Paper

Lorentzian Operator for Angular Source Localization with Large Array

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Abstract—Source localization problem consists of an ensemble of techniques that are used to obtain spatial information of present radiation in given medium of propagation, with a constraint of the antenna geometry and the characteristics of radiating sources. This condition gives multitude of cases to study, hence several methods were proposed in the literature. In this paper, a new algorithm for estimating the Direction of Arrival (DoA) of narrowband and far field punctual sources is introduced. By exploiting the spectrum of covariance matrix of received data, the Lorentzian function on spectral matrix to filter the eigenvalues is applied. This filtering process eliminates the eigenvalues belonging to signal subspace. Parameters of Lorentz function are adjusted using first and second statistics of eigenvalues. The algorithm requires the knowledge of minimum eigenvalue and is performing when the dimension of antenna is relatively large which is confirmed by several Monte Carlo simulations.

Keywords—array processing, Direction of Arrival, narrowband, operator.

1. Introduction

In the context of array signal processing, source localization [1] refers to the techniques implemented to detect the location of present radiation in space. The origin of these radiations is often considered to be punctual sources due to far field assumption. The radius of propagation is larger than the maximum antenna dimension [2]. These techniques are valid for both electromagnetic and acoustic waves, thus a source can be cosmic, a cell phone, a seismic wave, sound in underwater and so on. Each source is characterized by its frequency, for example it can be narrowband [2] or wideband [3]. Due to this diversity, this field of research has attracted more interest due to its usefulness in many applications including radioastronomy [4], geolocalization such as Global Positioning System (GPS) [5], localization of mobile stations [6], radar and sonar [7] in both civilian and military applications, underwater acoustics [8], medical signal processing and seismology.

Most of localization techniques exploit the space-time diversity, some methods are based on time delay, known as Time of Arrival (ToA) [1]. This concept requires synchronization between the transmitters and the receivers. Other methods use the properties of propagating wavefront along the antenna to calculate the Angle of Arrival (AoA) of radiating sources [2]. This mechanism has the advantage of no requirement of synchronization. In fact, to compute the angle of single source, at least two aerials are needed and the distance between them is a function of wavelength of incoming wave. In case of multiple sources, the resulting wavefield is a superposition of each radiation. In this situation, an antenna with larger number of sensors must be used, thus the antenna beamwidth becomes narrower, which gives the ability to separate two sources with small angular difference.

In some cases, the problem of localization becomes difficult, for example, when some sources with different signal power are present, or the propagating signals have different carrier frequencies or when the source signals are highly correlated [9]. The preliminary solutions are based on preprocessing techniques [2] to decorrelate the waveforms. Additionally, some of these problems are caused by the transmission channel. During the propagation many phenomena can occur [10], for example a wave can be scattered, when it hits objects having dimensions smaller than the wavelength. This condition is known as Rayleigh scattering [10]. When a wavefield enters a medium with different electrical properties than the previous one, a refraction occurs [10]. Another type of deviation can happen when wave faces a smooth surface like metal, a reflection takes place with the Angle of Incidence (AoI) equals the angle of reflection in this situation.

The problem of localization depends on the environment and search dimensions, one dimensional scan focuses only on azimuth angle, this type requires only one dimensional arrays geometry. For two dimensional localization, it is mandatory to use two dimensional arrays such as circular [11], rectangular [2], L-shaped [12] and fractal arrays.

Concerning the mathematical aspects, some Direction of Arrival (DoA) techniques are based on extracting signals information from second order statistics [2]–[4], in the other hand alternative approaches use high order statistics [13]. Covariance based methods (also called spectral matrix or cross correlation matrix) can be divided into beamforming and eigendecomposition techniques [2], subspace based techniques have resolution power that is able to locate sources under angular limit resolution of array. These techniques use several spectral decomposition which are eigendecomposition [2], QR or LU factorization and Singular Value Decomposition (SVD) [14].

Recent researches are focused on enhancing the eigen-based approaches, when external factors impact the electrical properties of sensors, such as temperature, humidity, pressure and vibration. These variables generate coupling effects between sensors, which degrade the performance of DoA methods [15]. Another type of ongoing researches offer a consistency of spectral techniques when the system dimensions become larger [16], precisely number of sensors and number of acquired samples. In the other hand, as the dimensions tend to infinity, a computational complexity increases, recent solution implements a non-regular sampling of impinging signals, this method is known as compressed/compressive sensing [17].

In this paper, authors introduce a new algorithm for localizing narrowband sources. Using the order of the covariance matrix spectrum, a new operator that performs a filtering operation on eigenvalues to isolate the noise subspace is introduced which is orthogonal to signal subspace. From mathematical definition this operator is the Lorentz function of Hermitian matrix, also known as Cauchy distribution [18]. This function needs adjustment of two parameters, the index of the function's peak, which corresponds to the smallest eigenvalue and the width that is related to a threshold between signal and noise eigenvalues. The authors use a theorem that offers a bound of minimum eigenvalue using first and second order statistics of spectral eigenvalues. The obtained bound is efficient, when the antenna contains large number of sensors comparatively to number of sources.

The proposed operator is validated through several Monte Carlo simulations along with other techniques.

In the Section 2 of this paper, the statistical signal model for DoA problem is described. In the Section 3, the author's contribution is presented and in the Section 4 some computer simulation results for performance analysis is shown.

2. Statistical Data Model

Let us consider a geometry given Fig. 1 which consists of one punctual radiating source and a uniform linear array of N sensors placed along y axis, the system source antennas are placed in the same horizontal plane $(x, y, z = cte)$.

The sensors are located in the farfield region relative to the source where the wavefront arriving are considered plane waves. The uniform distance between the sensors is half the wavelength of the emitting source $d = \lambda/2$ and the farfield condition implies that the Line of Sight (LoS) r_0 is much larger than the length of the array $r_0 \gg L_\lambda = (N-1)d$. The propagation model [21] is given by the equation:

$$\vec{\nabla}^2 \vec{E}_i(\vec{r},t) = \frac{1}{c^2}\frac{\partial^2 \vec{E}_i(\vec{r},t)}{\partial t^2}, \tag{1}$$

where c denotes the velocity of propagation $c = (\mu\varepsilon)^{\frac{-1}{2}}$ and $\vec{\nabla}^2$ is Laplace operator $\vec{\nabla}^2 = \Delta = \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} + \frac{\partial^2}{\partial z^2}$. By

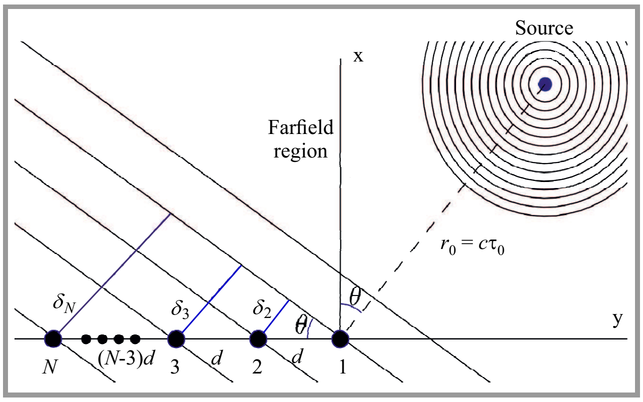

Fig. 1. A farfield punctual source emitting radiations received by a uniform linear array of sensors with azimuth angle θ relative to the reference.

assuming the transverse mode of propagation by considering the z component of the wave vector $\vec{E}_i = (E_x = 0, E_y = 0, E_z \neq 0)$, the solution of the i-th source is given by:

$$E_z(\vec{r},t) \simeq s_i(t) e^{j(\omega t - \vec{k}_i.\vec{r})}. \tag{2}$$

The solution is only an approximation because the variation of the function $s_i(t)$ is temporally negligible than the oscillation of the carrier wave with frequency $\omega = 2\pi f_c$ where $c = \lambda f_c$ [21]. This is also known as Slowly Varying Envelope Approximation (SVEA) in other fields. The $\vec{k}_i$ is the wave vector having the components in spherical coordinates as:

$$\vec{k}_i = \frac{-2\pi}{\lambda_i}\begin{pmatrix} \sin\varphi_i \cos\theta_i \\ \sin\varphi_i \sin\theta_i \\ \cos\varphi_i \end{pmatrix}, \tag{3}$$

where (θ_i, φ_i) are the azimuth and elevation of the i-th source respectively. The received wavefront is a superposition of all existing sources and the magnitudes of the collected signals are proportional to $\sum_{i=1}^{P} \vec{E}_i$. During the acquisition time $T = T_s K$, where T_s is the sampling period and K is the number of measurements, the carrier frequencies [21] terms are removed from the signals $e^{j\omega t}$, so for any measurement at instant $t \in \{1, \ldots, K\}$, the signal at the m-th sensor with position $\vec{r}_m$, is given by:

$$x_m(t) = \sum_{i=1}^{P} s_i(t) e^{-j\vec{k}_i.\vec{r}_m} + n_m(t), \tag{4}$$

where $n_m(t)$ is the additive noise at the m-th sensor considered complex and random process with zero mean. While considering the uniform linear array (ULA), the complex vector of signals at instant t is:

$$x(t) = A(\theta)s(t) + n(t), \tag{5}$$

with $s(t) \in \mathbb{C}^{P\times 1}$ is the source waveforms, $n(t) \in \mathbb{C}^{N\times 1}$ is the noise waveforms and $A(\theta) \in \mathbb{C}^{N\times P}$ is the steering matrix given by:

$$A = \begin{pmatrix} 1 & \dots & 1 \\ e^{-j\mu_1} & \dots & e^{-j\mu_P} \\ \dots & \dots & \dots \\ e^{-j(N-1)\mu_1} & \dots & e^{-j(N-1)\mu_P} \end{pmatrix}, \quad (6)$$

where $\mu_i = 2\pi d\lambda^{-1}\sin(\theta_i)$ and θ_i is the AoA of the i-th punctual source. The rank of the steering matrix A in Eq. (6) is P such that the P sources are located in different angular positions θ_i. In a compact form, the matrix of received signals is $X(t) = A(\theta)S(t) + N(t)$ with the dimensions $X(t) \in \mathbb{C}^{N\times K}$, $S(t) \in \mathbb{C}^{P\times K}$ and $N(t) \in \mathbb{C}^{N\times K}$. The objective is to calculate a localization function $f(\theta)$ from which positions θ_i can be derived. Most of high resolution DoA techniques are based on second order statistics of $X(t)$, the spectral matrix $< x(t)x^{+}(t) >$ has the following theoretical expression:

$$\Gamma = \Gamma_s + \Gamma_n, \quad (7)$$

where $\Gamma_s = A\Gamma_{ss}A^{+}$, $\Gamma_n = \sigma^2 I_N$ and $\Gamma_{ss} =< s(t)s^{+}(t) >$ is the correlation matrix of sources, if the waveforms are not correlated, then $[\Gamma_{ss}]_{ij} = \delta_{ij}\sigma_i^2$, where σ_i^2 is the power of the i-th signal. In decreasing order, the spectrum of matrix Γ is given as:

$$\sigma_\Gamma = \left\{\lambda_1 \geqslant \lambda_2 \geqslant \dots \geqslant \lambda_P > \lambda_{P+1} \simeq \dots \simeq \lambda_N = \sigma^2\right\}.$$

For example one of the high resolution DoA operators [19] exploits the threshold between the smallest signal eigenvalue λ_P and the largest noise eigenvalue λ_{P+1} to calculate the projector in the noise subspace. This latter is obtained by spectral decomposition, indeed the spectral matrix is decomposed as $\Gamma = U\Lambda U^{+}$, where Λ is a diagonal matrix of eigenvalues and $U \in \mathbb{C}^{N\times N}$ is the orthonormal matrix such as $\|U\|_F = \sqrt{Tr\{UU^{+}\}} = \sqrt{N}$. The first P columns of U correspond to the largest eigenvalues to form a base of signal subspace $U_s \in \mathbb{C}^{N\times P}$ and the remaining $N-P$ columns form a noise subspace $U_n = [u_{P+1}, \dots, u_N]$. The projector into the noise subspace $P_n \in \mathbb{C}^{N\times N}$ is defined by the relation $P_n = U_n U_n^{+}$. For given steering vector $a(\theta)$ with testing angle $\theta \in \Omega = [\theta_{min}, \theta_{max}]$, the localization function verifies

$$f(\theta) = a^{+}(\theta)P_n a(\theta) = \begin{cases} 0 & \text{if } \theta \text{ is DoA,} \\ \neq 0 & \text{otherwise.} \end{cases} \quad (8)$$

After performing an angular scan in the region Ω, the indexes of the peaks of $f(\theta)$ indicate the angles of arrival of radiating sources.

3. Lorentzian DoA Algorithm

For real variable $x \in \mathbb{R}$, the single peak normalized Lorentz function centered at x_0 is defined by [18]:

$$f(x) = \frac{1}{\pi}\frac{\beta}{(x-x_0)^2 + \beta^2}, \quad (9)$$

with parameters (x_0, β) such that:

$$\int_{\mathbb{R}} f(x)dx = 1.$$

At the abscissa x_0, the function has maximum value of $f(x_0) = \frac{1}{\beta\pi}$ and equals half maximum at $x = x_0 \pm \beta$, which makes the Full Width at Half Maximum (FWHM) to be 2β, the inflection points x_c occur when the second derivative is null:

$$\frac{\partial^2 f(x)}{\partial x^2} = \frac{-2\beta}{\pi((x-x_0)^2+\beta^2)^2} + \frac{8\beta(x-x_0)^2}{\pi((x-x_0)^2+\beta^2)^3} = 0. \quad (10)$$

This equation has a solution of $x_c = x_0 \pm \frac{\beta}{\sqrt{3}}$, and $f(x_c) = \frac{3}{4\pi\beta}$. Remark that at the inflection points the magnitude is reduced, comparatively to the maximum value, by a factor of 0.75.

Given the condition that the spectral matrix Γ is positive definite, then $\sigma_\Gamma \in \mathbb{R}^{+}$. Let us denote $\lambda \in \mathbb{R}^{+}$ the scalar function representing the eigenvalues and $\lambda_{\min}$ its lowest value, the spectrum is considered to be binary $\{\lambda_n \simeq \lambda_{\min}, \lambda_s\}$. We search for function that normalizes the eigenvalue $\lambda_{\min}$ and forces any signal eigenvalue λ_s to zero, for this purpose the following version of Lorentz function is used:

$$f(\lambda) = \frac{1}{\alpha(\lambda - \lambda_{\min})^2 + 1}, \quad (11)$$

where α is the scaling parameter of the width, this principle is illustrated in Fig. 2. From the Eq. (11), we need to

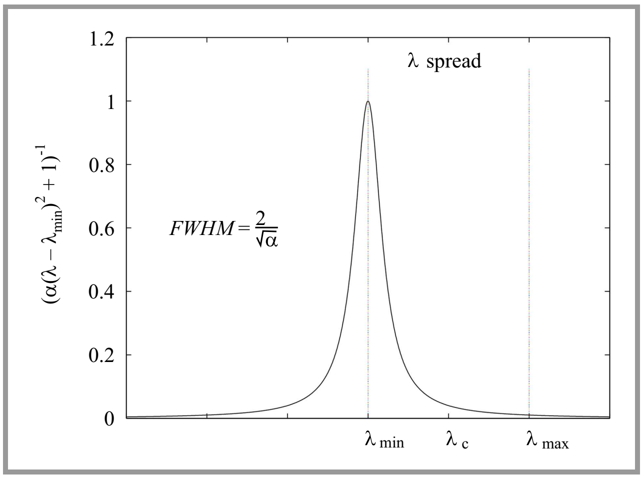

Fig. 2. Lorentzian function with parameters $\{\alpha, \lambda_{\min}\}$ applied to the spectrum of operator Γ.

calculate two parameters. The minimum eigenvalue can be estimated using the power method, first the largest eigenvalue $\lambda_{\max} = \lambda_1$ is computed, next the $\lambda_{\min}$ is calculated using the condition number $\tau = \lambda_{\max}/\lambda_{\min}$. The random vector $\phi \in \mathbb{C}^{N\times 1}$ with norm $||\phi||_\infty = 1$ is chosen, and for $m \geq 2$ the following iterations are performed:

$$\begin{aligned} \phi_{m+1} &= \Gamma\phi_m, \\ \mu_m &= \phi_m^{+}\phi_{m+1}, \\ \phi_m &= \frac{\phi_{m+1}}{\phi_{m+1}^{+}\phi_{m+1}}. \end{aligned} \quad (12)$$

When $\mu_m \to \lambda_{\max}$, the minimum eigenvalue is calculated by the following equation:

$$\lambda_{\min} = \frac{\lambda_{\max}}{\tau} = \frac{\lambda_{\max}}{||\Gamma||_2||\Gamma^{-1}||_2}. \tag{13}$$

The inversion of spectral matrix is required, the scaling parameter α is necessarily related to a threshold λ_c that differentiates the two subsets $\{\lambda_s, \lambda_n\}$. The authors impose the condition that at abscissa λ_c, the function equals the value $\varepsilon = 10^{-3}$, this is equivalent to $\alpha(\lambda_c - \lambda_{min})^2 \simeq \varepsilon^{-1}$, consequently the chosen scaling parameter is given by:

$$\alpha = \frac{\varepsilon^{-1}}{(\lambda_c - \lambda_{min})^2} = \frac{10^3}{(\lambda_c - \lambda_{min})^2}. \tag{14}$$

The threshold λ_c is proposed as the bound of minimum eigenvalue $\lambda_{\min}$, this theoretical bound can be calculated using only the trace of spectral matrix. The theorem of the smallest eigenvalue bounds [20] is based on mean and standard deviation of σ_Γ, before announcing the theorem, the following variables are defined:

$$< \lambda >= \frac{tr(\Gamma)}{N} = \frac{1}{N}\sum_{i=1}^{N}\Gamma_{ii}, \tag{15}$$

$$\Delta\lambda = \sqrt{< \lambda^2 > - < \lambda >^2} = \sqrt{\frac{tr(\Gamma^2)}{N} - \left(\frac{tr(\Gamma)}{N}\right)^2}. \tag{16}$$

Using these two statistics and for matrix with real eigenvalues, the bounds for smallest and largest eigenvalues are given by the Theorem 1.

Theorem 1: For Hermitian matrix $\Gamma \in \mathbb{C}^{N\times N}$ $(\Gamma^+ = \Gamma)$, the extremum eigenvalues are bounded by:

$$\left(< \lambda > -\Delta\lambda\sqrt{N-1}\right) \leq \lambda_{\min} \leq \left(< \lambda > -\frac{\Delta\lambda}{\sqrt{N-1}}\right), \tag{17}$$

$$\left(< \lambda > +\frac{\Delta\lambda}{\sqrt{N-1}}\right) \leq \lambda_{\max} \leq \left(< \lambda > +\Delta\lambda\sqrt{N-1}\right). \tag{18}$$

The proof is presented in [20]. The smallest signal eigenvalue satisfies $\lambda_s > \lambda_{\min}$, for a relatively large array $(N > 2P)$. The proposed threshold is given by:

$$\lambda_c =< \lambda > -\frac{\Delta\lambda}{\sqrt{N-1}}. \tag{19}$$

Note: While having only two sensors $\Gamma \in \mathbb{C}^{2\times 2}$ and eventually single source the eigenvalues are exactly $\lambda_{\min} = < \lambda > -\Delta\lambda$, $\lambda_{\max} =< \lambda > +\Delta\lambda$. This special case can be used in the presence of single source $P = 1$ and N sensors where the $N/2$ spectral matrices Γ_i for $i = 1, \ldots, N/2$ can be calculated, and theirs eigenvalues using the above equations are computed.

After describing the theoretical expressions for the couple $\{\alpha, \lambda_{min}\}$, the Lorentz function is now adaptive to the variation of parameters describing the physical system. The application of the proposed function on self adjoint operator Γ acts on its eigenvalues $f(\Gamma) = Uf(\Lambda)U^+$, then we have the following result.

Proposition: Given Hermitian matrix $\Gamma = K^{-1}XX^+$, The Lorentz operator defined by:

$$f(\Gamma) = \left(\alpha(\Gamma - H)^2 + I_N\right)^{-1} \tag{20}$$

is an approximation to the projector into the noise subspace, with $H = \lambda_{\min}I_N$, $\alpha = \frac{10^3}{(\lambda_c - \lambda_{\min})^2}$ and $\lambda_c =< \lambda > -\frac{\Delta\lambda}{\sqrt{N-1}}$. Indeed, developing the above equation, based on the relation $f(\Gamma) = Uf(\Lambda)U^+$, yields to the decomposition:

$$\begin{aligned} f &= U\frac{I_N}{\alpha(\Lambda - \Gamma_0)^2 + I_N}U^+ = \sum_{g=1}^{N}\frac{u_g u_g^+}{\alpha(\lambda_g - \lambda_{\min})^2 + 1} \\ &= \sum_{i=1}^{P}\frac{u_i u_i^+}{\alpha(\lambda_i - \lambda_{\min})^2 + 1} + \sum_{j=P+1}^{N}\frac{u_j u_j^+}{\alpha(\lambda_j - \lambda_{\min})^2 + 1} \\ &\simeq \sum_{j=P+1}^{N} u_j u_j^+ \simeq P_n. \end{aligned} \tag{21}$$

From numerical experiments, the obtained operator is not an exact a projector because either the noise eigenvalues are not normalized and have some fluctuating errors (example $f(\lambda_n) = 0.98$), or the signal eigenvalues are not totally annihilated (e.g. $f(\lambda_s) = 10^{-3}$). The Algorithm 1 summarizes the proposed method.

Algorithm 1: Lorentzian operator algorithm

Input: $\Gamma \in \mathbb{C}^{N\times N}$ $(N > P)$.

1. Compute $\lambda_{\min}$ using power method for example.
2. Compute statistics of operator Γ
   $m = \frac{Tr(\Gamma)}{N}$ and $s = \sqrt{\frac{Tr(\Gamma^2)}{N} - m^2}$.
3. Compute parameters $\lambda_c = m - \frac{s}{\sqrt{N-1}}$,
   $\alpha = \frac{10^3}{(\lambda_c - \lambda_{\min})^2}$ and $H = \lambda_{\min}I_N$.
4. Calculate $P_n = \left(\alpha(\Gamma - H)^2 + I_N\right)^{-1}$.

4. Results and Discussion

4.1. Simulation Results

In this section, some computer simulations using a configuration of Uniform Linear Array of $N = 11$ sensors considered isotropic and identical are performed. The available range for this type of array is $\Omega = [\frac{-\pi}{2}, \frac{\pi}{2}]$.

The distance between the sensors is half the wavelength of the carrier waves. It is assumed the presence of $P = 4$ narrowband and far field sources impinging on array from directions $-80°$, $15°$, $20°$ and $56°$, the number of samples is set to $K = 200$. The signals are chosen to be ergodic complex and zero mean random processes with uniform power of 1 W. In all the tests, the perturbative noise is additive and also complex zero mean random process uncorrelated between the sensors and independent of $s(t)$. The noise power is derived from $SNR = 20\log(\frac{1}{\sigma})$. Figure 3 represents an average of $L = 100$ Monte Carlo trials of Lorentzian localization function, with $N = 11$, $P = 4$, $\theta = [-80°, 15°, 20°, 56°]$, $K = 200$, $d = \lambda/2$, $s(t) \sim \mathscr{CN}(0, I_4)$ and $SNR = 5$ dB.

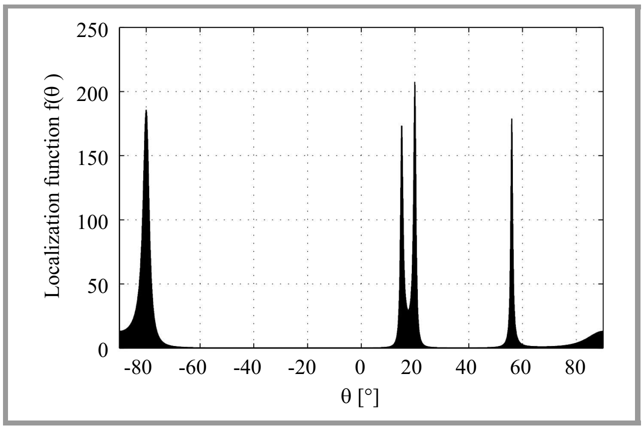

Fig. 3. Average of 100 trials of the proposed operator.

The obtained result proves that the function has the ability to separate the sources. In the second test, the proposed operator with several spectra is compared. The authors choose a critical situation where the transmitted signals and noise signals are equipowered $SNR = 0$ dB. Figure 4 presents an average of $L = 100$ trials of Lorentzian function against the first three spectra, which are based on subspace computation with $L = 100$ trials, $N = 11$, $P = 4$, $\theta = [-80°, 15°, 20°, 56°]$, $K = 200$, $d = \lambda/2$ and $s(t) \sim \mathscr{CN}(0, I_4)$ and $SNR = 0$ dB.

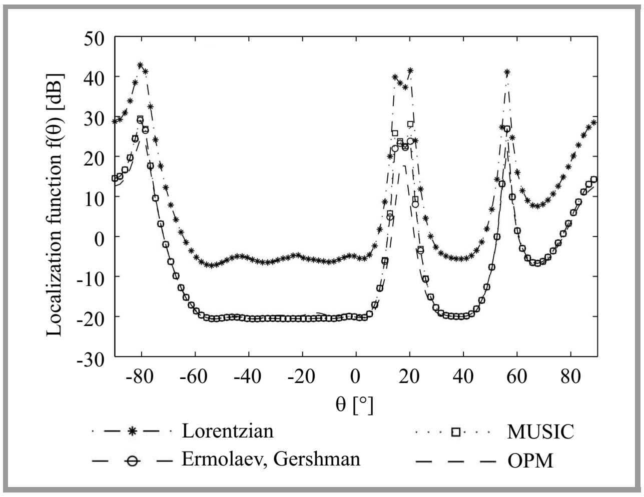

Fig. 4. Lorentzian operator against three different spectra.

Lorentzian spectrum is compared with Multiple Signal Classification (MUSIC) method [23], Ermolaev and Gershman subspace [19] with parameter $m = 10$, and orthonormalized propagator [24]. Schmidt's method and Ermolaev subspace are identical in this case. They are successful in locating all angle indexes, the OPM could locate the farthest source at 80° but did not separate sources at 15° and 20°. Lorentzian spectrum identifies all the AoAs with higher magnitudes.

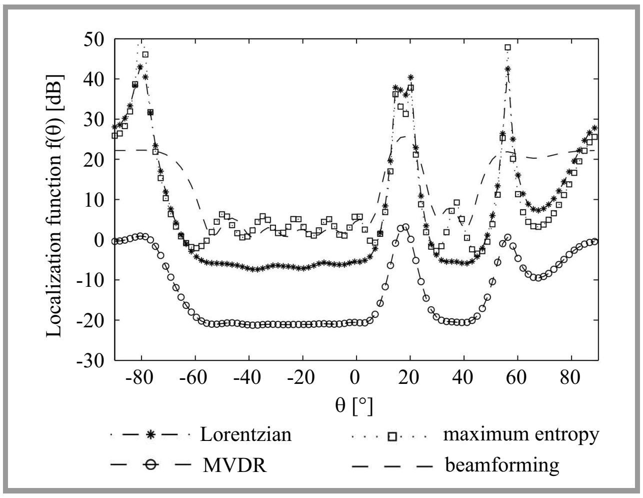

Fig. 5. Lorentzian operator against three different spectra.

In Fig. 5 the presented function is compared with three other techniques (with the same conditions as in Fig. 4). The authors realize that Capon's method (MVDR) [2]–[10] failed to separate sources at 15°, 20° and the shape of its localization function is similar to the beamforming [2] in these conditions. Maximum entropy method [10] with parameter $l = 1$ and Lorentzian are approximately the same.

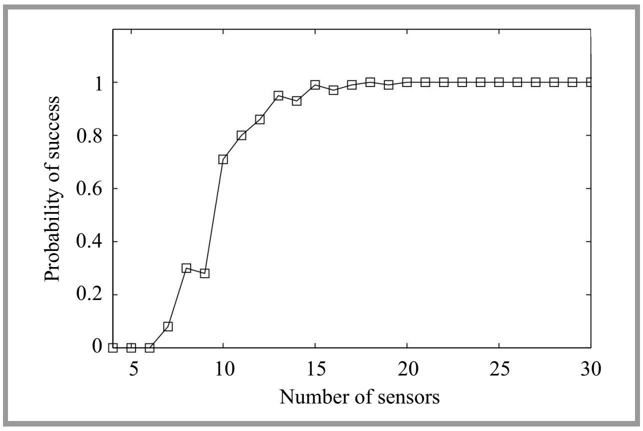

Fig. 6. Probability of success of Lorentzian operator.

To evaluate the performance with respect to the dimension of the antenna in Fig. 6 the probability of success over $L = 100$ trials with varying number of sensors starting from $N = 4$ is presented, with $L = 100$ trials, $P = 4$, $\theta = [-80°, 15°, 20°, 56°]$, $K = 200$, $d = \lambda/2$, $s(t) \sim \mathscr{CN}(0, I_4)$ and $SNR = 5$ dB. The result informs that the bound λ_c in Eq. (27) is not valid unless the number of sensors N is about triple the number of sources P, in fact the probability of detection reaches 90% when $N = 12$.

4.2. Experimental Results

In this second part of performance evaluation, the resolution power of the proposed operator using underwater acoustical data obtained from linear array of hydrophones [25] is tested. The received echoes are generated by two acoustic sources. The Table 1 summarizes the data description.

Table 1
Description of experimental underwater acoustic data

Data	Value
Number of hydrophones	$N = 6$
Interelement spacing	$d = 0.9$ m
Length of the array	$L_\lambda = 4.5$ m
Number of samples	$K = 4096$
Sources wavelength	$\lambda = 5.32$ m
Average power of data $X(t)$	$Tr(\Gamma)/N = 0.99$ W
Eigenvalues of Γ	[4.3648, 1.4835, 0.1225, 0.0220, 0.0051, 0.0007]
Number of sources	$P = 2$
Angular step	$d\theta = 0.1^\circ$ in the range $[-\pi/2, \pi/2]$
Estimated noise power	$\sigma^2 \simeq 0.0376$ W
Estimated powers of sources	$\sigma_1^2 \simeq 0.7188$ W and $\sigma_2^2 \simeq 0.3092$ W
Estimated signal to noise ratios	$SNR1 \simeq 25.62$ dB and $SNR2 \simeq 18.30$ dB

The noise power or minimum eigenvalue is computed using the equation:

$$\sigma^2 = \frac{1}{4}\sum_{j=3}^{6} \lambda_j \,. \tag{22}$$

The powers of sources are calculated using the beamforming as:

$$f_{BF}(\theta) = \frac{1}{N^2} a^{+}(\theta)\Gamma a(\theta)\,, \tag{23}$$

where the values of two largest peaks are approximately equal to the powers of sources σ_1^2 and σ_2^2. The implemented steering vector $a(\theta) \in \mathbb{C}^{6\times 1}$ is defined by the relations:

$$\begin{cases} a(\theta) = e^{-2\pi j r^T \lambda^{-1} \sin(\theta)} \\ r \quad\;\; = [0.00, 0.90, 1.80, 2.70, 3.60, 4.50] \end{cases} .$$

The seven DoA spectral techniques are applied to identify the locations of the acoustic sources which are the Lorentzian operator, MUSIC projector [23], Orthonormal Propagator (OPM) [24], Ermolaev and Gershman operator [19], Maximum Entropy Method (MEM) [10], where the operator is computed using the fourth column of Γ^{-1}, Minimum Variance Distortionless Response operator (MVDR) [2]–[10] and partial propagator method (PAR) [26]. Figure 7 presents the obtained results [25].

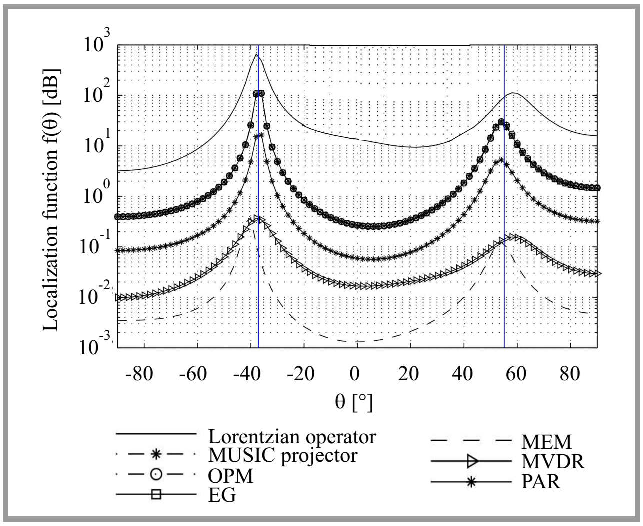

Fig. 7. DoA localization functions $f(\theta)$ of acoustic sources from experimental data.

The Lorentzian localization function has the highest values of peaks and the majority of the spectra present some deviations of locations. To quantify these fluctuations, in Table 2 the estimated DoAs using peak detection algorithm are presented.

Table 2
Estimated DoAs of acoustic sources

Spectral technique	Source 1 θ_1 [°]	Source 2 θ_2 [°]
Lorentzian	–37.70	58.60
MUSIC	–37.00	54.00
OPM	–37.00	54.20
EG	–37.00	54.00
MEM	–40.80	53.80
MVDR	–37.50	58.20
PAR	–36.90	53.60
Mean values	–37.70	55.20

The three subspace techniques MUSIC, OPM and EG operators identify the acoustic sources with same values of θ_1 and θ_2, the MVDR and Lorentzian functions present the same result where the angular position of the second source is different than the result of the first three subspace techniques by 4°. This difference is reduced for the partial propagator method where $\theta_2 = 53.60°$. The Maximum Entropy Method is efficient if the fourth column is chosen as reference, however the peak of the first source is deviated by approximately 3°.

5. Research Perspectives

The proposed approach for DoA problem is based on band-pass filter using single shaped Lorentzian function. The similar solution was proposed using Gaussian function with exponential operator [27]. Another solution consists of using Heaviside function as low pass filter of eigenvalues, one of the approximations is given by the equation:

$$f(\lambda) = 1 - \frac{1}{e^{-\alpha(\lambda-\lambda_c)}+1}, \tag{24}$$

where λ_c is the threshold such as $\lambda_{P+1} < \lambda_c < \lambda_P$ and α is a constant that controls the rate of decay as illustrated in Fig. 8 where larger value gives fast transition.

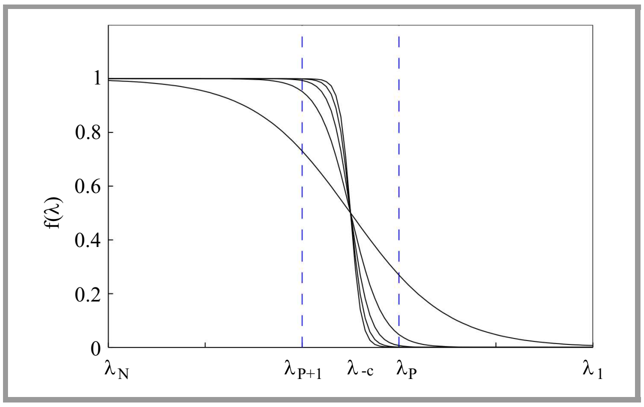

Fig. 8. Approximation of rectangular function $f(\lambda)$ applied to eigenvalues of spectral matrix Γ with parameters λ_c, $\alpha = \{1,3,5,7,9\}$.

As perspective, a theoretical value of threshold λ_c and fast approximation of exponential operator $e^{-\alpha(\Gamma-\lambda_c I_N)}$ may provide accurate results comparatively to conventional DoA spectra.

6. Conclusion

In this paper, a new high resolution algorithm for narrowband source localization problem using large array is proposed. The main idea consisted of applying Lorentz function on spectral matrix of received data such as low pass filter, where the cut-off value is the threshold between signal and noise eigenvalues, this mechanism requires a priori knowledge of minimum eigenvalue, which is the index of function's peak. Theoretical threshold and scaling parameter of Lorentz function were derived using first and second order statistics of eigenvalues using only the trace.

Several computer simulations demonstrated the resolution power of the proposed algorithm when the dimension of the antenna is relatively large.

References

[1] E. Xu, Z. Ding, and S. Dasgupta, "Source localization in wireless sensor networks from signal time-of-arrival measurements", *IEEE Trans. Sig. Process.*, vol. 59, no. 6, pp. 2887–2897, 2011.

[2] Z. Chen, G. Gokeda, and Y. Yu, *Introduction to Direction-of-Arrival Estimation*. Norwood, MA, USA: Artech House, 2010.

[3] Y.-S. Yoon, L. M. Kaplan, and J. H. McClellan, "TOPS: new DOA estimator for wideband signals", Signal Processing, *IEEE Trans. Sig. Process.*, vol. 54, no. 6, pp. 1977–1989, 2006.

[4] R. Feliachi, "Spatial processing of cyclostationary interferers for phased array radio telescopes", Ph.D. thesis, Université d'Orléans, Orléans, France, 2010.

[5] B. Yang, F. He, J. Jin, H. Xiong, and G. Xu, "DOA estimation for attitude determination on communication satellites", *Chinese J. Aeronautics*, vol. 27, no. 3, pp. 670–677, 2014.

[6] Y.-H. Ko, Y.-J. Kim, H.-I. Yoo, W.-Y. Yang, and Y.-S. Cho, "DoA estimation with cell searching for mobile relay stations with uniform circular array", in *Proc. IEEE 20th Int. Symp. Person., Indoor Mob. Radio Commun.*, Tokio, Japan, 2009, pp. 993–997.

[7] M. Jiang, J. Huang, W. Han, and F. Chu, "Research on target DOA estimation method using MIMO sonar", in *Proc. 4th IEEE Conf. on Indust. Elec. Appl. ICIEA 2009*, Xi'an, China, 2009, pp. 1982–1984.

[8] C. Shao-hua, Z. Wei, and L. Hui-bin, "Improved DOA estimation of underwater target with acoustic cross array", in *Proc. IEEE 11th Int. Conf. Sig. Process. ICSP 2012*, Beijing, China, 2012, vol. 3, pp. 2071–2074.

[9] I.-K. Rhee and H.-S. Kim, "Improved DOA estimation of correlated signals in correlated antenna noises environment", in *Proc. Int. Conf. Inform. Netw. ICOIN 2013*, Bangkok, Thailand, 2013, pp. 66–70.

[10] F. B. Gross, *Smart Antennas for Wireless Communications with Matlab*. New York, NY, USA: McGraw-Hill Professional, 2005.

[11] P. Tan, P. Wang, Y. Luo, Y. Zhang, and H. Ma, "Study of 2D DOA estimation for uniform circular array in wireless location system", *Int. J. Comp. Netw. Inform. Secur.* (*IJCNIS*), vol. 2, no. 2, pp. 54–60, 2010.

[12] L. Liu, Q. Ji, and Y. Jiang, "Improved Fast DOA Estimation Based on Propagator Method", in *Proc. APSIPA Ann. Summit and Conf. APSIPA ASC 2011*, Xi'an, China, 2011.

[13] Y.-H. Chen and Y.-S. Lin, "Fourth-order cumulant matrices for DOA estimation", *IEE Proc. Radar, Sonar and Navig.*, vol. 141, no. 3, pp. 144–148, 1994.

[14] R. Roy and T. Kailath, "ESPRIT-estimation of signal parameters via rotational invariance techniques", *IEEE Trans. Acoust., Speech and Sig. Process.*, vol. 37, no. 7, pp. 984–995, 1989.

[15] J. Dai, W. Xu, and D. Zhao, "Real-valued DOA estimation for uniform linear array with unknown mutual coupling", *Sig. Process.*, vol. 92, no. 9, 2012.

[16] X. Mestre and M.-A. Lagunas,"Modified subspace algorithms for DoA Estimation with large arrays", *IEEE Trans. Sig. Process.*, vol. 56, no. 2, pp. 598–614, 2008. doi: 10.1109/TSP.2007.907884.

[17] Y. Wang, G. Leus, and A. Pandharipande, "Direction estimation using compressive sampling array processing", in *Proc. IEEE/SP 15th Worksh. on Statis. Sig. Process. SSP 2009*, Cardiff, UK, 2009, pp. 626–629. doi: 10.1109/SSP.2009.5278497.

[18] S. A. Clough and F. X. Kneizys, "Convolution algorithm for the Lorentz function", *Applied Optics*, vol. 18, no. 13, pp. 2329–2333, 1979.

[19] V. T. Ermolaev and A. B. Gershman, "Fast algorithm for minimum-norm direction-of-arrival estimation", *IEEE Trans. Sig. Process.*, vol. 42, no. 9, pp. 2389–2394, 1994. doi: 10.1109/78.317860.

[20] H. Wolkowicz and G. P. H. Styan, "Bounds for eigenvalues using traces", *Linear Algebra and its Applications*, vol. 29, pp. 471–506, 1980.

[21] H. Krim and M. Viberg,, "Two decades of array signal processing research: the parametric approach", *IEEE Sig. Process. Mag.*, vol. 13, no. 4, pp. 67–94, 1996.

[22] Y. Khmou, S. Safi, and M. Frikel, "Comparative study between several direction of arrival estimation methods", *J. Telecommun. Inform. Technol.*, no. 1, pp. 41–48, 2014.

[23] R. O. Schmidt, "Multiple emitter location and signal parameter estimation", *IEEE Trans. Antenn. Propag.*, vol. 34, no. 3, pp. 276–280, 1986.

[24] S. Marcos, A. Marsal, and M. Benidir, "The propagator method for source bearing estimation", *Sig. Process.*, vol. 42, no. 2, pp. 121–138, 1995.

[25] P. Stoica and R. Moses, *Spectral Analysis of Signals*. Upper Saddle River, NY, USA: Prentice-Hall, 2005.

[26] J. Chen, Y. Wu, H. Cao, and H. Wang, "Fast algorithm for DOA estimation with partial covariance matrix and without eigendecomposition", *J. Sig. Inform. Process.*, vol. 2 no. 4, pp. 266–269, 2011.

[27] Y. Khmou, S. Safi, and M. Frikel, "Exponential operator for bearing estimation", *Int. J. Adv. Sci. Technol.* (*IJAST*), vol. 74, pp. 1–10, 2015.

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