Paper

An Inclusive Performance Analysis of Single-branch Single-relay AF Transmission in a Mixed Rayleigh-Nakagami-*m* Fading Environment

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Abstract—In this paper, the end-to-end performance of a single-branch two-hop amplify-and-forward (AF) relaying network in a mixed Rayleigh-Nakagami-m fading environment, is investigated. Four different fading scenarios and three standard relay configurations for each scenario are considered. Exact analytical expressions for the outage probability and tight upper bounds for the ergodic capacity are derived. Results of Monte Carlo simulations are provided to verify the accuracy of the analytical results.

Keywords— *amplify-and-forward (AF), decode-and-forward (DF), two/dual-hop relaying, outage probability, ergodic capacity, signal-to-noise ratio (SNR).*

1. Introduction

In a challenging wireless communication environment, cooperative relaying may be used to extend the coverage area and to enable high speed information transfers [1]–[4]. In relay networks, a source node (S) communicates with a destination node (D) through one or several intermediate nodes (R) referred to as relays. The two main relaying protocols are regenerative and non-regenerative relaying [5]. Decode-and-forward (DF) relaying [6], [7] is the most commonly used regenerative relaying protocol, where the relay detects and decodes the signal received from the source and then re-encodes and re-transmits it towards the destination. Amplify-and-forward (AF) relaying [8], [9] is a straightforward and popular non-regenerative relaying protocol. In AF relaying, the relay simply amplifies the signal received from the source and forwards it straight to the destination.

In a practical scenario, because of the geographical location of the relay node and its distances with respect to the source node and the destination node, different links may experience different fading conditions and, rarely, similar fading conditions. For example, one link may be in a line-of-sight (LOS) situation, while the other link may be in a non-LOS situation. Both of them may operate in a LOS or a non-LOS scenario as well. In the literature, the former situation has been described as an asymmetric or mixed fading scenario, while the latter has been described as a symmetric fading scenario.

Initially, most works focusing on single-branch single-relay cooperative networks considered the symmetric scenario [8], [10]–[12]. Later, increasing interest in the asymmetric scenarios was observed. Authors of [3] derived exact and lower bound expressions for the outage probability and the average bit error probability for a two-hop AF relaying network over a mixed Rayleigh-Rician fading environment. After that and considering the same fading environment, more cooperative relaying models were studied, for instance the two-hop AF models in [13], [14] and the two-hop DF model in [15].

A mixed Nakagami-Rician fading environment was considered in [16], while the authors of [17] studied the performance of AF cooperative relaying in a mixed Rayleigh-generalized Gamma fading environment. The work [18] studied the performance two-hop AF relaying in an asymmetric Nakagami-lognormal fading environment, deriving novel closed-form expressions for the outage probability and the symbol error rate. In a recent study on two-hop AF relaying [19], a non-identical Rician fading environment was considered. Kumat *et al.* analyzed, in [20], the performance of QAM in AF cooperative networks over Rayleigh fading channels. In [21], the authors developed a unified framework to evaluate the performance of single-branch two-hop AF relaying, in the presence of transceiver hardware impairments, in a symmetric Nakagami-*m* fading environment.

A crucial design issue affecting AF relaying is the selection of the relay amplification factor based on the noise power and the source-to-relay channel state-information (CSI). Three standard AF relay configurations are known in the literature, namely channel-noise-assisted (CNA) relaying [22]–[24], channel-assisted (CA) relaying [11] and blind relaying [3], [12].

In the previously mentioned works and in numerous other studies related to the analysis of performance of single-branch dual-hop AF relaying over fading environments, either their fading conditions are limited to one fading symmetric or asymmetric scenario, or their relay functioning is limited to just one configuration. In this study, four different fading scenarios are considered, where each of the two hops' links may experience either a Rayleigh or a Nakagami-*m* fading. Under each of the four fading scenarios, three different relay configurations are considered, namely CNA, CA and blind relaying. Such an approach renders twelve different fading scenario-relay configurations. The contribution of the paper consists in deriving an exact analytical expression for the outage probability and a tight upper bound for the ergodic capacity for each of the twelve cases referred to above.

The remainder of the paper is organized as follows. Section 2 presents the system and channel model. In Section 3, exact analytical expressions for the outage probability and tight upper bounds for the ergodic capacity are derived. Some Monte Carlo simulations are carried out and their results are provided in Section 4. Finally, Section 5 concludes the paper.

For quick reference, we wish to provide an explanation of the common notations used in this paper. $K_v(.)$ is the modified Bessel function of the second kind [30] of order v. $W_{\mu,v}(.)$ denotes the Whittaker function [30]. ${}_2F_1(\alpha,\beta;\gamma;.)$ represents the Gauss hyper-geometric function [30]. $f_X(x)$, $F_X(x)$, $\bar{F}_X(x)$ and $M_X(s)$ denote PDF, CDF, complementary CDF and the MGF of a continuous RV X, respectively. Z^+ denotes the set of positive integers. The gamma function $\Gamma(n)$ of integer n satisfies $\Gamma(n) = (n-1)!$.

2. System and Channel Model

As shown in Fig. 1, this work considers a single-branch single-relay (i.e. a two-hop) AF cooperative relay network, where the transmission from source S to destination D via relay R takes place in two time slots. In the first time slot, S sends its signal to R. In the second time slot, R amplifies the signal received from S by a gain factor G, and forwards the resultant signal to D [19], [25], and [26]. It is assumed here that there is no direct or indirect path other than the $S \to R \to D$ path. For the end-to-end (E2E) signal-to-noise ratio (SNR) Λ of this system, the model proposed in [25] is considered:

$$\Lambda = \frac{\Lambda_1 \Lambda_2}{a\Lambda_1 + \Lambda_2 + b} , \tag{1}$$

where $\Lambda_i = \frac{P_i}{N_i}|h_i|^2$ represents the instantaneous SNR of the i-th hop, with P_i being the transmission power of s_i, N_i being the power of the additive white Gaussian noise (AWGN) component n_i.

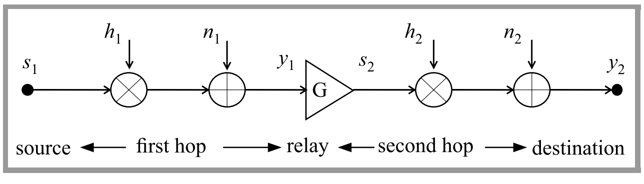

Fig. 1. Block diagram of single-branch dual-hop AF relaying.

The channel magnitude $\{|h_i|\}$ is modeled either as a Rayleigh or a Nakagami-*m* distributed RV, so as stated in Table 1, four different fading scenarios will be considered.

Table 1
Four different fading scenarios considered in this work

Case	$S-R$ link $\{\lvert h_1\rvert\}$	$R-D$ link $\{\lvert h_2\rvert\}$
Scenario 1	Nakagami-*m*	Nakagami-*m*
Scenario 2	Nakagami-*m*	Rayleigh
Scenario 3	Rayleigh	Nakagami-*m*
Scenario 4	Rayleigh	Rayleigh

The instantaneous SNR, Λ_i, is either exponential or Gamma distributed. Their PDFs are $f^{Exp}_{\Lambda_i}(x) = (1/\bar{\lambda}_i)\mathrm{e}^{\frac{-x}{\bar{\lambda}_i}}$; $x \geq 0$, and $f^{\Gamma}_{\Lambda_i}(x) = (x^{\alpha_i - 1}/\Gamma(\alpha_i)\beta_i^{\alpha_i})\mathrm{e}^{\frac{-x}{\beta_i}})$; $x \geq 0$, respectively.

The E2E SNR in Eq. (1), corresponds to a gain given by:

$$G^2 = \frac{P_2}{a|h_1|^2 P_1 + bN_1} , \tag{2}$$

where $a, b \geq 0$.

The setting $P_1 = P_2 = 1$ in Eq. (2) corresponds to the model considered in [25]. Note that the choice of the values of the parameters a and b reflects the relay configuration with $(a,b) \in \{(1,1),(1,0),(0,1)\}$ representing CNA, CA, and the blind relay configuration, respectively.

3. Performance Analysis

Given the E2E SNR, Λ in Eq. (1), we provide here exact closed-form expressions for the outage probability and tight upper bounds for the ergodic capacity of the system in consideration. All the expressions will be given as functions of parameters a and/or b.

3.1. Outage Probability

The outage probability denoted by $P_{out}(x)$ is defined as the probability that the channel fading makes the effective end-to-end SNR fall below a certain threshold x characteristic of acceptable communication quality:

$$P_{out}(x) = \Pr(\Lambda \leq x) = \Pr\left[\frac{\Lambda_1 \Lambda_2}{a\Lambda_1 + \Lambda_2 + b} \leq x\right] . \tag{3}$$

After applying some algebraic manipulations, $P_{out}(x)$ may be expressed as:

$$P_{out}(x) = 1 - \int_0^{\infty} \bar{F}_{\Lambda_2}\left(ax + \frac{ax^2 + bx}{z}\right) f_{\Lambda_1}(z + x)\mathrm{d}z \tag{4}$$

where $\bar{F}_{\Lambda_i}(x) = 1 - F_{\Lambda_i}(x)$ is the complementary CDF of Λ_i.

First Scenario:

- For the CNA relay configuration, the outage probability $P_{out}(x)$ in Eq. (4), will be given for $x \geq 0$ by

$$P_{out}(x) = 1 - 2\mathrm{e}^{-\left(\frac{1}{\beta_1}+\frac{a}{\beta_2}\right)x} \sum_{n=0}^{\alpha_1-1} \sum_{k=0}^{\alpha_2-1} \sum_{m=0}^{k} c_1(n,k,m) \times K_{n-m+1}\, 2\sqrt{\frac{(ax+b)x}{\beta_1\beta_2}} \left(a+\frac{b}{x}\right)^{\frac{n+m+1}{2}} \times x^{\alpha_1+k}, \quad (5)$$

where $c_1(n,k,m) = \frac{a^{k-m}\beta_1^{\frac{n-m+1-2\alpha_1}{2}}\beta_2^{\frac{m-n-1-2k}{2}}}{m!(k-m)!n!(\alpha_1-1-n)!}$.

- For the CA relay configuration, the outage probability $P_{out}(x)$ in Eq. (4), will be given for $x \geq 0$ by:

$$P_{out}(x) = 1 - 2\mathrm{e}^{-\left(\frac{1}{\beta_1}+\frac{a}{\beta_2}\right)x} \sum_{n=0}^{\alpha_1-1} \sum_{k=0}^{\alpha_2-1} \sum_{m=0}^{k} c_1(n,k,m) \times K_{n-m+1}\, 2\sqrt{\frac{ax}{\beta_1\beta_2}} a^{\frac{n+m+1}{2}} x^{\alpha_1+k}. \quad (6)$$

- For the blind relay configuration, the outage probability $P_{out}(x)$ in Eq. (4), for $x \geq 0$ will be:

$$P_{out}(x) = 1 - 2\mathrm{e}^{-\frac{1}{\beta_1}x} \sum_{n=0}^{\alpha_1-1} \sum_{k=0}^{\alpha_2-1} c_2(n,k) \times K_{n-k+1}\, 2\sqrt{\frac{bx}{\beta_1\beta_2}} x^{\frac{2\alpha_1+k-n-1}{2}}, \quad (7)$$

where $c_2(n,k) = \frac{\beta_1^{\frac{n-k+1-2\alpha_1}{2}}\left(\frac{b}{\beta_2}\right)^{\frac{n+k+1}{2}}}{n!k!(\alpha_1-n-1)!}$.

Proof: See the appendix. ■

Second Scenario:

- For the CNA relay configuration, the outage probability $P_{out}(x)$ in Eq. (4), will be given for $x \geq 0$ by:

$$P_{out}(x) = 1 - 2\mathrm{e}^{-\left(\frac{1}{\beta_1}+\frac{a}{\gamma_2}\right)x} \sum_{n=0}^{\alpha_1-1} c_3(n) \times K_{n+1}\, 2\sqrt{\frac{(ax+b)x}{\beta_1\gamma_2}} \left(a+\frac{b}{x}\right)^{\frac{n+1}{2}} x^{\alpha_1}, \quad (8)$$

where $c_3(n) = \frac{\beta_1^{\frac{n+1-2\alpha_1}{2}}\gamma_2^{-\frac{n+1}{2}}}{n!(\alpha_1-1-n)!}$.

- For the CA relay configuration, the outage probability $P_{out}(x)$ in Eq. (4), will be given for $x \geq 0$ by:

$$P_{out}(x) = 1 - 2\mathrm{e}^{-\left(\frac{1}{\beta_1}+\frac{a}{\gamma_2}\right)x} \sum_{n=0}^{\alpha_1-1} c_3(n) \times K_{n+1}\, 2\sqrt{\frac{a}{\beta_1\gamma_2}xa^{\frac{n+1}{2}}} x^{\alpha_1}. \quad (9)$$

- For the blind relay configuration, the outage probability $P_{out}(x)$ in Eq. (4), for $x \geq 0$ will be:

$$P_{out}(x) = 1 - 2\mathrm{e}^{-\frac{1}{\beta_1}x} \sum_{n=0}^{\alpha_1-1} c_3(n) K_{n+1}\, 2\sqrt{\frac{bx}{\beta_1\gamma_2}} \times b^{\frac{n+1}{2}} x^{\frac{2\alpha_1-n-1}{2}}. \quad (10)$$

Proof: See the appendix. ■

Third Scenario:

- For the CNA relay configuration, the outage probability $P_{out}(x)$ in Eq. (4), will be given for $x \geq 0$ by:

$$P_{out}(x) = 1 - 2\mathrm{e}^{-\left(\frac{1}{\beta_1}+\frac{a}{\beta_2}\right)x} \sum_{k=0}^{\alpha_2-1} \sum_{q=0}^{k} c_4(k,q) \times K_{1-q}\, 2\sqrt{\frac{(ax+b)x}{\gamma_1\beta_2}} \left(a+\frac{b}{x}\right)^{\frac{q+1}{2}} x^{k+1}, \quad (11)$$

where $c_4(k,q) = \frac{a^{k-q}\gamma_1^{-\frac{q+1}{2}}\beta_2^{\frac{q-1-2k}{2}}}{q!(k-q)!}$.

- For the CA relay configuration, the outage probability $P_{out}(x)$ in Eq. (4), for $x \geq 0$ will be:

$$P_{out}(x) = 1 - 2\mathrm{e}^{-\left(\frac{1}{\gamma_1}+\frac{a}{\beta_2}\right)x} \sum_{k=0}^{\alpha_2-1} \sum_{q=0}^{k} c_4(k,q) \times K_{1-q}\, 2\sqrt{\frac{a}{\gamma_1\beta_2}xa^{\frac{q+1}{2}}} x^{k+1}. \quad (12)$$

- For the blind relay configuration, the outage probability $P_{out}(x)$ in Eq. (4), for $x \geq 0$ will be:

$$P_{out}(x) = 1 - 2\mathrm{e}^{-\frac{1}{\gamma_1}x} \sum_{k=0}^{\alpha_2-1} c_5(k) K_{1-q}\, 2\sqrt{\frac{bx}{\gamma_1\beta_2}} \times x^{\frac{k+1}{2}}, \quad (13)$$

where $c_5(k) = \frac{\gamma_1^{-\frac{q+1}{2}}\frac{b}{\beta_2}^{\frac{k+1}{2}}}{k!}$.

Proof: See the appendix. ■

Fourth Scenario:

- For the CNA relay configuration, the outage probability $P_{out}(x)$ in Eq. (4), for $x \geq 0$ is:

$$P_{out}(x) = 1 - 2\sqrt{\frac{(ax+b)x}{\gamma_1\gamma_2}}\mathrm{e}^{-\left(\frac{1}{\gamma_1}+\frac{a}{\gamma_2}\right)x} \times K_1\, 2\sqrt{\frac{(ax+b)x}{\gamma_1\gamma_2}}\,. \quad (14)$$

- For the CA relay configuration, the outage probability $P_{out}(x)$ in Eq. (4), for $x \geq 0$ will be:

$$P_{out}(x) = 1 - 2\sqrt{\frac{a}{\gamma_1\gamma_2}}x\mathrm{e}^{-\left(\frac{1}{\gamma_1}+\frac{a}{\gamma_2}\right)x} \times K_1\, 2\sqrt{\frac{a}{\gamma_1\gamma_2}}x\,. \quad (15)$$

- For the CA relay configuration, the outage probability $P_{out}(x)$ in Eq. (4), for $x \geq 0$ will be:

$$P_{out}(x) = 1 - 2\sqrt{\frac{bx}{\gamma_1\gamma_2}}\mathrm{e}^{-\frac{1}{\gamma_1}x} \times K_1\, 2\sqrt{\frac{a}{\gamma_1\gamma_2}}x\,. \quad (16)$$

Proof: See the appendix. ■

For $a = 1$, Eq. (15) reduces to Eq. (27) in [10].
For $b = c$, Eq. (16) reduces to Eq. (9) in [12].

3.2. The Ergodic Capacity

For a single-branch single-relay AF relaying network, the ergodic capacity (in bits/channel used) can be expressed as [26]:

$$C_{erg} \triangleq \frac{1}{2}E\{\log_2(1+\Lambda)\}, \quad (17)$$

where Λ is the end-to-end SNR in Eq. (1).

The ergodic capacity in Eq. (17) can be upper bounded as follows:

$$C_{erg} \leq \frac{1}{2}\log_2\left(1 + \int_0^\infty (P_{out}(x) - 1)\mathrm{d}x\right). \quad (18)$$

Proof: See the appendix. ■

First Scenario:

- For the CNA relay configuration, the ergodic capacity C_{erg} in Eq. (18), will be:

$$\begin{aligned} C_{erg} \leq \frac{1}{2}\log_2\Bigg(1 - 2\sum_{n=0}^{\alpha_1-1}\sum_{k=0}^{\alpha_2-1}\sum_{m=0}^{k}\sum_{q=0}^{n+m+2} c_1(n,k,m) \\ \times a^{\frac{n+m+1}{2}}\binom{n+m+2}{q}\left(\frac{b}{a}\right)^q \\ \times \frac{\sqrt{a\beta_1\beta_2}\Gamma(n+2)\Gamma(m+1)}{2b(-1)^{\alpha_1+k-q+1}} \\ \times \frac{d^{\alpha_1+k-q+1}}{dp^{\alpha_1+k-q+1}}\Bigg\{\mathrm{e}^{\frac{bp}{2a}} \\ \times W_{-\frac{n+m+2}{2},\frac{n-m+1}{2}}\frac{b\left(p-\sqrt{p^2-\frac{4a}{\beta_1\beta_2}}\right)}{2a} \\ \times W_{-\frac{n+m+2}{2},\frac{n-m+1}{2}}\frac{b\left(p+\sqrt{p^2-\frac{4a}{\beta_1\beta_2}}\right)}{2a} \\ \Bigg\}_{|p=\frac{1}{\beta_1}+\frac{a}{\beta_2}}\Bigg). \quad (19) \end{aligned}$$

- For the CA relay configuration, the ergodic capacity C_{erg} in Eq. (18), will be given by:

$$\begin{aligned} C_{erg} \leq \frac{1}{2}\log_2\Bigg(1 - 2\sum_{n=0}^{\alpha_1-1}\sum_{k=0}^{\alpha_2-1}\sum_{m=0}^{k} c_1(n,k,m)a^{\frac{n+m+1}{2}} \\ \times \frac{\sqrt{\pi}\Gamma(\alpha_1+k+n-m+2)\Gamma(\alpha_1+k-n+m)}{\Gamma(\alpha_1+k+3/2)} \\ \times \left(\frac{16a}{\beta_1\beta_2}\right)^{\frac{n-m+1}{2}} \\ \times \frac{{}_2F_1\left(\alpha_1+k+n-m+2; n-m+3/2; \alpha_1+k+3/2; \bar{s}\right)}{\left(\left(\sqrt{\frac{a}{\beta_2}}+\sqrt{\frac{1}{\beta_1}}\right)^2\right)^{\alpha_1+k+n-m+2}}\Bigg), \end{aligned} \quad (20)$$

where $\bar{s} = \left(\left(\sqrt{\frac{a}{\beta_2}}-\sqrt{\frac{1}{\beta_1}}\right)^2 / \left(\sqrt{\frac{a}{\beta_2}}+\sqrt{\frac{1}{\beta_1}}\right)^2\right)$.

- For the blind relay configuration, the ergodic capacity C_{erg} in Eq. (18), will be given by:

$$\begin{aligned} C_{erg} \leq \frac{1}{2}\log_2\Bigg(1 - 2\sum_{n=0}^{\alpha_1-1}\sum_{k=0}^{\alpha_2-1} c_2(n,k) \\ \times \frac{\Gamma(\alpha_1+1)\Gamma(\alpha_1+k-n)}{2\sqrt{\frac{b}{\beta_1\beta_2}}}\left(\frac{1}{\beta_1}\right)^{\frac{n-k-2k}{2}} \\ \times e^{\frac{b}{2\beta_2}}W_{-\frac{n+m+2}{2},\frac{n-m+1}{2}}\left(\frac{b}{\beta_2}\right)\Bigg). \quad (21) \end{aligned}$$

Proof: See the appendix. ■

Second Scenario:

- For the CNA relay configuration, the ergodic capacity C_{erg} in Eq. (18) will be:

$$C_{erg} \leq \frac{1}{2}\log_2\Bigg(1-2\sum_{n=0}^{\alpha_1-1}\sum_{q=0}^{n+2} c_1(0,0,0)a^{\frac{n+1}{2}}\binom{n+2}{q} \times \left(\frac{b}{a}\right)^q \frac{\sqrt{a\beta_1\gamma_2}\Gamma(n+2)\Gamma(1)}{2b(-1)^{\alpha_1+k-q+1}}\frac{d^{\alpha_1-q+1}}{dp^{\alpha_1-q+1}}\Bigg\{\mathrm{e}^{\frac{bp}{2a}} \times W_{-\frac{n+2}{2},\frac{n+1}{2}}\frac{b(p-\sqrt{p^2-\frac{4a}{\beta_1\gamma_2}})}{2a} \times W_{-\frac{n+2}{2},\frac{n+1}{2}}\frac{b(p+\sqrt{p^2-\frac{4a}{\beta_1\gamma_2}})}{2a}\Bigg\}_{|p=\frac{1}{\beta_1}+\frac{a}{\gamma_2}}\Bigg). \tag{22}$$

- For the CA relay configuration, the ergodic capacity C_{erg} in Eq. (18) may be derived from:

$$C_{erg} \leq \frac{1}{2}\log_2\Bigg(1-2\sum_{n=0}^{\alpha_1-1} c_3(n)a^{\frac{n+1}{2}} \times \frac{\sqrt{\pi}\Gamma(\alpha_1+n+2)\Gamma(\alpha_1-n)}{\Gamma(\alpha_1+3/2)}\left(\frac{16a}{\beta_1\gamma_2}\right)^{\frac{n+1}{2}} \times \frac{{}_2F_1\left(\alpha_1+n+2;n+3/2;\alpha_1+3/2;\bar{s}\right)}{\left(\left(\sqrt{\frac{a}{\gamma_2}}+\sqrt{\frac{1}{\beta_1}}\right)^2\right)^{\alpha_1+n+2}}\Bigg). \tag{23}$$

- For the blind relay configuration, the ergodic capacity C_{erg} in Eq. (18) take the form:

$$C_{erg} \leq \frac{1}{2}\log_2\Bigg(1-2\sum_{n=0}^{\alpha_1-1} c_3(n)b^{\frac{n+1}{2}} \times \frac{\Gamma(\alpha_1+1)\Gamma(\alpha_1-n)}{2\sqrt{\frac{b}{\beta_1\gamma_2}}}\left(\frac{1}{\beta_1}\right)^{\frac{n-2\alpha_1}{2}} \times \mathrm{e}^{\frac{b}{2\gamma_2}} W_{\frac{n-2\alpha_1}{2},\frac{n+1}{2}}\frac{b}{\gamma_2}\Bigg). \tag{24}$$

Proof: See the appendix. ■

Third Scenario:

- For the CNA relay configuration, the ergodic capacity C_{erg} in Eq. (18) is:

$$C_{erg} \leq \frac{1}{2}\log_2\Bigg(1-2\sum_{n=0}^{\alpha_1-1}\sum_{k=0}^{\alpha_2-1}\sum_{q=0}^{m+2} c_1(0,k,m)a^{\frac{m+1}{2}} \times \binom{m+2}{q}\left(\frac{b}{a}\right)^q \frac{\sqrt{a\gamma_1\beta_2}\Gamma(2)\Gamma(m+1)}{2b(-1)^{2+k-q}} \times \frac{d^{k-q+2}}{dp^{k-q+2}}\Bigg\{\mathrm{e}^{\frac{bp}{2a}} \times W_{-\frac{m+2}{2},\frac{-m+1}{2}}\frac{b(p-\sqrt{p^2-\frac{4a}{\gamma_1\beta_2}})}{2a} \times W_{-\frac{m+2}{2},\frac{-m+1}{2}}\frac{b(p+\sqrt{p^2-\frac{4a}{\gamma_1\beta_2}})}{2a}\Bigg\}_{|p=\frac{1}{\gamma_1}+\frac{a}{\beta_2}}\Bigg). \tag{25}$$

- For the CA relay configuration, the ergodic capacity C_{erg} in Eq. (18), will be given by:

$$C_{erg} \leq \frac{1}{2}\log_2\Bigg(1-2\sum_{n=0}^{\alpha_1-1}\sum_{k=0}^{\alpha_2-1} c_4(k,m)a^{\frac{m+1}{2}} \times \frac{\sqrt{\pi}\Gamma(k-m+3)\Gamma(k+m+1)}{\Gamma(k+5/2)}\left(\frac{16a}{\gamma_1\beta_2}\right)^{\frac{1-m}{2}} \times \frac{{}_2F_1\left(k-m+3;-m+3/2;k+5/2;\bar{s}\right)}{\left(\left(\sqrt{\frac{a}{\beta_2}}+\sqrt{\frac{1}{\gamma_1}}\right)^2\right)^{k-m+3}}\Bigg). \tag{26}$$

- For the blind relay configuration, the ergodic capacity C_{erg} in Eq. (18) may be derived by:

$$C_{erg} \leq \frac{1}{2}\log_2\Bigg(1-2\sum_{k=0}^{\alpha_2-1} c_5(k)\frac{\Gamma(2)\Gamma(k+1)}{2\sqrt{\frac{b}{\gamma_1\beta_2}}} \left(\frac{1}{\gamma_1}\right)^{\frac{-k-2}{2}} \mathrm{e}^{\frac{b}{2\beta_2}} W_{\frac{-k-2}{2},\frac{-k+1}{2}}\frac{b}{\beta_2}\Bigg). \tag{27}$$

Proof: See the appendix. ■

Fourth Scenario:

- For the CNA relay configuration, the ergodic capacity C_{erg} in Eq. (18), will be given by:

$$
\begin{aligned}
C_{erg} \leq \frac{1}{2}\log_2 \Bigg(& 1 - 2\sum_{q=0}^{2} c_1(0,0,0) a^{\frac{1}{2}} \binom{2}{q} \left(\frac{b}{a}\right)^q \\
& \times \frac{\sqrt{a\gamma_1\gamma_2}\,\Gamma(2)\Gamma(1)}{2b(-1)^{2-q}} \\
& \times \frac{d^{2-q}}{dp^{2-q}} \Bigg\{ \mathrm{e}^{\frac{bp}{2a}} W_{-1,\frac{1}{2}} \frac{b\left(p - \sqrt{p^2 - \frac{4a}{\gamma_1\gamma_2}}\right)}{2a} \\
& \times W_{-1,\frac{1}{2}} \frac{b\left(p + \sqrt{p^2 - \frac{4a}{\gamma_1\gamma_2}}\right)}{2a} \Bigg\}_{|p=\frac{1}{\gamma_1}+\frac{a}{\gamma_2}} \Bigg). \qquad (28)
\end{aligned}
$$

- For the CA relay configuration, the ergodic capacity C_{erg} in Eq. (18), will be:

$$
\begin{aligned}
C_{erg} &\leq \frac{1}{2}\log_2 \left(1 - 2\sqrt{\frac{a}{\gamma_1\gamma_2}} \frac{\sqrt{\pi}\,\Gamma(3)\Gamma(1)}{\Gamma(5/2)} \left(\frac{16a}{\gamma_1\gamma_2}\right)^{\frac{1}{2}} \right. \\
&\qquad \left. \times \frac{{}_2F_1(3;3/2;5/2;\bar{s})}{\left(\left(\sqrt{\frac{1}{\gamma_1}}+\sqrt{\frac{a}{\gamma_2}}\right)^2\right)^3} \right) \\
&= \frac{1}{2}\log_2 \left(1 - \frac{64}{3}\frac{a}{\gamma_1\gamma_2} \frac{{}_2F_1(3;3/2;5/2;\bar{s})}{\left(\sqrt{\frac{1}{\gamma_1}}+\sqrt{\frac{a}{\gamma_2}}\right)^6} \right). \qquad (29)
\end{aligned}
$$

- For the blind relay configuration, the ergodic capacity C_{erg} in Eq. (18) is:

$$
C_{erg} \leq \frac{1}{2}\log_2 \left(1 - \gamma_1 \mathrm{e}^{\frac{b}{2\gamma_2}} W_{-1,\frac{1}{2}} \frac{b}{\gamma_2} \right). \qquad (30)
$$

Proof: See the appendix. ■

4. Numerical Results

In this section, we carry out some Monte Carlo simulations to verify the theoretical considerations. The four different scenarios, as mentioned previously, are used. For a Nakagami-m faded link, $\alpha_1 = \alpha_2 = 2$, and $\beta_1 = \beta_2 = 1$, while for a Rayleigh faded link, $\bar{\gamma} = 1$ was taken. We consider also the three standard relay configurations: CNA, CA and blind relaying.

First, in Fig. 2, we show the outage probability, $P_{out}(\lambda_{th})$, as a function of the average SNR for two different thresholds $\lambda_{th} = 2^2 - 1 = 3$ (i.e. 2 bits/channel used) and $\lambda_{th} = 2^5 - 1 = 31$ (i.e. 5 bits/channel used)). From this figure, we can clearly see that the analytical expressions (represented by lines) match the results of Monte Carlo simulations (represented by symbols). For all four scenarios, CNA and CA relaying is characterized by almost the same outage probability which is higher than that of blind relaying, except for the second scenario where all three relay configurations perform almost exactly in the same manner.

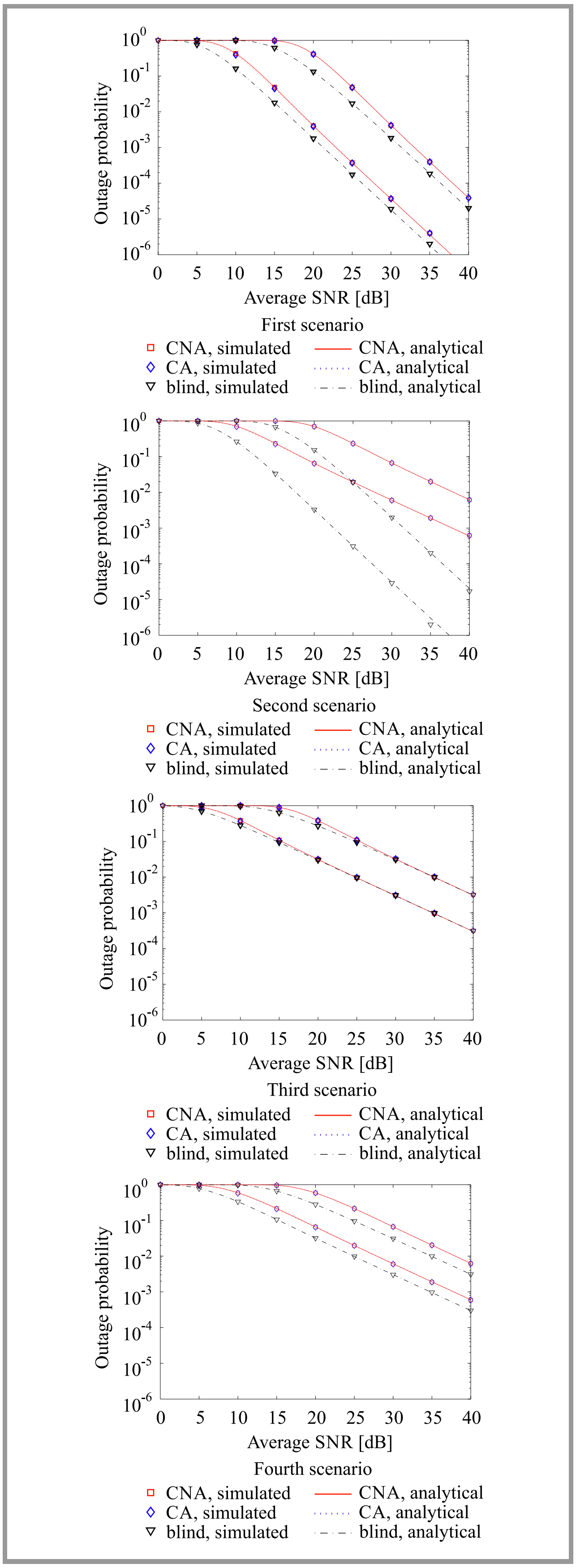

Fig. 2. Outage probability $P_{out}(\lambda_{th})$ for AF relaying; simulated and analytical results for CNA, CA, and blind relay configurations.

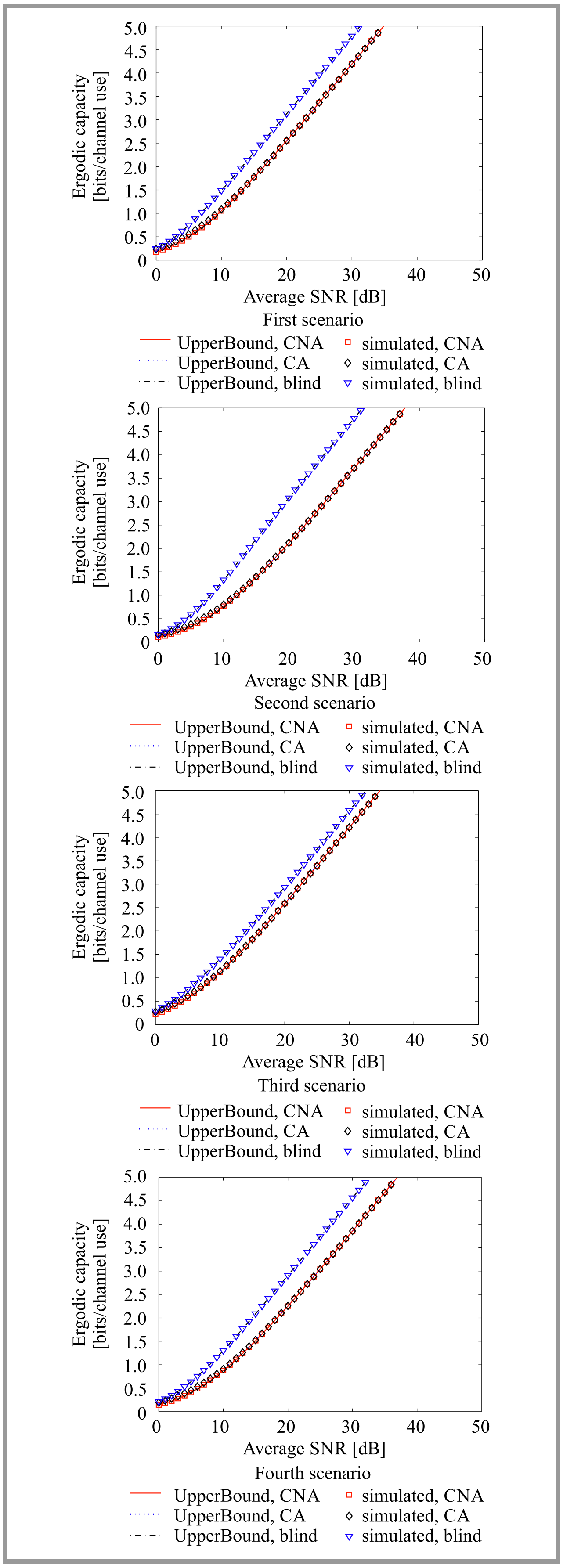

Fig. 3. Ergodic capacity C_{erg} for AF relaying; simulated and analytical results for CNA, CA, and blind relay configurations.

Figure 3 shows the ergodic capacity C_{erg} of AF relaying as a function of the average SNR. As shown, blind relaying, which suffered a lower outage probability, exhibits a higher ergodic capacity compared to CNA and CA relaying types. As a general comment, for both the outage probability and the ergodic capacity, the curves of CNA and CA relaying agree firmly. This approves the use of the former to approximate the latter in many applications.

5. Conclusion

In this paper, we derived novel closed-form expressions for the outage probability and tight upper bounds for the ergodic capacity of a single-branch two-hop relaying network under four different fading scenarios, with three standard relay configurations considered for each scenario. Monte Carlo simulations have been carried out to verify the accuracy of the analytical results.

Appendix

Proof of Results Shown in Section 3

Here, we provide the proof of results shown in Section 3. As stated in Section 2, the channel magnitude $\{|h_i|\}$ of the i-th hop is modeled either as a Rayleigh or a Nakagami-m distributed RV, so the instantaneous SNR, Λ_i, is to be modeled either as an exponential or a gamma distributed RV. Their CCDFs and PDFs are given for $i \in \{1,2\}$ by:

$$\begin{cases} f_{\Lambda_i}^{Exp}(x) = \frac{1}{\lambda_i}\mathrm{e}^{-\frac{x}{\lambda_i}}, \quad x \geq 0 \\ \bar{F}_{\Lambda_i}^{Exp}(x) = \mathrm{e}^{-\frac{x}{\lambda_i}}, \quad x \geq 0. \end{cases} \tag{31}$$

$$\begin{cases} f_{\Lambda_i}^{Gamma}(x) = \frac{x^{\alpha_i - 1}\mathrm{e}^{-\frac{x}{\beta_i}}}{\Gamma(\alpha_i)\beta_i^{\alpha_i}}, \quad x \geq 0 \\ \bar{F}_{\Lambda_i}^{Gamma}(x) = \sum_{k=0}^{\alpha_i - 1} \frac{\mathrm{e}^{-\frac{x}{\beta_i}}}{k!}\left(\frac{x}{\beta_i}\right)^k, \quad x \geq 0 \end{cases} \tag{32}$$

1. Proof of the outage probability results (Subsection 3.1)

Let $Pr\{\Lambda\}$ denote the probability of event Λ. The outage probability $P_{out}(x)$ can be derived as:

$$\begin{aligned} P_{out}(x) &= \Pr\left\{\frac{\lambda_1\lambda_2}{a\lambda_1 + \lambda_2 + b} \leq x\right\} \\ &= \int_0^x \Pr\left\{\lambda_2 \geq \frac{(a\lambda_1 + b)x}{\lambda_1 - x}\right\} . f_{\lambda_1}(\lambda_1)\mathrm{d}\lambda_1 \\ &+ \int_x^\infty \Pr\left\{\lambda_2 \leq \frac{(a\lambda_1 + b)x}{\lambda_1 - x}\right\} . f_{\lambda_1}(\lambda_1)\mathrm{d}\lambda_1 \\ &= 1 - \int_x^\infty \bar{F}_{\lambda_2}\left(\frac{(a\lambda_1 + b)x}{\lambda_1 - x}\right) . f_{\lambda_1}(\lambda_1)\mathrm{d}\lambda_1 . \end{aligned} \tag{33}$$

Simplifying with $z = \lambda_1 - x$, leads to Eq. (4).

First Scenario:

CNA case: substituting from Eq. (32) in Eq. (33) gives:

$$P_{out}(x) = 1 - \sum_{k=0}^{\alpha_2-1} \frac{e^{\left(\frac{ax}{\beta_2}+\frac{x}{\beta_1}\right)}}{k!\beta_2^k\Gamma(\alpha_1)\beta_1^{\alpha_1}} \left(ax^2+bx\right)^k$$
$$= 1 - \int_0^{\infty} (z+x)^{\alpha_1-1} \left(\frac{1}{z}+\frac{ax}{ax^2+bx}\right)^k \times e^{-\left(\frac{(ax^2+bx)/\beta_2}{z}+\frac{z}{\beta_1}\right)} dz\,. \quad (34)$$

The use of lemma 4 from [26] yields Eq. (5) after some simplifications.
CA case: Eq. (6) can be directly deduced from Eq. (5) by setting $b = 0$.
Blind case: By setting $a = 0$ Eq. (34) reduces to:

$$P_{out}(x) = 1 - \frac{e^{-\frac{x}{\beta_1}} x^{\alpha_1-1}}{\Gamma(\alpha_1)\beta_1^{\alpha_1}} \sum_{k=0}^{\alpha_2-1} \frac{\left(\frac{x}{\beta_2}\right)^k}{k!}$$
$$\times \int_0^{+\infty} \left(\frac{b}{z}\right)^k \left(1+\frac{z}{x}\right)^{\alpha_1-1} e^{-\left(\frac{z}{\beta_1}+\frac{bx}{\beta_2 z}\right)} dz\,. \quad (35)$$

The binomial expansion of the second term in the integral and the use of [28, 3.471.9] yields Eq. (7).

Second Scenario:

CNA case: substituting from Eq. (31) and Eq. (32) in Eq. (4) gives:

$$P_{out}(x) = 1 - \frac{e^{-\left(\frac{a}{\gamma_2}+\frac{1}{\beta_1}\right)}}{\Gamma(\alpha_1)\beta_1^{\alpha_1}}$$
$$\times \int_0^{\infty} (z+x)^{\alpha_1-1} e^{-\left(\frac{(ax^2+bx)/\gamma_2}{z}+\frac{z}{\beta_1}\right)} dz\,. \quad (36)$$

The binomial expansion of the term $(z+x)^{\alpha_1-1}$ and the use of [28, 3.471.9] yields Eq. (8) after some simplifications.
CA and blind cases: Eqs. (9) and (10) can be directly deduced from Eq. (8) by setting $b = 0$ and $a = 0$, respectively.

Third Scenario:

CNA case: substituting from Eqs. (31) and (32) in Eq. (4) gives:

$$P_{out}(x) = 1 - \sum_{k=0}^{\alpha_2-1} \frac{e^{-(\frac{1}{\gamma_1}+\frac{a}{\beta_2})x}}{k!\beta_2^k\gamma_1} (ax^2+bx)^k$$
$$\times \int_0^{+\infty} \left(\frac{ax}{ax^2+bx}+\frac{1}{z}\right)^k e^{-\left(\frac{(ax^2+bx)/\beta_2}{z}+\frac{z}{\gamma_1}\right)} dz\,. \quad (37)$$

The binomial expansion of the term $\left(\frac{ax}{ax^2+bx}+\frac{1}{z}\right)^k$ and the use of [28, 3.471.9] yields Eq. (11) after some simplifications.

CA case: Eq. (12) can be directly deduced from Eq. (11) by setting $b = 0$.
Blind case: by setting $a = 0$, Eq. (37) reduces to:

$$P_{out}(x) = 1 - \sum_{k=0}^{\alpha_2-1} \frac{e^{-\frac{x}{\gamma_1}}}{k!\beta_2^k\gamma_1} (bx)^k$$
$$\times \int_0^{+\infty} \left(\frac{1}{z}\right)^k e^{-\left(\frac{bx/\beta_2}{z}+\frac{z}{\gamma_1}\right)} dz\,. \quad (38)$$

The use of [28, 3.471.9] to evaluate the integral in Eq. (38), yields Eq. (13).

Fourth Scenario:

CNA case: substituting from Eq. (31) in Eq. (4) gives:

$$P_{out}(x) = 1 - \int_0^{\infty} e^{-\frac{1}{\gamma_2}\left(ax+\frac{ax^2+bx}{z}\right)} \frac{1}{\gamma_1} e^{-\frac{1}{\gamma_1}(z+x)} dz\,. \quad (39)$$

The use of [28, 3.324.1] yields Eq. (14) after some simplifications.
CA and blind cases: Eqs. (15) and (16) can be directly deduced from Eq. (14) by setting $b = 0$ and $a = 0$, respectively.

2. Proof of ergodic capacity results (Subsection 3.2)

Let $E\{\Lambda\}$ denote the expected value of event Λ. The ergodic capacity C_{erg} can be expressed as in Eq. (17).
We note that the function $\log_2(1+\Lambda)$ in Eq. (17), is twice-differentiable and its second derivative is:

$$-\frac{1}{\ln(2)(1+\Lambda)^2} < 0\,. \quad (40)$$

Jensen's inequality could be then applied to get:

$$C_{erg} \leq \frac{1}{2}\log_2\left(1+E\{\Lambda\}\right). \quad (41)$$

We know that the expected value $E\{\Lambda\}$ of an event Λ is linked to its moment-generating function by:

$$E\{\Lambda\} = \frac{d}{ds} M_\Lambda(s)|_{s=0}\,. \quad (42)$$

Substituting Eq. (6) from [25] in Eq. (42) and substituting the result in Eq. (41) yields Eq. (18).

First Scenario:

CNA case: substituting from Eq. (5) in Eq. (18) gives:

$$C_{erg} \leq \frac{1}{2}\Bigg(1 - 2\sum_{n=0}^{\alpha_1-1}\sum_{k=0}^{\alpha_2-1}\sum_{m=0}^{k} c_1(n,k,m)$$
$$\times \int_0^{+\infty} e^{-\left(\frac{1}{\beta_1}+\frac{a}{\beta_2}\right)x} \left(a+\frac{b}{x}\right)^{\frac{n+m+1}{2}}$$
$$\times x^{\alpha_1+k} K_{n-m+1}\, 2\sqrt{\frac{x(ax+b)}{\beta_1\beta_2}}\, dx\Bigg)\,. \quad (43)$$

From the integral in Eq. (43), we separate factor $(a+(b/x))^{n+m+2}$ and we perform a binomial expansion on it. Then we re-arrange the result into a Laplace transformation that can be evaluated and simplified using the identities shown in [29, 4.1.6] and [29, 4.17.20] to yield Eq. (19).
CA case: substituting from Eq. (6) in Eq. (18) gives:

$$C_{erg} \leq \frac{1}{2}\log\Bigg(1-2\sum_{n=0}^{\alpha_1-1}\sum_{k=0}^{\alpha_2-1}\sum_{m=0}^{k} c_1(n,k,m) \times \int_0^{+\infty} \mathrm{e}^{-\left(\frac{1}{\beta_1}+\frac{a}{\beta_2}\right)x} a^{\frac{n+m+1}{2}} x^{\alpha_1+k} \times K_{n-m+1}\, 2\sqrt{\frac{a}{\beta_1\beta_2}x}\,\mathrm{d}x\Bigg). \quad (44)$$

Evaluating the integral in Eq. (44) using [28, 6.621.3] yields Eq. (20).
Blind case: substituting from Eq. (7) in Eq. (18) and applying [28, 6.621.3] yields Eq. (21).
NB. The results in three other scenarios may be proven by following exactly the same steps as shown for the first scenario.

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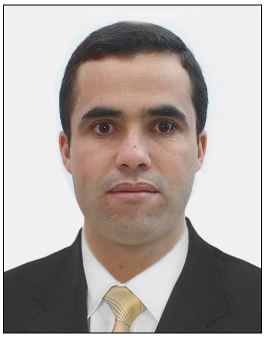

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