Paper

Multicommodity Auction Model for Indivisible Network Resource Allocation

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Abstract—In this paper we present the multicommodity auction model BCBT-I that allocates indivisible network resources among bidders. The approach can be considered as a generalization of the basic multicommodity model for balancing communication bandwidth trade (BCBT). The BCBT model assumes that offers concerning inter-node links and point-to-point bandwidth demands can be realized partially. However, in the real-world trade there might be a need to include capacity modularity in the market balancing process. Thus we state the model for balancing communication bandwidth trade that takes into account the indivisibility of traded bandwidth modules. This requires to solve a mixed integer problem and increases computational complexity. Furthermore, the pricing issue appears nontrivial, as the dual prices cannot be longer used to set fair, competitive market prices. For clearing the market, we examine the multicommodity pricing mechanism based on differentiation of buy and sell market prices.

Keywords— ***bandwidth allocation, indivisible commodities, modularity, multicommodity trade, pricing.***

1. Introduction

In this paper the bandwidth trading is considered from the viewpoint of network operators, service providers and other wholesale active market players, buying and selling bandwidth. For the purpose of modeling trade of bandwidth resources in the communication networks, the network consists of nodes connected by links. The capacity of an inter-node link is the elementary commodity on the bandwidth market. However, network resources being traded can be more complex and can be composed of many links, i.e., paths, subnetworks.

It is well recognized that the base for bandwidth trading can be standardized contracts, that use prespecified amount of bandwidth [1]. This requires to take into account modularity of capacity in the trading models. Indivisibility can be associated with bandwidth sell offers concerning links or/and bandwidth buy offers concerning end-to-end network paths. Modularity requirements can be applied in trading resources of any layer of a communication network architecture (for example, optical links and synchronous digital hierarchy (SDH) containers). The size of indivisible unit of bandwidth may differ depending on the market considered or even depending on individual offers. For example, there may be portfolio of synchronous transport module (STM) contracts. Moreover, buyers may need to buy a set of different bandwidth links to establish connections. They should not be exposed to risk of buying some but not all links or risk of buying different quantities of bandwidth on different links.

Requirements of bandwidth market participants are difficult to satisfy using bilateral agreements, which are currently the most popular form of communication bandwidth trading. Other mechanisms, such as simple auctions and exchanges aim mainly in facilitating buyer-seller contacts. Thus, the efficient bandwidth trade requires development of advanced business tools [2], [3].

To cope with the problem of providing bidders with the possibility to submit offers for bundles of elementary commodities when auctioning indivisible units of bandwidth, researchers have proposed various rules and approaches. They can be assigned into two classes: simultaneous, single link auctions [2], [4]–[6]; and combinatorial auctions [7]. In simultaneous, separate auctions for individual links a user that wants to buy a certain path must put simultaneous bids at all relevant auctions. Then special, iterative mechanisms are required to coordinate individual links-auctions. This aspect, as well as possible suboptimality are the main roots of our criticisms for these methods. Combinatorial auctions based approaches may be seen as the best suited approaches for bandwidth trading. However they are proved to be NP-hard (non-deterministic polynomial-time hard) which is the main disadvantage pointed out also by other researchers. Lastly, all the over mentioned approaches require buyers to specify the particular links that constitute a desired path. This may lead to welfare inefficiency, as was shown in [8].

In this paper we state a multicommodity auction model BCBT-I that allocates indivisible network resources among bidders and provides efficient allocation of indivisible units of traded bandwidth resources. The model falls into a class of the multicommodity exchange models, that can provide efficient resources allocation by solving global economic surplus (welfare) maximization problem. The basic balancing communication bandwidth trade (BCBT) model proposed in [8] was the preliminary step in designing efficient multicommodity bandwidth exchange. The distinguishing feature of BCBT is that it allows bidders to place buy offers not for bundled links, but rather for end-to-end connections. This model is in the form of a linear programming problem in which many elementary buy and sell offers are simultaneously considered. Prices in BCBT model can be set according to values of appropriate dual prices. However, the basic BCBT model treats bandwidth as fully divisible commodity.

In the paper we provide a generalization of BCBT model, allowing us to consider capacity modularity requirements.

The issue is nontrivial, as the new model BCBT-I allows participants to declare different sizes of indivisible units of bandwidth to be traded. Moreover, the pricing issue appears as dual prices can not be longer used to set fair, competitive market prices. We examine application of multicommodity clearing mechanism based on differentiation of buy and sell competitive market prices.

The paper is organized as follows. The description of the BCBT-I model is given in Section 2. Section 3 discusses application of multicommodity balancing mechanism to fair distribution of social welfare. Section 4 presents simple examples illustrating the main features of BCBT-I model. Section 5 is the summary of the paper.

2. The BCBT-I Model

The BCBT-I model falls into a class of the multicommodity exchange models. It provides a considerable functional extension to the BCBT model [8], which treats bandwidth as fully divisible commodities. Buy and sell offers considered state capacity (appropriately demanded or supplied) and unit price (appropriately sell or buy). Realization of sell and buy offers is given by a non-negative variable less or equal to offered capacity.

Contrary, the BCBT-I model assumes that bandwidth is traded in indivisible amounts. Besides capacity and unit price, market participant may declare the size of indivisible unit of bandwidth in which the submitted offer should be realized, for example, 155.52 Mbit/s corresponding to one STM-1 contract. Such a feature may be very valuable in real trading practises, however it was not addressed by other researchers dealing with bandwidth indivisibility (see, for example, [5], [9]).

Market clearing with indivisible bandwidth requires to solve a mixed integer problem. This leads to integer variable problems, increasing computational complexity of the model comparing to simple BCBT model, as problem changes character from P to NP-hard. Thus applying BCBT-I for large networks with many market participants may require some aggregation mechanisms – an issue introduced in [10]. Below we give the statement of mixed integer formulation of BCBT-I model.

2.1. Mathematical Programming Formulation

The BCBT-I model defines three sets: network nodes (V), buy offers (D) and sell offers (E). Each buy offer $d \in D$ defines maximum capacity to be bought h_d, unit price E_d and size of indivisible unit in which bandwidth has to be purchased M_d. Each buy offer $d \in D$ concerns end-to-end path, described by the source node s_d and sink node t_d. Similarly, each sell offer $e \in E$ defines maximum capacity y_e, unit price S_e and size of indivisible unit in which bandwidth is offered for sale M_e. Sell offers concern particular links. This relationship is reflected by parameters a_{ve} defined for each pair $(v,e) \in V \times E$. Parameter a_{ve} accepts three values: 1 if a link connected with offer e originates in node v, –1 if a link connected with e terminates in node v and 0 otherwise.

It is assumed that offers can be realized in the indivisible units of bandwidth, thus x_d is the integer variable stating the number of units M_d realized for buy offer d, x_e is the integer variable stating the number of units M_e realized for sell offer e. Non-negative variable x_{ed} is continuous and denotes the bandwidth capacity allocated to sell offer e to serve buy offer d. The model BCBT-I is formulated as a mathematical linear program presented below:

$$\hat{Q} = \max\left(\sum_{d\in D} E_d M_d x_d - \sum_{e\in E} S_e M_e x_e\right), \quad (1)$$

$$0 \le M_d x_d \le h_d, \quad \forall_{d\in D}, \quad (2)$$

$$0 \le M_e x_e \le y_e, \quad \forall_{e\in E}, \quad (3)$$

$$\sum_{d\in D} x_{ed} \le M_e x_e, \quad \forall_{e\in E}, \quad (4)$$

$$0 \le x_{ed}, \quad \forall_{e\in E, d\in D}, \quad (5)$$

$$\sum_{e\in E} a_{ve} x_{ed} = \begin{cases} M_d x_d & v = s_d \\ 0 & v \neq s_d, t_d \\ -M_d x_d & v = t_d \end{cases}, \quad \forall_{v\in V, d\in D}, \quad (6)$$

$$x_d \in \mathbb{Z}, \quad \forall d \in D, \quad (7)$$

$$x_e \in \mathbb{Z}, \quad \forall e \in E. \quad (8)$$

The aim of the BCBT-I model is to maximize the economic welfare, which is the market surplus defined as a difference between buyers incomes and sellers costs – objective function (1). First and second group of constraints set upper and lower bounds on accepted volume of supply constr. (2) and demand constr. (3). Next two group of constraints ensure that total bandwidth flow at particular link will not be greater than realization of sell offer concerning this link constr. (4) and that bandwidth flow at all links will be non-negative constr. (5). Constraints (6) assert appropriate bandwidth flow for demand realization of each buy offer at each node and can be seen as an analogue to the Kirchhoff's current law. Two last groups of constraints impose indivisibility of demand (7) and supply (8) realization.

The general BCBT-I model (1)–(8) can be considered in a few versions, depending on the indivisibility requirements, that may appear only on supply or demand side. For example, if we discard (7) and set $M_d = 1$, for each $d \in D$, then obtained variant of BCBT-I model considers bandwidth indivisibility only from supply point of view, allowing for fully divisible demands realization. Of course, a symmetrical variant can be created by removing constr. (8) and assigning $M_e = 1$, for each $e \in E$. One can notice then, that BCBT-I is an extension of BCBT model, as it would result in the same statement if constrains (7) and (8) would be discarded and all parameters M_e and M_d would be set to 1.

2.2. Main Features of the Model

The BCBT-I is an effective model of bandwidth exchange. Effectiveness is here conceived in the sense of maximizing global economic surplus (market surplus). It is achieved by joint optimization of all submitted buy and sell offers. For given offers the BCBT-I chooses the best allocation of bandwidth determining volumes of accepted offers.

For profit maximizing market players, very important individual goals are the values of economic profits (surplus) they could get. Moreover, from individual player points of view, an important feature of the exchange is the "transparency" and fairness conditions of clearing, which encourage players to place sincere offers and to use truthful bidding strategies reflecting their underlying values. The basic linear BCBT model provides transparent and fair conditions of clearing, since the dual prices in the optimal solution enables setting the competitive market prices for all bandwidths resources on individual links.

In the case of BCBT-I model, the optimal solution determines the realizations of offers that provides efficiency by maximizing the global surplus. However, the MILP (mixed integer linear programming) optimization problem BCBT-I does not provide prices to distribute the surplus between market participants. Thus, a special pricing mechanism for fair economic surplus distribution should be provided. For this purpose the multicommodity balancing mechanism is applied. It is presented in the section below.

3. Multicommodity Balancing Mechanism

A good market mechanism should fulfill many different requirements. From the viewpoint of individual market player several desired properties can be claimed: maximization of individual outcome, individual rationality, impartiality, fairness and simplicity in available strategies [11]. Also from the global point of view, some features of market mechanism are strongly preferable: maximization of social surplus, enabling high competitiveness, budget-balancing, limiting market power, preventing entry deterrence and predation, incentive compatibility. Meeting all these requirements is impossible, what was already proofed in the field of mechanism design theory (Myerson-Satterthwaite impossibility theorem [12]).

In [13] a novel *generic* approach for clearing and fair social welfare distribution in general multicommodity auctions with indivisibility and non-convexities was developed. The method is based on considering two vectors of competitive market clearing sell prices and buy prices of commodities and services. The sell and buy prices are differentiated to share the (non-negative) costs of necessary compensations paid to unfairly priced participants. Sharing of the compensation costs allows the market operator to treat all market participants without discrimination and is justified by incentives that should be given to market participants to bid fairly. The aim is to offset the financial losses and to provide profit optimality to market participants so that they break even. The total compensation cost is calculated in addition to the profit (social welfare) objective function.

The balancing mechanism consists of two steps: allocation and pricing. In the first step a quantity balancing model of the multicommodity auction is solved – the BCBT-I model in our case. In the second step of a balancing mechanism, in order to obtain the best sell and buy competitive prices, the compensation cost is minimized by solving a payment problem.

3.1. Allocation and Payment Rule

Both phases of multicommodity balancing mechanism are performed consecutively. The quantitative balancing is done first. In terms of auction theory it can be treated as a provider of allocation rule that determines an optimal selection of sell and buy bids to realization. The optimal solution maximizes the social welfare and assures zero global profit-optimality loss.

The price determination model can be applied as a separate pricing step of the market clearing procedure, after allocating the resources. It allows the market operator to fairly redistribute the social welfare among market participants, by computing the best buy and sell prices that minimize the costs of necessary (non-negative) compensations. As it was mentioned before, these compensations should be paid to some market participants to avoid individual profit-optimality losses, that may occur due to non-convexities existing in the market.

Problem of setting sell and buy market prices can be formulated as a linear programming task which can be found in [13]. In this article we only present the basic concept of this mechanism.

3.2. Cost of Compensations

There are conflicts between the centralized maximum welfare goal and the profit-driven goals of independent market participants. In particular, the centrally imposed allocation may require some costly offers to be accepted, while rejecting other competitive offers, even though these would have make profit selling the bandwidth under market prices. The rationality assumption of self-interested market participants under competition is that neither buyer or seller can willingly accept a loss of profit, if such loss can be avoided, for example, by reducing consumption, or by making some links unavailable.

On some markets there may exist constraints that significantly limit the trade. In such a case rejecting all non-competitive offers and accepting all competitive offers may lead to suboptimal solutions according to global welfare criterion. In order to maximize social surplus there may be reasonable to impose acceptance (rejection) of some offers that would be rejected (accepted) in the case of market without constraints.

In the analyzed mechanism, there may happen that a market participant is forced by the market operator to buy (sell)

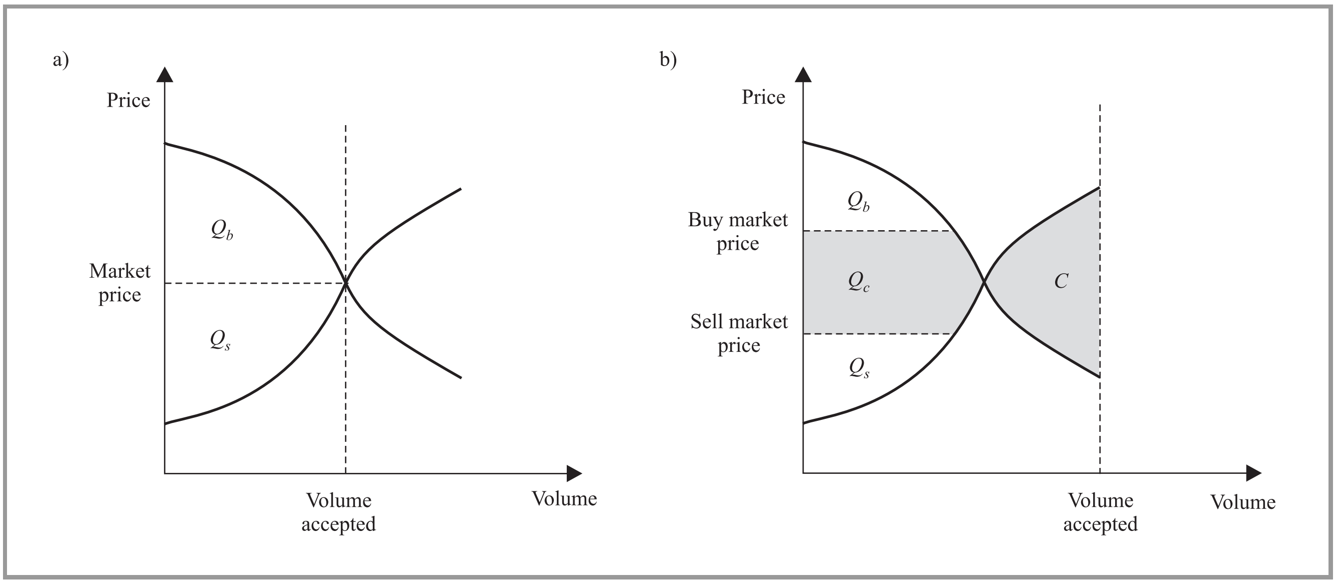

Fig. 1. Supply and demand charts in two cases: equality of sell and buy market price – zero system cost (a); differentiation of sell and buy market prices – non-zero system cost (b).

bandwidth when the market price is higher (lower) than its offered price. To offset the financial losses under market conditions, and to provide the profit optimality to assure break even, the market participant gets the compensation equal exactly to the deficit he or she would have under the market conditions.

Analogously, if a buyer (seller) is competitive with offered price greater (lower) than the market price, but, due to the market operator decisions, the allocated amount of bandwidth is less then expected, then the market participant should get compensation equal to the lost opportunity of gaining surplus.

The overmentioned (non-negative) compensations paid to some market participants to avoid individual profit-optimality losses are minimized in the optimization model, that determines also differentiated market sell and buy price. The prices are differentiated to cover the costs of compensations and to assure budget-balanced property of market mechanism.

3.3. Differentiation of Buy and Sell Market Prices

Different values of sell and buy market price determine which part of social surplus is dedicated for covering the compensations system cost. If system cost of fair global market distribution is zero, then sell and buy market price are the same, corresponding to uniform price at perfect market without any constraints. However, if system cost is non-zero, then buy market price should be greater than sell market price in order to fix appropriate part of social welfare for covering this cost.

In Fig. 1 the supply and demand chart of single commodity is presented. Two different situations are considered: zero system cost and non-zero system cost. In the first case whole social surplus is divided between buyers (Q_b) and sellers (Q_s). There is no need for differentiation of sell and buy market prices, so only one uniform price is set. In the second case system cost of compensations C is not equal zero. Therefore global welfare should be divided for three parts: buyers surplus (Q_b), sellers surplus (Q_s) and surplus dedicated for covering system cost (Q_c). In order to balance the budget, condition $Q_c = C$ must be meet. Determining appropriate size of Q_c can be achieved by differentiation of sell and buy market price of each commodity.

4. Case Study

In this section we present simple examples illustrating the main features of BCBT-I model. Let us consider network with four nodes and four links – Fig. 2. For each link

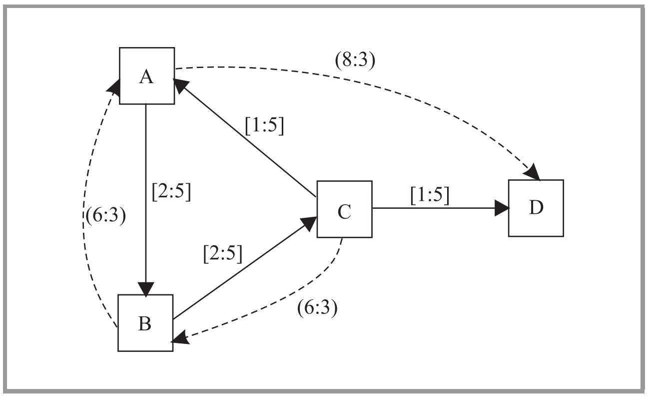

Fig. 2. Network resources with sell and buy offers.

depicted by solid arrow there is one sell offer. Parameters of sell offers are given in the square bracket. First number is an unit price and the second number is a maximum capacity. For example, sell offer with unit price 2 and maximum volume 5 is submitted for link connecting nodes A and B. There are also three buy offers, each connected with path depicted by dotted arrows. Parameters of buy offers are given in the parenthesis. First number is an offered price and the second number is a maximum capacity. For example, buy offer with unit price 8 and maximum capacity 3 is submitted for path connecting nodes A and D.

Proposed bandwidth exchange model will be considered in three variants, depending on the bandwidth indivisibility requirements. In order to solve particular variant we use the BCBT and BCBT-I, accordingly.

4.1. Fully Divisible Bandwidth

This variant considers bandwidth as fully divisible commodity. For market balancing, the BCBT model is used. Obtained solution is presented in Fig. 3. For each sell offer the unit price at which seller has sold the bandwidth and accepted volume are given in the brackets – first number denotes price and the second accepted volume. For each buy offer the unit price buyer has to pay and accepted volume are given in parenthesis – first number denotes price and the second accepted volume.

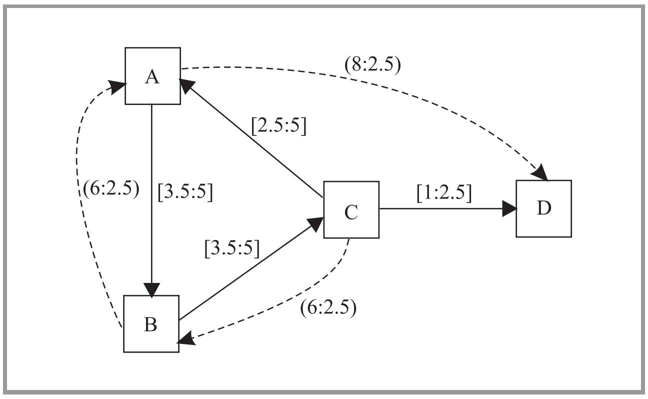

Fig. 3. Solution obtained by the BCBT model in the case of fully divisible bandwidth.

Global welfare obtained in this variant equals 22.5. Its division among market players is fair. Offer at link C-D is partially accepted, therefore price of that link equals market price. Market prices of other links are higher than appropriate offered prices because offers connected with them are fully accepted. All buy offers are partially accepted. Thus, prices that buyers have to pay equal their offered prices.

4.2. Indivisible Bandwidth

This variant considers bandwidth as commodity comprised of several indivisible units. It is assumed that all buy and sell offers are realized in the multiple of unit of size 1, hence all parameters M_e and M_d are set to the value of 1. The solution is presented in Fig. 4. Notation of results is the same as in Fig. 3. Differences between this solution and solution given in previous variant are marked by the bold fold.

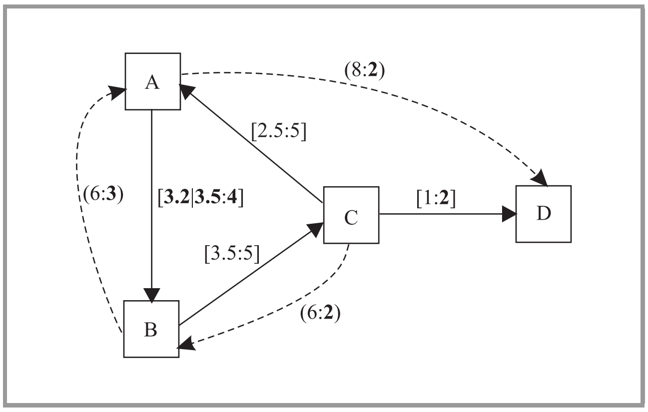

Fig. 4. Solution obtained by the BCBT-I model in the case of indivisible bandwidth.

Global welfare obtained in this variant equals 21. It is lower than in the previous variant, due to bandwidth indivisibility requirement. Market price of link A-B is differentiated: sell market price equals 3.2 and buy market price equals 3.5. Because volume of offer submitted for that link equals 4, the part of social welfare dedicated for covering system cost equals $4 \cdot (3.5 - 3.2) = 1.2$. It is used to pay compensation to owner of link A-B. Note that although the offered sell price (2) is lower than market sell price of that link (3.2), the sell offer is not fully accepted. The maximum volume is 5 while accepted volume is 4. Owner of this offer faces profit opportunity loss equals $(5-4) \cdot (3.2-2) = 1.2$. The multicommodity balancing mechanism results in maximal global welfare achieved with relatively small system cost.

4.3. Indivisible Bandwidth on the Supply Side Only

This variant assumes that the constraint of bandwidth indivisibility is required only on the supply side. All sell offers are realized in the multiple of unit of size 1, hence all parameters M_e equal 1. Modified variant of BCBT-I, without constraints (7) and with all parameters M_d set to 1, gives the solution presented in Fig. 5. Notation of results is the same as in Fig. 3. Differences between this solution and solution given by the BCBT model are marked by the bold fold.

Global welfare obtained in this variant equals 22. It is lower than in the first variant considering fully divisible bandwidth and greater than in the second variant considering indivisible bandwidth on both supply and demand side. When we compare the allocations, we can see that the accepted volume at link C-D equals 2.5 in case of the BCBT model and 3 in this case. All others values of volumes are the same in both cases. As total demand equals total supply in the case of the BCBT model, then in this case supply of bandwidth is higher than bandwidth demand. The cost of the excessive supply equals $(3-2.5)\cdot 1 = 0.5$.

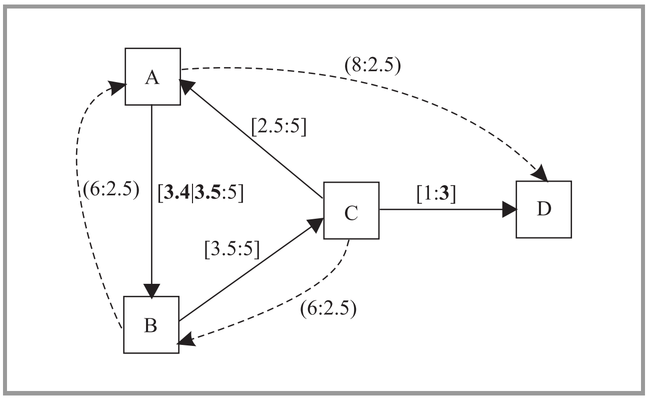

Fig. 5. Solution obtained by variant of the BCBT-I model assuming indivisible bandwidth on the supply side only.

When we compare market price of link A-B we can see that in the first variant price equals 3.5. In this variant market price of that link is differentiated: sell market price equals 3.4 and buy market price equals 3.5. Although there are no system cost of fair global welfare division, the differentiation of buy an sell market price allows to cover cost $5\cdot(3.5-3.4) = 0.5$ of superfluous supply.

5. Summary

In this paper we presented the multicommodity auction model BCBT-I for indivisible network resource allocation. It is an extension of the BTBT market model that considers bandwidth as fully divisible commodity. Constraints assuring bandwidth indivisibility causes that the BCBT-I model is formulated as a mixed-integer problem. The BCBT-I model ensures determining optimal (according to global welfare) volume of accepted offers. For fair distribution of social surplus additional mechanism, we examined the balancing mechanism based on differentiation of sell and buy market price. Illustrative examples confirm the proposed model accuracy. Further works are needed to analyze the computational complexity of the proposed model. The pricing issues introduced in this paper needs also further, comprehensive studies.

Acknowledgments

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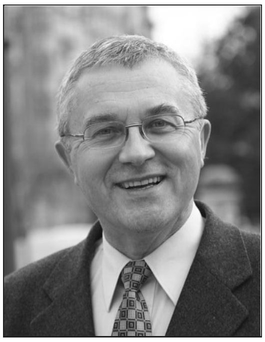

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