right-} = +\infty$. The superscripts (1), (2) designate the zone to which the corresponding quantity relates. Thus, the symbol $\widehat{TE}_{0\hat{n}}$ is a general notation for two waves $\widehat{TE}^{(1)}_{0\hat{n}}$ and $\widehat{TE}^{(2)}_{0\hat{n}}$, supported in the first and second regions, respectively.

10.3. Phase characteristics

Taking into account the propagation conditions and repeating the procedure, described in Subsection 9.2 with the roots $\hat{\zeta}^{(3)}_{\hat{k},1}$ of Eq. (16), the $\bar{\hat{\beta}}^{(1)}_{-}(\bar{r}^{(1)}_{0-})$ and $\bar{\hat{\beta}}^{(2)}_{-}(\bar{r}^{(2)}_{0-})$ – characteristics with $\hat{\alpha}^{(1)}_{-}$ and $\hat{\alpha}^{(2)}_{-}$ as parameters for the slow $\widehat{TE}^{(1)}_{01}$ and $\widehat{TE}^{(2)}_{01}$ modes, respectively in the structure are computed and presented with dashed curves of infinite length in Figs. 9 and 10, respectively. Thus, transmission takes place for $\hat{\alpha}_{-} < 0$ in two unlimited from above frequency bands. An envelope (dotted line), labelled with $\hat{E}n_{1-}$ (for the co-ordinates of the points of which $(\bar{r}^{(1)}_{0en-}, \bar{\hat{\beta}}^{(1)}_{en-})$ it is valid $\bar{r}^{(1)}_{0en-} = \bar{r}^{(1)}_{0left-}$ and $\bar{\hat{\beta}}^{(1)}_{en-} = \bar{\hat{\beta}}^{(1)}_{left-}$), restricts from the left the area of propagation

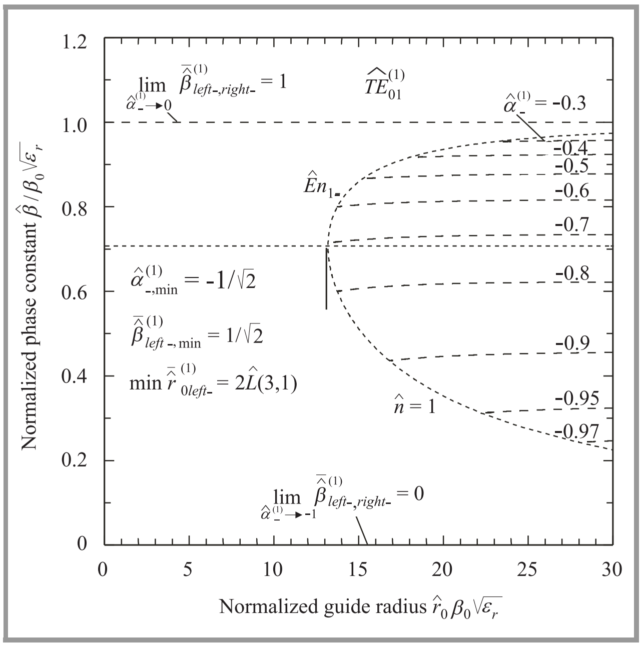

Fig. 9. Phase curves $\bar{\hat{\beta}}^{(1)}_{-}(\bar{r}^{(1)}_{0-})$ of the slow $\widehat{TE}^{(1)}_{01}$ mode in the circular ferrite waveguide for $-1 < \hat{\alpha}^{(1)}_{-} < 0$.

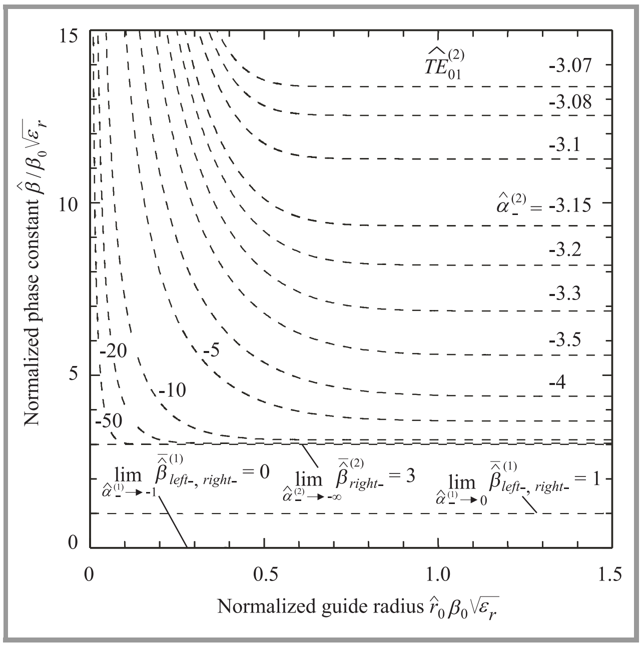

Fig. 10. Phase curves $\bar{\hat{\beta}}^{(2)}_{-}(\bar{r}^{(2)}_{0-})$ of the slow $\widehat{TE}^{(2)}_{01}$ mode in the circular ferrite waveguide for $\hat{\alpha}^{(2)}_{-} < -3$.

in case of weak anisotropy (Fig. 9). It has a minimum $\min \bar{r}_{0left-} = 2\hat{L}(3,1)$ at $\hat{\alpha}^{(1)}_{-,\min} = -1\sqrt{2}$ and $\bar{\hat{\beta}}^{(1)}_{left-,\min} = 1\sqrt{2}$. A comparison of both sets of curves shows that a large slowing down is provided if the anisotropy is strong, especially in case $\bar{r}^{(2)}_0$ is small (see Fig. 10). Ferrite switches and isolators are the possible applications of the structure.

11. Areas of TE_{01} mode propagation

The joint consideration of the results of analysis of the anisotropic waveguide shows that in case of positive (counterclockwise) magnetization there is one (densely hatched) area in which normal TE_{01} mode is supported (Fig. 11). If the magnetization is negative (clockwise), the areas are already three: one (densely hatched) of normal and two (sparsely hatched) of slow wave propagation (Fig. 12).

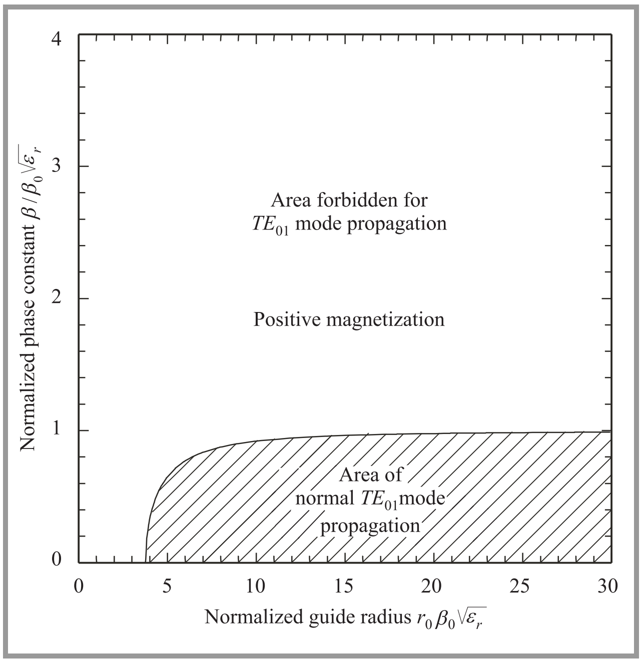

Fig. 11. Areas of TE_{01} mode propagation in case of positive magnetization.

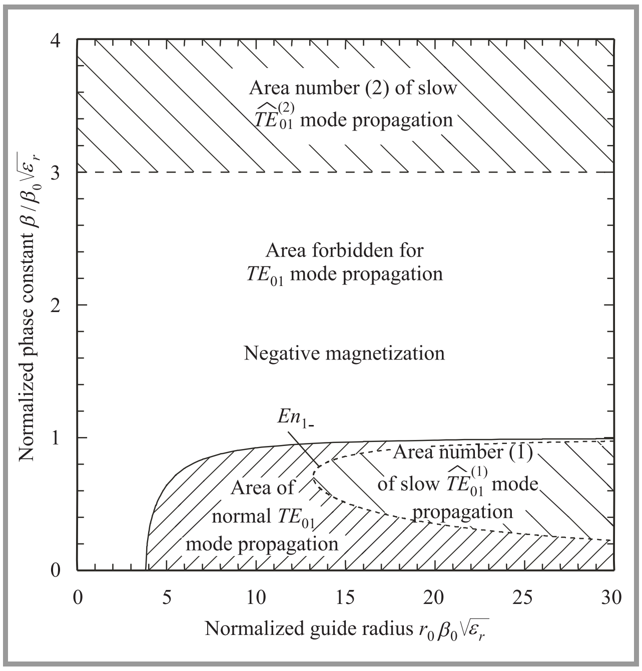

Fig. 12. Areas of TE_{01} mode propagation in case of negative magnetization.

An important corollary of Theorem 1 is the coincidence of the envelopes of characteristics for $M_r < 0$ of the normal (Fig. 8) and the slow mode (Fig. 9) in one curve (the dotted line in Fig. 12, labelled En_{1-}) which does not belong to any of the zones and serves as their common border, delimiting them. Indeed, since in view of Eq. (12) $L(3,1) = \hat{L}(3,1)$, the points $(\overline{r}_{0en-},\ \overline{\beta}_{en-})$ and $(\overline{\hat{r}}_{0en-}^{(1)},\ \hat{\overline{\beta}}_{en-}^{(1)})$ in the $\overline{r}_0 - \overline{\beta}$ phase plane, forming the En_{1-} and $\hat{E}n_{1-}$ – characteristics for the TE_{01} and $\widehat{TE}_{01}^{(1)}$ modes, respectively, are identical for all values of parameters $\alpha_- = \hat{\alpha}_-^{(1)}$ whose intervals of variation, determined by the corresponding propagation conditions in Sections 9.3 and 10.2, are the same. Area number (2) for the slow wave is separated from aforesaid two ones by a region where no transmission is allowed.

12. Conclusions

Some basic concepts of the theory, the special cases and examples of the use of the CHFs in different fields of physics are considered. The opinion is declared that a universal mathematical procedure, based on them would successfully substitute the methods for analysis of a large number of tasks, utilizing the numerous functions which are their special cases. This approach would make possible to reveal the interior connections among plenty of phenomena and would facilitate the physical interpretation of the results from their description, as well as the process of their generalization.

The problems for normal and slow rotationally symmetric TE modes in the circular waveguide, uniformly filled with azimuthally magnetized ferrite are threshed out as a sphere of microwave application of the complex, and real Kummer CHFs rescpectively. The propagation conditions and phase characteristics of the structure are obtained, using various properties of the wave function, established analytically and/or numerically. The main result of the study is that for positive (negative) magnetization one area of normal (three areas – one of normal and two of slow) TE_{01} mode propagation exists (exist). The region of normal and the first one of slow waves transmission in case of negative magnetization are demarcated by an envelope curve which can be traced by means of a numerically proved theorem for identity of the zeros of certain Kummer functions. The areas mentioned are separated from the second one for slow wave propagation by a zone in which no fields can be sustained. The phase behaviour reveals the potentialities of the structure as a remanent phaser (for normal waves) or as a current controlled switch and isolator (for both kinds of waves). The criterion for phaser operation of waveguide is deduced as a direct corollary of the aforesaid theorem for the zeros. A large number of configurations, containing a central ferrite rod of azimuthally magnetized ferrite, coated by an arbitrary number of dielectric layers could be described, extending the analysis method based on the Kummer CHFs.

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