Paper

Performance Evaluation of Two-Link Multirate Loss Models with Restricted Accessibility

Irene P. Keramidi[1], Ioannis D. Moscholios[1], Panagiotis G. Sarigiannidis[2], and Michael D. Logothetis[3]

[1] *Department of Informatics and Telecommunications, University of Peloponnese, Tripolis, Greece*
[2] *Department of Electrical and Computer Engineering, University of West Macedonia, Kozani, Greece*
[3] *WCL, Department of Electrical and Computer Engineering, University of Patras, Patra, Greece*

https://doi.org/10.26636/jtit.2020.142320

Abstract—We consider a two-link communication system with restricted accessibility that services Poisson arriving calls of many service-classes and propose a multirate teletraffic loss model for its analysis. In a restricted accessibility system, call blocking occurs even if available resources do exist at the time of a call's arrival. In the two-link system under consideration, each link has two thresholds (offloading and support) which express the in-service calls in a link. The offloading threshold represents the point from which a link offloads calls. The support threshold (which is lower than the offloading threshold) defines the point up to which a link supports offloaded calls. The two-link system with restricted accessibility is modeled as a loss system whose steady state probabilities do not have a product form solution. However, approximate formulas for the determination of call blocking probabilities are proposed. In addition, we also provide a corresponding analysis related to the case of quasi-random traffic (i.e. traffic generated by a finite number of users). The accuracy of all formulas is verified through simulation and is found to be quite satisfactory.

Keywords—accessibility, blocking, non-product form, Poisson, quasi-random, threshold.

1. Introduction

Bandwidth sharing policies are quality of service (QoS) guarantee mechanisms that are necessary for the provision of bandwidth required by calls in a communication link. Assuming that the link is modeled as a loss system carrying call-level traffic, the most common bandwidth sharing policy is the complete sharing (CS) policy. In the CS policy, a new call is blocked if its required bandwidth units (b.u.) exceed the link's available b.u. In addition to the "CS policy" term adopted in this paper, other equivalent terms are also used in the literature, such as "full accessibility" or "full availability" [1], [2]. The latter, however, usually refers to the proportion of time over which the link is available [3].

The simplest loss system that adopts the CS policy is the Erlang loss system [4]. In this system, with its analysisbased on the Erlang loss model, new calls follow a Poisson process, require one b.u. in order to be accepted by the system and have a generally distributed service time. Call blocking occurs if all b.u. are occupied at the time arrival of a given call. The fact that call blocking probabilities (CBP) are determined according to the Erlang B formula has led to an extensive amount of Erlang loss model extensions for the call-level analysis of wired (e.g. [5]–[20]), wireless (e.g. [21]–[33]), satellite (e.g. [34]–[36]) and optical networks (e.g. [37]–[43]).

In [29], a two-link loss system servicing Poisson traffic is considered and studied. Arriving calls belong to a single service-class and each call requests a single b.u. in order to be accepted in a link. Each link may service calls offloaded from the other link. An offloaded call is a new call that arrives in a link but will be served by the other link, subject to bandwidth availability. This offloading mechanism operates with the aid of a high and a low threshold per link, expressing the number of calls serviced by each link. The high threshold is the offloading threshold, while the low threshold is the support threshold (see Fig. 1). The latter expresses the point up to which the link is capable of supporting offloaded calls (from the other link). The offloading threshold determines the point from which call offloading between the two links may start.

Due to the offloading mechanism, there is no local balance (LB) between adjacent states and, therefore, the steady state probabilities of this system do not have a product form solution (PFS) (see the tutorial example of [44]). Thus, the CBP determination can be based either on the accurate but complex method of solving a set of linear global balance (GB) equations, or on an approximate but efficient method that relies on the Erlang B formula and on the assumption (approximation) that the links are independent.

Such an offloading scheme may find a potential application in mobile/Wi-Fi networks. To manage the increasing traffic in mobile networks, traffic may be offloaded to Wi-Fi networks [45], [46]. In order to increase the bandwidth of Wi-Fi access links, recent research focuses on bandwidth

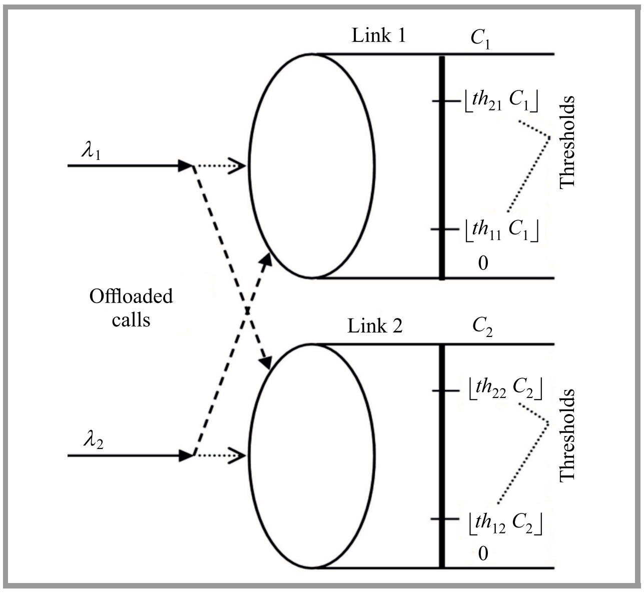

Fig. 1. The two-link loss system.

sharing policies that should be adopted and on the aggregation of backhaul access link capacities. The impact that such aggregation exerts on CBP may be studied via the single-rate model of [29].

In this paper, we extend the model of [29] by considering that the system accommodates Poisson arriving calls of numerous service-classes. Nowadays, this consideration is a sine-qua-non condition in multidimensional network traffic environments. In addition, in the proposed new model we incorporate the notion of restricted accessibility – not only in the case of Poisson traffic, but also in the case of quasi-random traffic (traffic generated by a finite number of users). In a restricted accessibility system, call blocking may occur even if b.u. are available at the time of arrival of a given call. The term "restricted accessibility" covers the following:

- Bandwidth sharing policies, such as the bandwidth (trunk) reservation policy [5], [10], [14], the threshold policy [16], [18] or the probabilistic threshold policy [30], [47]. In the bandwidth reservation policy, call blocking can occur if the available b.u. of the system are reserved at the time of an arrival of a call. In the threshold and the probabilistic threshold policies, a predefined threshold (different for each service-class) is set in order to express the number of in-service calls (of each service-class). If the acceptance of a new call leads to a value that is above that threshold, then call blocking always occurs (threshold policy) or it occurs with a certain probability (probabilistic threshold policy).

- The case where each state of the system (excluding the state where there are no calls in the system) is associated with a blocking probability. Such an approach may be useful when modeling interference between neighboring cells (e.g. in CDMA systems) [3], [48]. In this paper, we focus on this type of restricted accessibility and propose an approximate method for the CBP calculation which is verified via simulation and is found to be quite satisfactory. The CBP calculation in the proposed two-link model is based on the Erlang multirate loss model (EMLM) [49], [50] which refers to a link that services Poisson traffic generated from different service-classes.

In the remainder of this paper, in Section 2, we review the model of [29]. In Section 3, we propose the extension of [29] which includes the case of many service-classes, as well as the notion of restricted accessibility. In Section 4, we present the corresponding analytical model for the case of multirate quasi-random traffic. In Section 5, we provide analytical and simulated CBP results for the proposed model, assuming the existence of Poisson traffic. We conclude in Section 6.

2. Review of the Two-link Single-rate System

We consider a two-link system of capacities C_1 and C_2 b.u. Each link services single-rate Poisson traffic. Arriving calls require one b.u. in order to be accepted in the system and have a generally distributed service-time with a mean of μ^{-1}. Let λ_l be the call arrival rate in link l $(l = 1,2)$ and let j_l be the occupied b.u. in link l. Then, $0 \le j_1 \le C_1$ and $0 \le j_2 \le C_2$. Note that j_l expresses the number of in-service calls in link l, since each call requires one b.u.

Each link l $(l = 1,2)$ has a support (low) threshold th_{1l} and an offloading (high) threshold th_{2l}, with $th_{1l} < th_{2l}$ and $0 \le th_{1l},\ th_{2l} \le 1$. By denoting the largest integer not exceeding x as $\lfloor x \rfloor$ and based on Fig. 1, the role of these thresholds in link l is described in the following manner:

- If $0 \le j_l < \lfloor th_{1l} C_l \rfloor$, then link l is in a *support mode* of operation. In that mode, the link can service new calls that arrive in link l and offloaded calls from link m $(m = 1,2,\ m \ne l)$.

- If $\lfloor th_{1l} C_l \rfloor \le j_l < \lfloor th_{2l} C_l \rfloor$, then link l is in a *normal mode* of operation. In that mode, the link does not service offloaded calls from the other link.

- If $\lfloor th_{2l} C_l \rfloor \le j_l$, then link l operates in an *offloading mode*. In that mode, a new call that initially arrives in link l is offloaded to link m. If that link is in *support mode* (i.e. $0 \le j_m < \lfloor th_{1m} C_m \rfloor$), then the call is accepted in link m. Otherwise, if $j_l \le C_l - 1$, the call is accepted in link l, whereas if $j_l > C_l - 1$, the call is blocked and lost.

Based on the above description, the call admission mechanism applicable to a call that arrives in link l $(l = 1,2)$ consists of two steps:

1. If $(0 \leq j_l < \lfloor th_{2l} C_l \rfloor)$, then the call is serviced via link l.

2. If $\lfloor th_{2l} C_l \rfloor \leq j_l$, then:

   - If $0 \leq j_m < \lfloor th_{1m} C_m \rfloor$, the call is offloaded to link m.
   - If $\lfloor th_{1m} C_m \rfloor \leq j_m$, then link m does not support offloaded calls from link l since it operates in *normal mode*. In that case, the call will be handled by link l. Thus, if $j_l \leq C_l - 1$, the call is accepted in link l. Otherwise, call blocking occurs.

Due to the *offloading* and *support modes* modes of the two links, there is no LB between adjacent system states and, therefore, the steady state distribution, $P(j) = P(j_1, j_2)$, of such a system cannot be described by a PFS. To determine $P(j_1, j_2)$, two methods exist in the literature.
The first method provides accurate CBP results (compared to simulation results) but is quite complex, since it requires the solution of a set of linear GB equations for each state $\boldsymbol{j} = (j_1, j_2)$ expressed as *rate into state* $\boldsymbol{j}$ *= rate out of state* $\boldsymbol{j}$:

$$\begin{aligned} \lambda_1(j_1-1, j_2)P(j_1-1, j_2) + \lambda_2(j_1, j_2-1)P(j_1, j_2-1) \\ +(j_1+1)\mu P(j_1+1, j_2) + (j_2+1)\mu P(j_1, j_2+1) \\ = \lambda_1(j_1, j_2)P(j_1, j_2) + \lambda_2(j_1, j_2)P(j_1, j_2) \\ +(j_1\mu + j_2\mu)P(j_1, j_2) , \end{aligned} \quad (1)$$

where:

$$l = 1,\, 2\,, \quad m \neq l$$

and

$$\lambda_l(j_1, j_2) = \begin{cases} \lambda_l + \lambda_m & \text{if } (j_l < \lfloor th_{1l} C_l \rfloor) \cap (j_m \geq \lfloor th_{2m} C_m \rfloor) \\ 0 & \text{if } (j_l \geq \lfloor th_{2l} C_l \rfloor) \cap (j_m < \lfloor th_{1m} C_m \rfloor) \\ 0 & \text{if } (j_1, j_2) \text{ is a boundary state} \\ \lambda_l & \text{otherwise} \end{cases} . \quad (2)$$

Having obtained $P(j_1, j_2)$, we can determine CBP in each link, $P_{b_1}^{'}$ and $P_{b_2}^{'}$ via Eqs. (3) and (4), respectively [29]:

$$P_{b_1}^{'} = \sum_{j_2 = \lfloor th_{12} C_2 \rfloor}^{C_2} P(C_1, j_2) , \quad (3)$$

$$P_{b_2}^{'} = \sum_{j_1 = \lfloor th_{11} C_1 \rfloor}^{C_1} P(j_1, C_2) . \quad (4)$$

Equation (3) expresses the fact that call blocking occurs in the first link if all b.u. are occupied (i.e. if $j_1 = C_1$) and, at the same time, the other link does not operate in the support mode (i.e. if $\lfloor th_{12} C_2 \rfloor \leq j_2$). Equation (4) may be interpreted accordingly.

To determine CBP in the system of [29], the following weighted summation can be used:

$$P_b^{'} = \frac{\lambda_1}{\lambda_1 + \lambda_2} P_{b_1}^{'} + \frac{\lambda_2}{\lambda_1 + \lambda_2} P_{b_2}^{'} . \quad (5)$$

Contrary to the first method, the second method is simpler but provides approximate CBP results. The approximation lies on the fact that each link l is modeled as an independent Erlang loss system of capacity C_l $(l = 1, 2)$.
The CBP in each link may be approximated by Eqs. (6) and (7), respectively:

$$P_{b_1} = P_1(C_1) P_2(j_2 \geq \lfloor th_{12} C_2 \rfloor) , \quad (6)$$

$$P_{b_2} = P_2(C_2) P_1(j_1 \geq \lfloor th_{11} C_1 \rfloor) , \quad (7)$$

where $P_l(C_l)$ is the CBP in link l $(l = 1, 2)$.
The values of $P_l(C_l)$ in Eqs. (6) and (7) may be determined via the Erlang B formula (see Eq. (8a) for the closed form or Eq. (8b) for the recurrent form):

$$P_l(C_l) = \frac{\frac{a_l^{C_l}}{C_l!}}{\sum\limits_{i=0}^{C_l} \frac{a_l^i}{i!}} , \quad a_l = \frac{\lambda_l}{\mu} , \quad (8a)$$

$$P_l(C_l) = \frac{a_l P_l(C_l - 1)}{C_l + a_l P_l(C_l - 1)} , \quad C_l \geq 1 , \quad P_l(0) = 1 . \quad (8b)$$

As far as the values of $P_l(j_l \geq \lfloor th_{1l} C_l \rfloor)$ in Eqs. (6) and (7) are concerned, they can be determined by:

$$P_l(j_l \geq \lfloor th_{1l} C_l \rfloor) = \sum_{j_l = \lfloor th_{1l} C_l \rfloor}^{C_l} P_l(j_l) , \quad (9)$$

where $P_l(j_l)$ is calculated according to the truncated Poisson distribution:

$$P_l(j_l) = \frac{\frac{a_l^{j_l}}{j_l!}}{\sum\limits_{i=0}^{C_l} \frac{a_l^i}{i!}} , \quad a_l = \frac{\lambda_l}{\mu} . \quad (10)$$

As far as the total blocking probability in the two-link system is concerned, it can be determined via Eq. (5), where $P_{b_1}^{\prime}$ and $P_{b_2}^{\prime}$ are replaced by P_{b_1} and P_{b_2}, determined in Eqs. (6) and (7), respectively.
An additional recursive way for the determination of $P_l(j_l)$, $j_l = 1, \ldots, C_l$ is based on the link independence assumption. In the Erlang loss model, used to describe each link l, there exist the following LB between states $j_l - 1$ and j_l [44]:

$$j_l P_l^{\prime}(j_l) = a_l P_l^{\prime}(j_l - 1) . \quad (11)$$

Based on Eq. (11), we can determine the unnormalized values of $P_l^{\prime}(j_l)$'s considering an initial value of $P_l^{\prime}(0) = 1$. Then, the normalized values of $P_l^{\prime}(j_l)$'s are given by:

$$P_l(j_l) = \frac{P_l^{\prime}(j_l)}{\sum\limits_{x=0}^{C_l} P_l^{\prime}(x)} . \quad (12)$$

Based on Eq. (12), we can compute P_{b_1}, P_{b_2} and, consequently, the total CBP, via Eqs. (6), (7) and (5), respectively.

3. The Proposed Multirate Loss Model – Poisson Case

In the proposed loss model, each link services Poisson arriving calls of K service-classes. New calls of service class k ($k = 1,\dots,K$) require b_k b.u. in order to be accepted in a link and have a generally distributed service-time with a mean of μ_k^{-1}. Let λ_{1k} and λ_{2k} be the arrival rates in each link of service-class k calls, respectively. We also denote, by j_1 and j_2, the b.u. occupied in each link. Similarly to Section 2, each link l ($l = 1,2$) has a support threshold th_{1l} and an offloading threshold th_{2l}, with $th_{1l} < th_{2l}$ and $0 \leq th_{1l}, th_{2l} \leq 1$.

To incorporate restricted accessibility into our model, we assume that each state j_l of link l, except for state $j_l = 0$ where link l is empty, is associated with a blocking probability, $pb_{l,k}(j_l)$. When there are no available b.u. for calls of service-class k in link l (i.e. when $j_l \geq C_l - b_k + 1$), then $pb_{l,k}(j_l) = 1$. Similarly, in the case of an empty system, $pb_{l,k}(0) = 0$.

The procedure of admitting a new service class k call that arrives in link l ($l = 1,2$) is the following:

1. If $(0 \leq j_l < \lfloor th_{2l}C_l \rfloor)$, then the call is handled by link l. In addition, if $j_l + b_k \leq C_l$, then the call is accepted in link l with probability $1 - pb_{l,k}(j_l)$.
2. If $\lfloor th_{2l}C_l \rfloor \leq j_l$, then:
   - If $(0 \leq j_m < \lfloor th_{1m}C_m \rfloor)$, the call is offloaded to link m and if $j_m + b_k \leq C_m$, the call is accepted in link m with probability $1 - pb_{m,k}(j_m)$.
   - If $\lfloor th_{1m}C_m \rfloor \leq j_m$, then link m operates in *normal mode* and, therefore, does not support offloaded calls. Thus, the call is handled by link l. If $j_l + b_k \leq C_l$, then the call is accepted in link l with probability $1 - pb_{l,k}(j_l)$. Otherwise, the call is blocked and lost.

To calculate the CBP of service-class k calls, we assume that each link is an independent EMLM system under restricted accessibility [48] and, therefore, the CBP of service-class k calls in the first and the second link can be given via Eqs. (13) and (14), respectively:

$$P_{\text{res},b_{1k}} = P_{\text{res},1k}(C_1)\, P_{\text{res},2}(j_2 \geq \lfloor th_{12}C_2 \rfloor) \ , \tag{13}$$

$$P_{\text{res},b_{2k}} = P_{\text{res},2k}(C_2)\, P_{\text{res},1}(j_1 \geq \lfloor th_{11}C_1 \rfloor) \ , \tag{14}$$

where $P_{\text{res},lk}(C_{lk})$ is the CBP of service-class k calls in link l ($l = 1,2$) and $P_{\text{res},l}(j_l \geq \lfloor th_{1l}C_l \rfloor)$ is the probability that link l does not operate in the support mode.

The values of $P_{\text{res},lk}(C_l)$ in Eqs. (13) and (14) can be given by:

$$P_{\text{res},lk}(C_l) = \sum_{j_l=1}^{C_l} G_l^{-1} q(j_l)\, pb_{l,k}(j_l) \ , \tag{15}$$

where $q(j_l)$ expresses the unnormalized values of the occupancy distribution of link l ($l = 1, 2$) while $G_l = \sum_{j_l=0}^{C_l} q(j_l)$ refers to the normalization constant.

In Eq. (15), the values of $q(j_l)$ can be computed via:

$$q(j_l) = \begin{cases} 1 & \text{for } j_l = 0 \\ \frac{1}{j_l} \sum_{k=1}^{K} a_{lk} b_k q(j_l - b_k) \times & \\ \quad [1 - pb_{l,k}(j_l - b_k)] & \text{for } j_l = 1,\dots,C_l \end{cases} \ , \tag{16}$$

where $a_{lk} = \lambda_{lk}/\mu_k$ is the total offered traffic-load of service-class k calls in link l.

Regarding the values of $P_{\text{res},l}(j_l \geq th_{1l}C_l)$, in (13) and (14), they can be calculated by:

$$P_{\text{res},l}(j_l \geq \lfloor th_{1l}C_l \rfloor) = \sum_{j_l=\lfloor th_{1l}C_l \rfloor}^{C_l} G_l^{-1} q(j_l) \ , \tag{17}$$

where $q(j_l)$ is given by (16).

Finally, Eq. (18) is proposed for determining the total blocking probability of service-class k calls in the two-link system:

$$P_{\text{res},b_k} = \frac{\lambda_{1k}}{\lambda_{1k} + \lambda_{2k}} P_{\text{res},b_{1k}} + \frac{\lambda_{2k}}{\lambda_{1k} + \lambda_{2k}} P_{\text{res},b_{2k}} \ . \tag{18}$$

4. The Proposed Multirate Loss Model – Quasi-Random Case

Contrary to the model proposed in Section 3, we now assume that each link services quasi-random traffic generated by K service-classes. New calls of service-class k ($k = 1, \dots, K$) in link l ($l = 1, 2$) are generated via a finite source population N_{lk} and require b_k b.u. Let $\lambda_{1k,\text{fin}}$ and $\lambda_{2k,\text{fin}}$ be the arrival rates of service-class k idle sources in the first and second link, respectively. Then, $\lambda_{lk,\text{fin}} = (N_{lk} - n_{1k} - n_{2k})\, v_{lk}$, where n_{lk} refers to the in-service calls of service-class k in link l and v_{lk} is the arrival rate per idle source of service-class k in link l. The corresponding offered traffic-load per idle source is $a_{lk,\text{idle}} = v_{lk}/\mu_k$. Note that if $N_{lk} \to \infty$ for all service-classes and the total offered traffic-load is constant, then calls arrive in the system according to a Poisson process, resulting in the model described in Section 3.

The process of admitting a new service-class k call that arrives in link l ($l = 1,2$) is similar to that described in Section 3 and is therefore omitted.

To calculate the time congestion probabilities of service-class k calls, we assume that each link is an independent Engset multirate loss model (EnMLM) under restricted accessibility and, therefore, time congestion probabilities of service-class k calls in the first and the second link can be determined via Eqs. (19) and (20), respectively:

$$P_{\text{fin-res},b_{1k}} = P_{\text{fin-res},1k}(C_1)\, P_{\text{fin-res},2}(j_2 \geq \lfloor th_{12}C_2 \rfloor) \ , \tag{19}$$

$$P_{\text{fin-res},b_{2k}} = P_{\text{fin-res},2k}(C_2)\, P_{\text{fin-res},1}(j_1 \geq \lfloor th_{11}C_1 \rfloor) \ , \tag{20}$$

where $P_{\text{fin-res},lk}(C_{lk})$ is the time congestion probability of service-class k calls in link l ($l = 1,2$) and

$P_{\text{fin-res},l}(j_l \geq \lfloor th_{1l}C_l \rfloor)$ is the probability that link l is not in the support mode.

The values of $P_{\text{fin-res},lk}(C_l)$ in Eqs. (19) and (20) can be determined via the following formula:

$$P_{\text{fin-res},lk}(C_l) = \sum_{j_l=1}^{C_l} G_l^{-1} q_{\text{fin}}(j_l) pb_{l,k}(j_l) \ , \qquad (21)$$

where $q_{\text{fin}}(j_l)$ expresses the unnormalized values of the occupancy distribution of link l ($l = 1,2$), while $G_l = \sum_{j_l=0}^{C_l} q_{\text{fin}}(j_l)$ is the corresponding normalization constant.

In Eq. (21), the values of $q_{\text{fin}}(j_l)$ can be calculated as follows:

$$q_{\text{fin}}(j_l) = \begin{cases} 1 & \text{for } j_l = 0 \\ \frac{1}{j_l} \sum_{k=1}^{K} (N_{lk} - Y_k) a_{lk,\text{idle}} b_k q_{\text{fin}} \times & \\ (j_l - b_k)[1 - pb_{l,k}(j_l - b_k)] & \text{for } j_l = 1, \ldots, C_l \end{cases} , \qquad (22)$$

where $Y_k = y_{1k}(j_1 - b_k) - y_{2k}(j_2 - b_k)$ and $y_{lk}(j_l)$ is the average number of service-class k calls in state j_l of link l, assuming that the system accommodates Poisson traffic.

The values of $y_{lk}(j_l)$ are given by:

$$y_{lk}(j_l) = \frac{a_{lk} q(j_l - b_k)[1 - pb_{l,k}(j_l - b_k)]}{q(j)} \ , \qquad (23)$$

where the values of $q(j)$ are computed via Eq. (16) (i.e. by the corresponding Poisson model).

The rationale behind Eqs. (22) and (23) is similar to that of the model from [51] that proposes an algorithm for the approximate determination of time congestion probabilities in the EnMLM.

Finally, regarding the values of $P_{\text{fin-res},l}(j_l \geq th_{1l}C_l)$, in Eqs. (19) and (20), they are given by:

$$P_{\text{fin-res},l}(j_l \geq \lfloor th_{1l}C_l \rfloor) = \sum_{j_l = \lfloor th_{1l}C_l \rfloor}^{C_l} G_l^{-1} q_{fin}(j_l) \ , \qquad (24)$$

where $q_{\text{fin}}(j_l)$ is determined via Eq. (22).

5. Numerical Examples

In this section, we consider two examples and provide simulation and analytical CBP results of the proposed model assuming Poisson traffic. Simulation results are based on Simscript III [52] and are mean values of 7 runs. In each run, ten million calls are generated. The first 5% of these generated calls are not considered in the CBP results so as to account for a warm-up period.

In the first example, we consider a system with the capacities of $C_1 = 24$ b.u. and $C_2 = 20$ b.u., accommodating two service-classes whose calls require $b_1 = 1$ and $b_2 = 2$ b.u., respectively. Let $\lambda_{11} = 9$ calls/min and $\lambda_{12} = 1$ call/min, for the first link. Similarly, let $\lambda_{21} = 7$ calls/min and

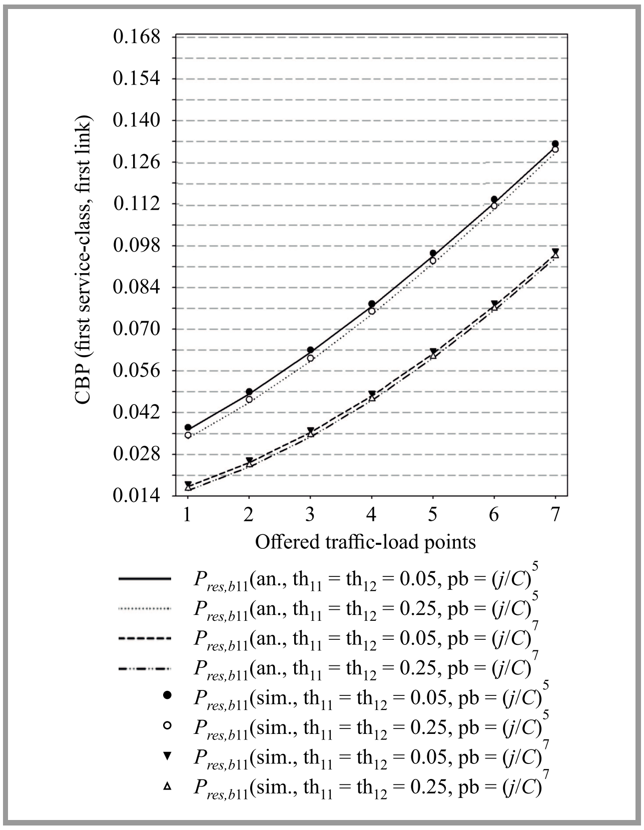

Fig. 2. CBP of the first service-class in the first link (example 1).

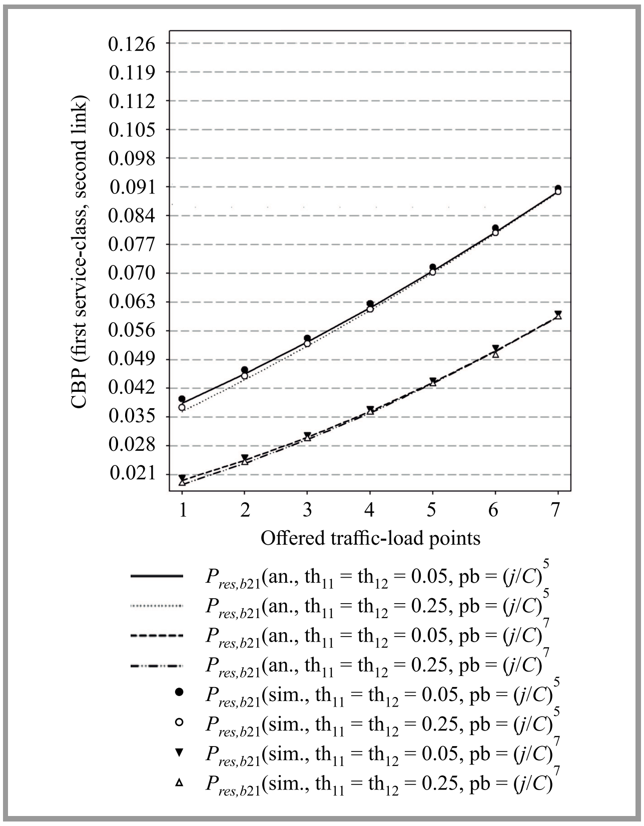

Fig. 3. CBP of the first service-class in the second link (example 1).

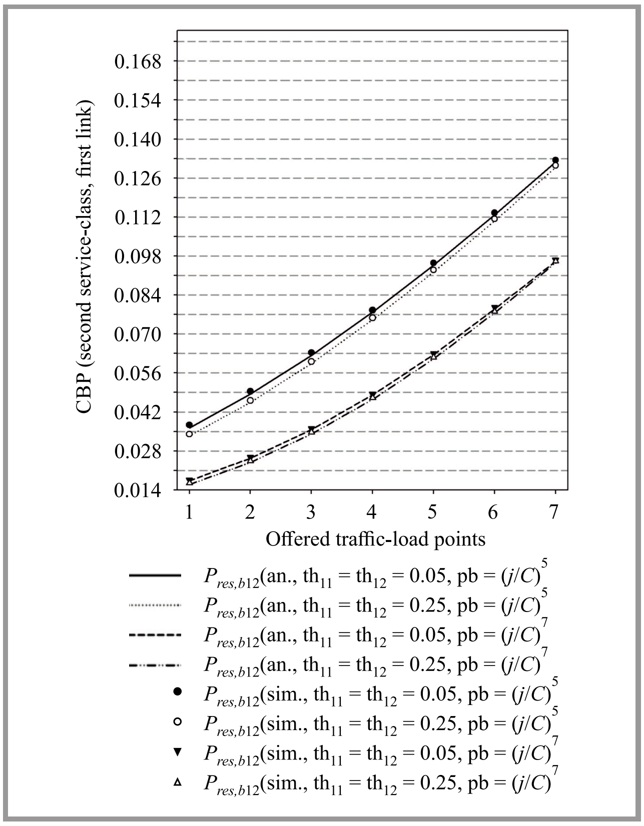

Fig. 4. CBP of the second service-class in the first link (example 1).

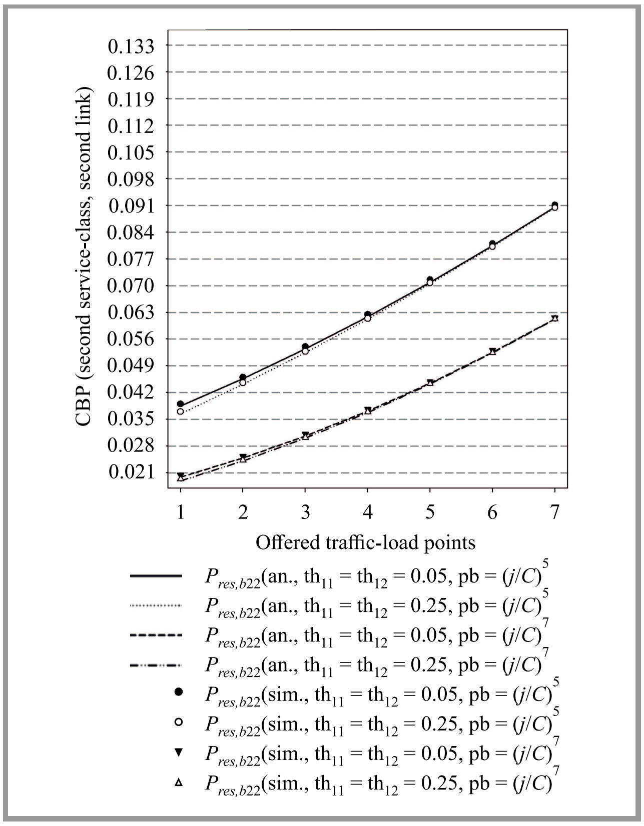

Fig. 5. CBP of the second service-class in the second link (example 1).

$\lambda_{22} = 1$ call/min, for the second link. Also let $\mu_1^{-1} = \mu_2^{-1} = 1.0$ min. Regarding the values of the thresholds, let the offloading thresholds equal $th_{21} = th_{22} = 0.7$, and let us consider two support threshold scenarios: (i) $th_{11} = th_{12} = 0.05$ and (ii) $th_{11} = th_{12} = 0.25$. Finally, regarding the restricted accessibility factors for each link, two sets are studied: (i) $pb_{l,1}(j_l) = pb_{l,2}(j_l) = (j_l/C_l)^5$ and (ii) $pb_{l,1}(j_l) = pb_{l,2}(j_l) = (j_l/C_l)^7$ where $l = 1,2$.

In the x-axis of Figs. 2–5, λ_{11} and λ_{21} increase in steps of 1.0 and 0.5, respectively. So, point 1 is: ($\lambda_{11} = 9.0$, $\lambda_{12} = 1.0$, $\lambda_{21} = 7.0$, $\lambda_{22} = 1.0$), while point 7 is: ($\lambda_{11} = 15.0$, $\lambda_{12} = 1.0$, $\lambda_{21} = 10.0$, $\lambda_{22} = 1.0$).

In Figs. 2–3, we present the CBP for the first service-class in each link, respectively. In Figs. 4–5, the corresponding CBP results for the second service-class are presented. Figures 2–5 show that the analytical CBP results:

- Are close to the simulation results, especially when the values of support thresholds th_{11} and th_{12} are at a reasonable level (e.g. 0.05 to 0.25). Depending on the system, higher values of th_{11} and th_{12} may increase the discrepancy between simulation and analytical CBP results. This behavior may also be observed in [29] and is anticipated due to the fact that both links work independently.
- The choice of $pb_{l,k}(j_l)$ greatly affects CBP. The higher values of set 1 leads to much higher CBP compared to the values of set 2.

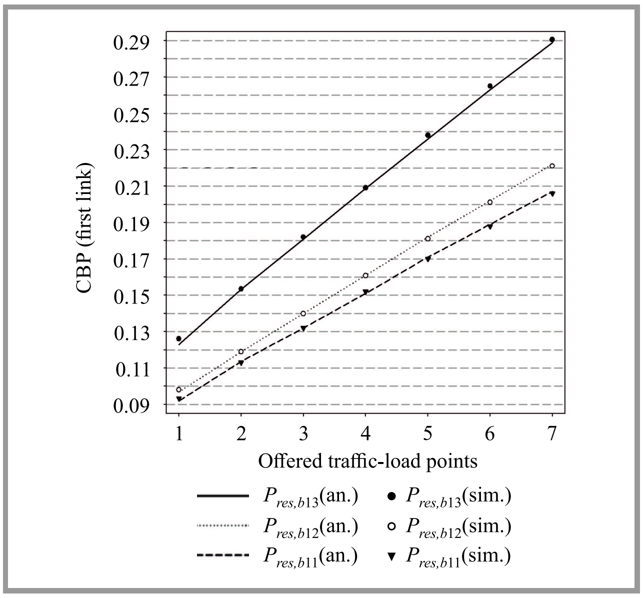

Fig. 6. CBP of each service-class in the first link (example 2).

In the second example, a larger system is considered. More precisely, we study a system of capacities $C_1 = 40$ b.u. and $C_2 = 45$ b.u. that services three service-classes whose calls require $b_1 = 1$, $b_2 = 3$ and $b_3 = 6$ b.u., respectively. Let $\lambda_{11} = 5$ calls/min, $\lambda_{12} = 3$ calls/min and $\lambda_{13} = 2$ calls/min, for the first link. Similarly, let $\lambda_{21} = 5$ calls/min, $\lambda_{22} = 3$ calls/min and $\lambda_{23} = 2$ calls/min, for the second link. Also

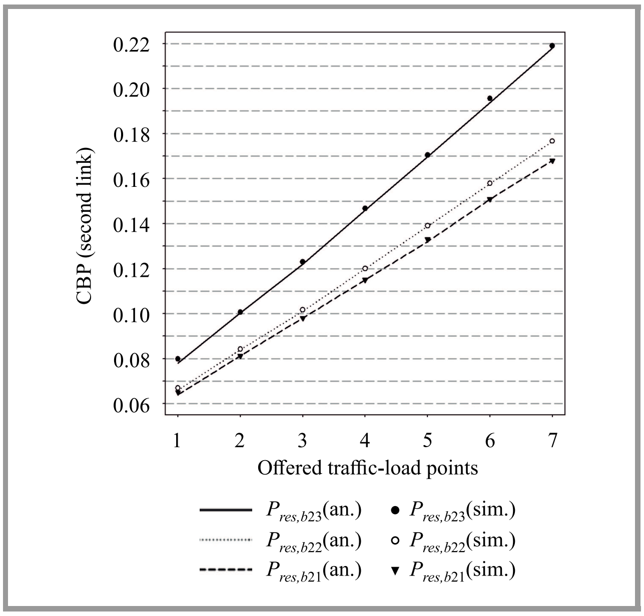

Fig. 7. CBP of each service-class in the second link (example 2).

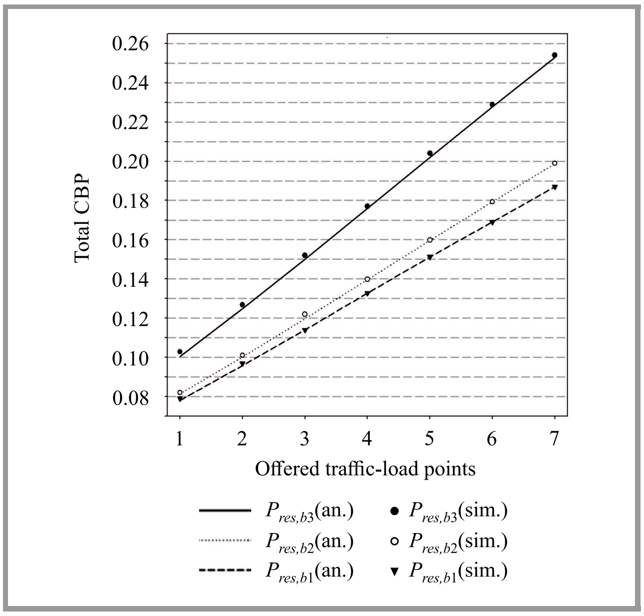

Fig. 8. Total CBP (example 2).

let $\mu_1^{-1} = \mu_2^{-1} = \mu_3^{-1} = 1$ min. Regarding the values of the thresholds, let the offloading thresholds equal $th_{21} = th_{22} = 0.7$ and let the support thresholds equal $th_{11} = th_{12} = 0.2$. Finally, regarding the restricted accessibility factors, let $pb_{l,1}(j_l) = pb_{l,2}(j_l) = (j_j/C_l)^7$ where $l = 1,2$.

In the x-axis of Figs. 6–8, $\lambda_{11}, \lambda_{12}, \lambda_{13}, \lambda_{21}, \lambda_{22}$ and λ_{23} increase in steps of 0.2, respectively. So, point 1 is: ($\lambda_{11} = 5.0$, $\lambda_{12} = 3.0$, $\lambda_{13} = 2.0$, $\lambda_{21} = 5.0$, $\lambda_{22} = 3.0$, $\lambda_{23} = 2.0$), while point 7 is: ($\lambda_{11} = 6.2$, $\lambda_{12} = 4.2$, $\lambda_{13} = 3.2$, $\lambda_{21} = 6.2$, $\lambda_{22} = 4.2$, $\lambda_{23} = 3.2$). In Figs. 6–7, we show the CBP for all service-classes in each link, respectively. In Fig. 8, we present the total CBP results for all service-classes. Figures 6–8 show that the analytical CBP results are again quite close to the corresponding simulation results. A similar degree of accuracy has been observed for various two-link systems that we studied.

6. Conclusion

We propose new multirate loss models for the call-level analysis of a two-link system with restricted accessibility that accommodates Poisson or quasi-random arriving calls of different service-classes. In this system, each link may support calls offloaded from the other link. The proposed models do not have a PFS for the steady state probabilities due to the restricted accessibility and existence of the offloading mechanism. However, we show that an approximate method does exist for the determination of blocking probabilities, achieving a satisfactory degree of accuracy compared to simulation. As a future work, we intend to analyze the case of interference between the two links, using the proposed model as a springboard for further considerations.

References

[1] M. Stasiak, M. Głąbowski, A. Wiśniewski, and P. Zwierzykowski, *Modeling and Dimensioning of Mobile Networks*. Wiley, 2011 (ISBN: 9780470665862).

[2] V. Iversen, *Teletraffic engineering and network planning*. DTU Photonic, Denmark, 2015 [Online]. Available: https://backend.orbit.dtu.dk/ws/portalfiles/portal/118473571/Teletraffic_34342_V_B_Iversen_2015.pdf

[3] V. Iversen, "Modelling restricted accessibility for wireless multi-service systems", in *Wireless Systems and Network Architectures in Next Generation Internet Second International Workshop of the EURO-NGI Network of Excellence, Villa Vigoni, Italy, July 13-15, 2005, Revised Selected Papers*, M. Cesana and L. Fratta, Eds. *LNCS*, vol. 3883, pp. 93–102, Springer, 2006 (DOI: 10.1007/11750673_8).

[4] I. Moscholios and M. Logothetis, *Efficient Multirate Teletraffic Loss Models Beyond Erlang*. Wiley & IEEE Press, 2019 (ISBN: 9781119426882).

[5] M. Stasiak and M. Głąbowski, "A simple approximation of the link model with reservation by a one-dimensional Markov chain", *Perform. Eval.*, vol. 41, no. 2–3, pp. 195–208, 2000 (DOI: 10.1016/S0166-5316(00)00008-0).

[6] I. Moscholios, M. Logothetis, and G. Kokkinakis, "Connection dependent threshold model: a generalization of the Erlang multiple rate loss model", *Perform. Eval.*, vol. 48, no. 1–4, pp. 177–200, 2002 (DOI: 10.1016/S0166-5316(02)00037-8).

[7] M. Głąbowski and M. Stasiak, "Point-to-point blocking probability in switching networks with reservation", *Annals of Telecommun.*, vol. 57, no. 7–8, pp. 798–831, 2002 (DOI: 10.1007/BF02995519).

[8] S. Rácz, B. Gerő, and G. Fodor, "Flow level performance analysis of a multi-service system supporting elastic and adaptive services", *Perform. Eval.*, vol. 49, no. 1–4, pp. 451–469, 2002 (DOI: 10.1016/S0166-5316(02)00115-3).

[9] M. Głąbowski and M. Stasiak, "Multi-rate model of the limited availability group with finite source population", in *Proc. Joint Conf. of the 10th Asia-Pacific Conf. on Commun. and the 5th Int. Symp. on Multi-Dimens. Mob. Commun. APCC/MDMC'04*, Beijing, China, 2004 (DOI: 10.1109/APCC.2004.1391716).

[10] I. Moscholios and M. Logothetis, "Engset multirate state-dependent loss models with QoS guarantee", *Int. J. of Commun. Syst.*, vol. 19, no. 1, pp. 67–93, 2006 (DOI: 10.1002/dac.748).

[11] V. Vassilakis, I. Moscholios, and M. Logothetis, "Call-level performance modelling of elastic and adaptive service-classes with finite population", *IEICE Transactions on Communications*, vol. E91-B, no. 1, pp. 151–163, 2008 (DOI: 10.1093/ietcom/e91-b.1.151).

[12] Q. Huang, King-Tim Ko, and V. Iversen, "Approximation of loss calculation for hierarchical networks with multiservice overflows", *IEEE Trans. on Commun.*, vol. 56, no. 3, pp. 466–473, 2008 (DOI: 10.1109/TCOMM.2008.060051).

[13] M. Stasiak, M. Sobieraj, J. Weissenberg, and P. Zwierzykowski, "Analytical model of the single threshold mechanism with hysteresis for multi-service networks", *IEICE Trans. on Commun.*, vol. E95-B, no. 1, pp. 120–132, 2012 (DOI: 10.1587/transcom.E95.B.120).

[14] I. Moscholios, J. Vardakas, M. Logothetis, and M. Koukias, "A quasi-random multirate loss model supporting elastic and adaptive traffic under the bandwidth reservation policy", *Int. J. on Adv. in Netw. and Serv.*, vol. 6, no. 3–4, pp. 163–174, 2013 [Online]. Available: http://citeseerx.ist.psu.edu/viewdoc/download?doi=10.1.1.442.3722&rep=rep1&type=pdf#page=56

[15] S. Yan, M. Razo, M. Tacca, and A. Fumagalli, "A blocking probability estimator for the multi-application and multi-resource constraint problem", in *Proc. Int. Conf. on Comput., Netw. and Commun. ICNC 2014*, Honolulu, Hawaii, 2014 (DOI: 10.1109/ICCNC.2014.6785460).

[16] I. Moscholios, M. Logothetis, J. Vardakas, and A. Boucouvalas, "Performance metrics of a multirate resource sharing teletraffic model with finite sources under both the threshold and bandwidth reservation policies", *IET Networks*, vol. 4, no. 3, pp. 195–208, 2015 (DOI: 10.1049/iet-net.2014.0050).

[17] Y. Huang, Z. Rosberg, K. Ko, and M. Zukerman, "Blocking probability approximations and bounds for best-effort calls in an integrated service system", *IEEE Trans. on Commun.*, vol. 63, no. 12, pp. 5014–5026, 2015 (DOI: 10.1109/TCOMM.2015.2494047).

[18] I. Moscholios, M. Logothetis, J. Vardakas, and A. Boucouvalas, "Congestion probabilities of elastic and adaptive calls in Erlang-Engset multirate loss models under the threshold and bandwidth reservation policies", *Comp. Networks*, vol. 92, no. 1, pp. 1–23, 2015 (DOI: 10.1016/j.comnet.2015.09.010).

[19] M. Głąbowski and M. Sobieraj, "Analytical modelling of multiservice switching networks with multiservice sources and resource management mechanisms", *Telecommun. Syst.*, vol. 66, no. 3, pp. 559–578, 2017 (DOI: 10.1007/s11235-017-0305-4).

[20] S. Sagkriotis, S. Pantelis, I. Moscholios, and V. Vassilakis, "Call blocking probabilities in a two-Link multi rate loss system for Poisson traffic", *IET Networks*, vol. 7, no. 4, pp. 233–241, 2018 (DOI: 10.1049/iet-net.2017.0223).

[21] I. Widjaja and H. Roche, "Sizing X2 bandwidth for Inter-connected eNBs", in *Proc. IEEE 70th Veh. Technol. Conf. VTC Fall 2009*, Anchorage, Alaska, USA, 2009 (DOI: 10.1109/VETECF.2009.5378834).

[22] M. Stasiak, P. Zwierzykowski, and D. Parniewicz, "Modelling of the WCDMA interface in the UMTS network with Soft Handoff Mechanism", in *Proc. IEEE Global Telecommun. Conf. GLOBECOM 2009*, Honolulu HI, USA, 2009 (DOI: 10.1109/GLOCOM.2009.5425329).

[23] B. Renard, S. Elayoubi, and A. Simonian, "A dimensioning method for the LTE X2 interface", in *Proc. IEEE Wirel. Commun. and Network. Conf. WCNC 2012*, Paris, France, 2012 (DOI:10.1109/WCNC.2012.6214261).

[24] M. Stasiak, D. Parniewicz, and P. Zwierzykowski, "Traffic engineering for multicast connections in multiservice cellular network", *IEEE Trans. on Indust. Inform.*, vol. 9, no. 1, pp. 262–270, 2013 (DOI: 10.1109/TII.2012.2188902).

[25] I. Moscholios, G. Kallos, M. Katsiva, V. Vassilakis, and M. Logothetis, "Call blocking probabilities in a W-CDMA cell with interference cancellation and bandwidth reservation", in *Proc. IEICE Inform. and Commun. Technol. Forum*, Poznań, Poland, 2014 (DOI:10.34385/proc.19.COMM1-4).

[26] I. Moscholios, G. Kallos, V. Vassilakis, and M. Logothetis, "Congestion probabilities in CDMA-based networks supporting batched Poisson input traffic", *Wirel. Personal Commun.*, vol. 79, no. 2, pp. 1163–1186, 2014 (DOI: 10.1007/s11277-014-1923-8).

[27] M. Khedr and R. Makki Hassan, "Opportunistic call admission control for wireless broadband cognitive networks", *Wirel. Networks*, vol. 20, no. 1, pp. 105–114, 2014 (DOI: 10.1007/s11276-013-0596-7).

[28] A. Machado de Medeiros and M. Yacoub, "BlockOut: blocking and outage in a single performance measure", *IEEE Trans. on Veh. Technol.*, vol. 63, no. 7, pp. 3451–3456, 2014 (DOI: 10.1109/TVT.2014.2299438).

[29] V. Burger, M. Seufert, T. Hossfeld, and P. Tran-Gia, "Performance evaluation of backhaul bandwidth aggregation using a partial sharing scheme", *Phys. Commun.*, vol. 19, pp. 135–144, 2016 (DOI: 10.1016/j.phycom.2016.01.005).

[30] I. Moscholios, V. Vassilakis, M. Logothetis, and A. Boucouvalas, "A probabilistic threshold-based bandwidth sharing policy for wireless multirate loss networks", *IEEE Wirel. Commun. Lett.*, vol. 5, no. 3, pp. 304–307, 2016 (DOI: 10.1109/LWC.2016.2547913).

[31] V. Vassilakis, I. Moscholios, and M. Logothetis, "Uplink blocking probabilities in priority-based cellular CDMA networks with finite source population", *IEICE Trans. on Commun.*, vol. E99-B , no. 1, pp. 1302–1309, 2016 (DOI: 10.1587/transcom.2015EUP0010).

[32] V. Vassilakis, I. Moscholios, and M. Logothetis, "Quality of service differentiation of elastic and adaptive services in CDMA networks: a mathematical modelling approach", *Wirel. Networks*, vol. 24, no. 4, pp. 1279–1295, 2018 (DOI: 10.1007/s11276-016-1411-z).

[33] P. Panagoulias and I. Moscholios, "Congestion probabilities in the X2 link of LTE networks", *Telecommun. Syst.*, vol. 71, no. 4, pp. 585–599, 2019 (DOI: 10.1007/s11235-018-00537-5).

[34] Z. Wang, P. Mathiopoulos, and R. Schober, "Performance analysis and improvement methods for channel resource management strategies of LEO-MSS with multiparty traffic", *IEEE Trans. on Veh. Technol.*, vol. 57, no. 6, pp. 3832–3842, 2008 (DOI: 10.1109/TVT.2008.919979).

[35] D. Yiltas and A. Zaim, "Evaluation of call blocking probabilities in LEO satellite networks", *Int. J. of Satell. Commun. and Network.*, vol. 27, no. 2, pp. 103–115, 2009 (DOI: 10.1002/sat.928).

[36] Z. Wang, P. Mathiopoulos, and R. Schober, "Channel partitioning policies for multi-class traffic in LEO-MSS", *IEEE Trans. on Aerospace and Electron. Syst.*, vol. 45, no. 4, pp. 1320–1334, 2009 (DOI: 10.1109/TAES.2009.5310301).

[37] J. Vardakas, I. Moscholios, M. Logothetis, and V. Stylianakis, "An analytical approach for dynamic wavelength allocation in WDM-TDMA PONs servicing ON-OFF traffic", *IEEE/OSA J. of Opt. Commun. and Network.*, vol. 3, no. 4, pp. 347–358, 2011 (DOI: 10.1364/JOCN.3.000347).

[38] Y. Deng and P. Prucnal, "Performance analysis of heterogeneous optical CDMA networks with bursty traffic and variable power control", *IEEE/OSA J. of Opt. Commun. and Network.*, vol. 3, no. 6, pp. 487–492, 2011 (DOI: 10.1364/JOCN.3.000487).

[39] J. Vardakas, I. Moscholios, M. Logothetis, and V. Stylianakis, "On code reservation in multi-rate OCDMA passive optical networks", in *Proc. 8th Int. Symp. on Commun. Sys., Netw. Digit. Signal Process. CSNDSP 2012*, Poznań, Poland, 2012 (DOI: 10.1109/CSNDSP.2012.6292722).

[40] J. Vardakas, I. Moscholios, M. Logothetis, and V. Stylianakis, "Performance analysis of OCDMA PONs supporting multi-rate bursty traffic", *IEEE Trans. on Commun.*, vol. 61, no. 8, pp. 3374–3384, 2013 (DOI: 10.1109/TCOMM.2013.061913.120798).

[41] V. Casares-Giner, "Some teletraffic issues in optical burst switching with burst segmentation", *Electron. Lett.*, vol. 52, no. 11, pp. 941–943, 2016 (DOI: 10.1049/el.2016.0137).

[42] Y. Guan, H. Jiang, M. Gao, S. Bose, and G. Shen, "Migrating elastic optical networks from standard single-mode fibers to ultra-low loss fibers: strategies and benefits", in *Proc. Opt. Fiber Commun. Conf. and Exhibi. OFC 2017*, Los Angeles, CA, USA, 2017 (DOI: 10.1364/OFC.2017.W4H.3).

[43] S. Hanczewski, M. Stasiak, and J. Weissenberg, "Non-full-available model of an EON node", *Optical Switch. and Network.*, vol. 33, pp. 131–142, 2019 (DOI: 10.1016/j.osn.2018.01.004).

[44] S. Pantelis, I. Moscholios, and S. Papadopoulos, "Call-level evaluation of a two-link single rate loss model for Poisson traffic", in *Proc. IEICE Inform. and Commun. Technol. Forum*, Poznań, Poland, 2017 (DOI:10.34385/proc.50.SESSION09_2).

[45] L. Mamatas, I. Psaras, and G. Pavlou, "Incentives and algorithms for broadband access sharing", in Proc. *ACM SIGCOMM Worksh. on Home Netwo.*, New Delhi, India, 2010 (DOI: 1851307.1851312).

[46] I. Psaras and L. Mamatas, "On demand connectivity sharing: Queuing management and load balancing for user-provided networks", *Comp. Networks*, vol. 55, no. 2, pp. 399–414, 2011 (DOI: 10.1016/j.comnet.2010.08.015).

[47] I. Moscholios, V. Vassilakis, M. Logothetis, and A. Boucouvalas, "State-dependent bandwidth sharing policies for wireless multirate loss networks", *IEEE Trans. on Wirel. Commun.*, vol. 16, no. 8, pp. 5481–5497, 2017 (DOI: 10.1109/TWC.2017.2712153).

[48] V. Iversen, S. Stepanov, and A. Kostrov, "Dimensioning of multiservice links taking account of soft blocking", in *Next Generation Teletraffic and Wired/Wireless Advanced Networking: 6th International Conference, NEW2AN 2006, St. Petersburg, Russia, May 29 – June 2, 2006*. Proceedings, Y. Koucheryavy, J. Harju, and V. B. Iversen, Eds. *LNCS*, vol. 4003, pp. 3–10, Springer, 2006 (DOI: 10.1007/11759355_3).

[49] J. Kaufman, "Blocking in a shared resource environment", *IEEE Trans. on Commun.*, vol. 29, no. 10, pp. 1474–1481, 1981 (DOI: 10.1109/TCOM.1981.1094894).

[50] J. Roberts, "A service system with heterogeneous user requirements", in *Performance of Data Communication Systems and Their Applications: Proceedings of the International Conference on Performance Data Communication System*, G. Pujolle, Ed. Amsterdam: North Holland, 1981, pp. 423–431 (ISBN: 9780444862839).

[51] M. Głąbowski and M. Stasiak, "An approximate model of the full-availability group with multi-rate traffic and a finite source population", in *Proc. 12th GI/ITG Conf. on Measur., Modell. and Eval. of Comp. and Commun. Syst. withy 3rd Polish-German Teletraff. Symp. MMB&PGTS 2004*, Dresden, Germany, 2004, pp. 195–204.

[52] Simscript III [Online]. Available: http://www.simscript.com/ (last accessed: Feb. 2020).

Irene P. Keramidi received her B.Sc. degree in Informatics and Telecommunications (2018) and the M.Sc. degree in Modern Wireless Communications (2019) from the University of Peloponnese, Tripolis, Greece. Her B.Sc. and M.Sc. theses were related to the application of queuing theory models in telecommunication systems. Currently, Keramidi is a Ph.D. student at the Department of Informatics and Telecommunications of the University of Peloponnese, where she focuses on "Vehicle-to-Everything" (V2X) communications. Her research interests include network performance evaluation and V2X systems.

https://orcid.org/0000-0003-4449-8712
E-mail: ekeramidi@uop.gr
Department of Informatics and Telecommunications
University of Peloponnese
221 00 Tripolis, Greece

Ioannis D. Moscholios – for biography, see this issue, p. 59.

Panagiotis G. Sarigiannidis, **Michael D. Logothetis** – for biographies, see this issue, p. 60.