Joint Optimization of Sum and Difference Patterns with a Common Weight Vector Using the Genetic Algorithm

Jafar Ramadhan Mohammed[1] and Duaa Alyas Aljaf[2]

[1]*Ninevah University, Mosul, Iraq,*
[2]*Mosul University, Mosul, Iraq*

Abstract — A monopulse searching and tracking radar antenna array with a large number of radiating elements requires a simple and efficient design of the feeding network. In this paper, an effective and versatile method for jointly optimizing the sum and difference patterns using the genetic algorithm is proposed. Moreover, the array feeding network is simplified by attaching a single common weight to each of its elements. The optimal sum pattern with the desired constraints is first generated by independently optimizing amplitude weights of the array elements. The suboptimal difference pattern is then obtained by introducing a phase displacement π to half of the array elements under the condition of sharing some sided elements weights of the sum mode. The sharing percentage is controlled by the designer, such that the best performance can be met. The remaining uncommon weights of the difference mode represent the number of degrees of freedom which create a compromise difference pattern. Simulation results demonstrate the effectiveness of the proposed method in generating the optimal sum and suboptimal difference patterns characterized by independently, partially, and even fully common weight vectors.

Keywords — common weight vector, difference pattern, genetic algorithm, monopulse radar antenna, sum pattern

1. Introduction

Target searching and tracking in monopulse radar antennas requires simultaneous formation of both sum and difference patterns. The estimated angle of the target can be computed by dividing the difference pattern by the sum pattern [1]. To achieve high angular accuracy, the sum pattern must have a narrow main beam and low sidelobes. Primarily, these two factors are reversely related. Thus, there is always a tradeoff between the main beam width and the sidelobe level. Many numerical algorithms have been proposed in the literature for optimizing the excitation amplitudes and/or phases of the array elements to get the required sidelobe level under the desired beam width constraints. For example, see the approaches presented in [2]–[8].

On the other hand, the difference patterns should be also optimized to ensure low sidelobe levels in order to suppress the undesired interfering signals that could affect the angular estimate of the target's location. There are many techniques in the literature that deal with such requirements, for example [9]–[11]. To cope with the requirements of optimal sum and difference patterns, ideally separate optimizations of two independent element weight vectors are needed [12]. However, these separate optimization methods were impractical, due to the use of two separate element weight vectors for one monopulse radar antenna. Moreover, their implementations are difficult and require a complex feeding network [13]–[14]. To tackle this problem, some researchers have been investigating the use of partially common weight vectors for optimizing sum and difference patterns. Ares *et al.* in [15] used simulated annealing to synthesize Taylor and Bayliss linear distributions with a common aperture tail. The common aperture tail technique has been successfully extended by the same authors [16] to the subarray configuration to obtain an optimum compromise between sum and difference patterns. Rocca *et al.* in [17] used convex optimization to optimally synthesize sum and difference patterns with arbitrary sidelobes and common excitation constraints. Then, the technique was further developed by the same authors to include the sparse theory [18] for the purpose of minimizing the number of array elements. In [19], Chun *et al.* used convex optimization again to synthesize asymmetric sum and difference patterns with a common complex weight vector. All the authors of the above-mentioned works assume that the problem is always convex and it can be solved by linear programming under some assumptions which cannot be sometimes satisfied especially when dealing with nonlinear problems such as phase-only synthesis problem and unequally spaced arrays. In fact, these synthesized problems are non-linear and non-convex and cannot be efficiently solved the use of with convex optimization methods. Therefore, global optimization approaches such as the genetic algorithm, particle swarm optimization, and evolutionary algorithms are more preferable than convex optimization methods due to the fact that they do not require any prior assumptions about the nature of the problems [20].

In [21], Keizer used iterative Fourier transform (IFT) to generate separately optimum sum and difference patterns, implicitly assuming that array elements are uniformly spaced at half the wavelength. In [22], Mohammed adopted the IFT technique to obtain the required sum and difference patterns

with a maximum allowable sharing percentage in the element excitations.

In light of the above discussions, there is a great need to find a general solution for optimizing the sum and difference patterns without any pre-specified assumptions or limitations. Also, both array patterns should be jointly optimized and the solution should be globally optimal. In this paper, a global genetic algorithm with a specific cost function that has the ability to jointly optimize both sum and difference patterns is considered to perform the required optimization process. The synthesis problem of the sum and difference patterns is first jointly formulated under the predefined constraint goals, such as beamwidth range, sidelobe envelope, and pattern nulling. Then, a single cost function is efficiently formulated to optimize the amplitudes of the common array weight vector. The proposed algorithm provides effective user-defined control over a wide range of sharing percentages ranging from 0% (i.e. independent weight vector) up to 100% (i.e. fully common weight vector).

A major novelty of our work is that the proposed optimization method can jointly synthesize arbitrary sum and difference patterns with a common weight vector, using a generalized genetic algorithm without any assumptions concerning convexity (as in [17]–[19]) and uniformity (as in [21]).

2. Problem Statement and Proposed Solution

Consider a linear array of an even number of isotropic elements $N = 2M$, which are equally spaced by $d = \lambda/2$ and symmetrically positioned with respect to the origin, along the x-axis. Also assume the indices of the elements on both sides of the array are: $-M, -M+1, \ldots, -1$ and $1, \ldots, M-1, M$ going from left-hand side to so right-hand side elements, as shown in Fig. 1. For such an antenna system, the array factor of the sum pattern can be written as [22]:

$$AF_{\text{Sum}}(u) = \sum_{n=-M}^{-1} a_n^s e^{j\frac{2n+1}{2}kdu} + \sum_{n=1}^{M} a_n^s e^{j\frac{2n-1}{2}kdu} \quad (1)$$

and in terms of the difference pattern, the array factor can be expressed by [22]:

$$AF_{\text{Dif}}(u) = \sum_{n=-M}^{-1} a_n^d e^{j\frac{2n+1}{2}kdu} - \sum_{n=1}^{M} a_n^d e^{j\frac{2n-1}{2}kdu}, \quad (2)$$

where $k = 2\pi/\lambda$, λ is the wavelength in free space, $u = \sin\theta$, and θ is the angle with respect to the normal direction of the array axis, and a_n^s and a_n^d are two separate weight vectors for the sum and difference patterns, respectively. First, the amplitude weights of a_n^s, $n = -M, \ldots M$ and $n \neq 0$ are independently optimized using the genetic algorithm to obtain the corresponding optimal sum pattern with the desired predefined constraints regarding main beam width, sidelobe level, and null control.

To proceed with the idea of the common weight vector, let us introduce a new parameter that specifies the common
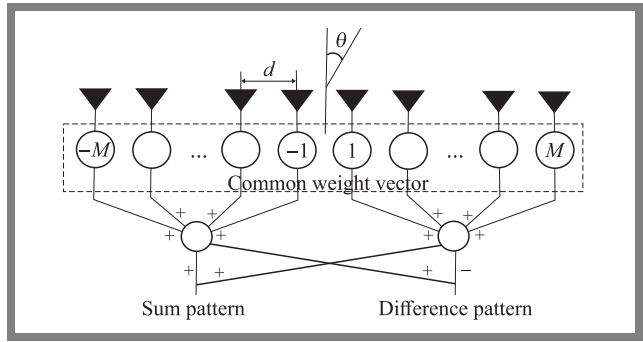

Fig. 1. Array configuration with a full common weight vector.

number of the array elements that share the same amplitude weights for the sum and difference modes, e.g. N_c. Thus, the remaining number of the uncommon array elements that will be available as degrees of freedom for optimizing the amplitudes of a_n^d to get the suboptimal difference pattern is $N - N_c$. To obtain the difference pattern, we also need to add a phase displacement of π to half of the elements of the array.

Now, depending on the chosen value of N_c, which is a user-defined parameter, three cases can be discussed. If $N_c = 0$, then two independent weight vectors for separately optimizing sum and difference patterns can be obtained. The second case involves a partially common weight vector with a certain sharing percentage that can be obtained for any value between $0 < N_c < N$. Finally, fully common weight vector between a_n^s and a_n^d can be obtained for $N_c = N$. Since the optimal amplitude weights for the sum and difference modes are usually very similar for the peripheral elements, one is inclined to start the value of N_c successively from the array ends. In this way, the searching spaces of the genetic optimizer are restricted, which helps significantly reduce the convergence speed of the optimizer and limit its run time by avoiding unnecessary random combinations of the array elements.

For symmetric amplitudes, the array factor of the difference pattern can be rewritten to include parameter N_c as follows:

$$AF_{\text{Dif}}(u) = \underbrace{\sum_{n=1}^{M-\frac{N_c}{2}} a_n^d \cos\left[\frac{2n-1}{2}kdu\right]}_{\text{Uncommon part } a_n^d \neq a_n^s} \quad (3)$$

$$+ \underbrace{\sum_{n=M-\frac{N_c}{2}+1}^{M} a_n^s \cos\left[\frac{2n-1}{2}kdu\right]}_{\text{Common part } a_n^d = a_n^s}.$$

Now, amplitudes of the two weight vectors a_n^s and a_n^d can be identified by minimizing the following cost function:

$$\text{cost} = \sum_{i=1}^{I} |AF_{\text{Sum}}(u_i) - UB_{\text{Sum}}(u_i)|^2 \quad (4)$$

$$+ \sum_{j=1}^{J} |AF_{\text{Dif}}(u_j) - UB_{\text{Dif}}(u_j)|^2,$$

Subject to:

$$|AF_{\text{Sum}}(u_o)|^2 = 1, \tag{5}$$

$$\text{if enforce symmetry then } a_n^s = a_{-n}^s, \ \ a_n^d = -a_{-n}^d \tag{6}$$

$$\text{for } n = 1, \dots M, \ \text{and}$$

$$a_n^d = a_n^s \ \ \text{for } M - N_{c/2} + 1 \leqslant n \leqslant M. \tag{7}$$

u_i and u_j represent the sampling points in the sidelobe region (i.e. exempting the main beam region) of the sum and difference patterns, respectively, i and j represent the patterns points for the sum and difference modes, I and J are the total pattern points which are both set to be equal to 512 with evenly spaced in u space, UB_{Sum} and UB_{Dif} are the upper bounds of the constraint masks including the upper sidelobe envelope of each pattern, u_o indicates the target direction in the sum pattern. Note that the first null-to-null beam widths of the sum and difference patterns are included in the constraint masks UB_{Sum} and UB_{Dif} of Eq. (4).

To obtain a simultaneous nulling capability in both sum and difference patterns while also maintaining the same sidelobe structures, the constraint masks of UB_{Sum} and UB_{Dif} in the sidelobe regions are set at the same levels of -30 dB. Moreover, the first and second term of Eq. (4) can be separated, to allow each of them to act as a single independent cost function for the design constraints given in Eqs. (5), (6), and (7).

Clearly, the cost function from Eq. (4) is the sum of the squares of the excess pattern magnitudes exceeding the specified sidelobe mask. This minimizes the excess sidelobe power of both patterns outside specified upper-bound goals. Generally, a better solution may be obtained for lower cost values and the optimization process will be considered as converged when the cost value becomes lower than a specific threshold value which is chosen here to be -40 dB [22].

According to the cost function (4) and for given sum and difference patterns, each pattern points i and j that lie outside the specified sidelobe bounds contribute a certain value to the cost function equal to the power difference between the upper bound goal and the generated patterns.

3. Simulation Results

To validate the effectiveness of the described method, a number of numerical experiments have been performed. In the following examples, the synthesis of equally spaced linear arrays composed of $N = 20$ and $M = 100$ elements is considered. For the genetic optimizer, an initial population of 50 random array weights is generated and is evolved for 10,000 generations. Then, 25 pairs of parents are chosen by means of a tournament at each iteration to produce 2 children using 2 crossovers. Thus, the number of produced children becomes 50. From the total of 100 individuals, only best 50 survive to the next generation. This process repeats until a specified number of iterations is reached [22].

Next the sum and difference patterns are generated by jointly optimizing the amplitude weights of the sum and difference modes. Since the amplitude weights are assumed to be sym-

metric with respect to the center of the array, only half of the weights need to be optimized. The amplitudes are restricted to lie between 0 and 1 and phases between $-180°$ and $180°$ (for the difference mode only).

In the first example, the synthesis of a linear array comprising $N = 2$ and $M = 20$ elements to independently generate optimum sum and difference patterns with two separate weight vectors a_n^s and a_n^d (i.e., the number of the common array elements $N_c = 0$) is considered. This case is considered as a benchmark for comparing other upcoming cases. The upper bounds of the sidelobe envelope of each pattern, i.e. UB_{Sum} and UB_{Dif} are set to -30 dB. The first null-to-null beam width (FNBW) of the sum pattern is constrained to be $u = \pm \frac{1}{N \cdot d}$, while that of the difference pattern is doubles. In addition, simultaneous two symmetric notches in the sum and difference sidelobe patterns centered at $u = \pm 0.57$ and ranging from $u = \pm 0.54$ to $u = \pm 0.6$ are placed.

The optimized sum and difference patterns along with their corresponding weights and cost functions are shown in Fig. 2. Clearly, implementation of such a feeding network system with these two separate weights (i.e. without a common weight vector) requires a large number of RF attenuators and phase shifters, equaling approx. From Fig. 2b, it can be observed that the optimum values of the excitation weights of the sum and difference modes are very similar to each other at both ends of the array. Accordingly, the amplitude weights of the side elements may be shared without any loss in system performance.

Table 1, shows the numerical results for the sharing percentage starting from 0 and reaching 100% (i.e. fully common weight vector), in incremental steps of 10% in each of the cases. For each case, performance measures related to taper efficiency, angle sensitivity K_r (1/rad), directivity, peak sidelobe level (i.e. peak sidelobe with respect to the maximum main beam), average sidelobes (i.e. area under the entire sidelobe region), and first null-to-null beam width (FNBW) are included. Further, the optimized weight vectors for the sum and difference patterns and for each considered case are presented as well. It can be observed that the greater the percentage of sharing weights, the poorer the difference side lobe pattern. Moreover, the remaining performance measures are slightly reduced as well when compared to the optimum values from the $N_c = 0\%$ scenario. The optimized sum and difference patterns for the two specific cases are highlighted in the following two examples.

In the second example, the results for the case of $N_c = 60\%$ and for a total number of array elements equal to $N = 20$ (i.e. 12 elements on both sides of the array are common for sum and difference modes) are shown in Fig. 3. In this case, the amplitude weights for the sum pattern are fixed at optimum levels (i.e. the same as in Fig. 1). The uncommon amplitude weights of the difference mode are optimized according to the cost function that was given in Eq. (4). Accordingly, little change in the sidelobe envelope of the difference pattern is noticed (Fig. 3a). However, the main beam shape and the null placement remain unchanged. Although the peak sidelobe level of the resulting difference pattern, -28 dB,
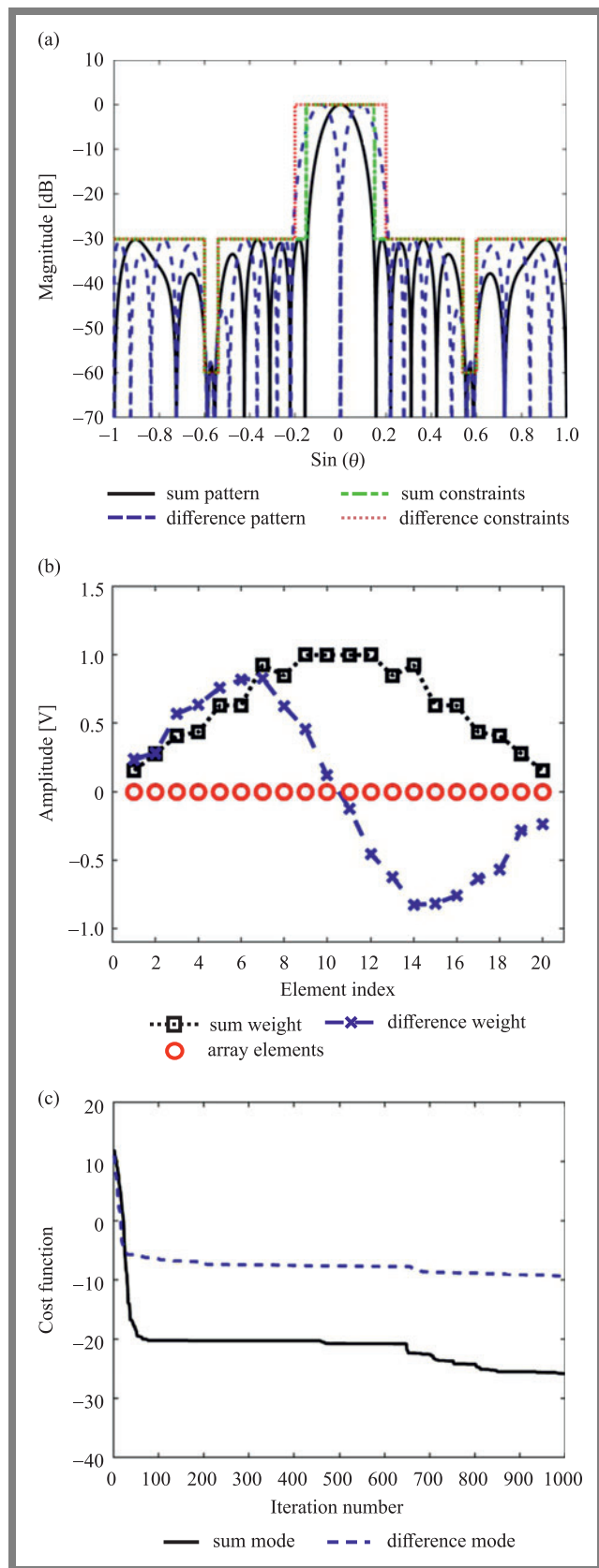
Fig. 2. Sum and difference patterns (a), their corresponding amplitude weights (b), and the cost function (c) for $N = 20$ and $N_c = 0\%$ (i.e. separate weights).
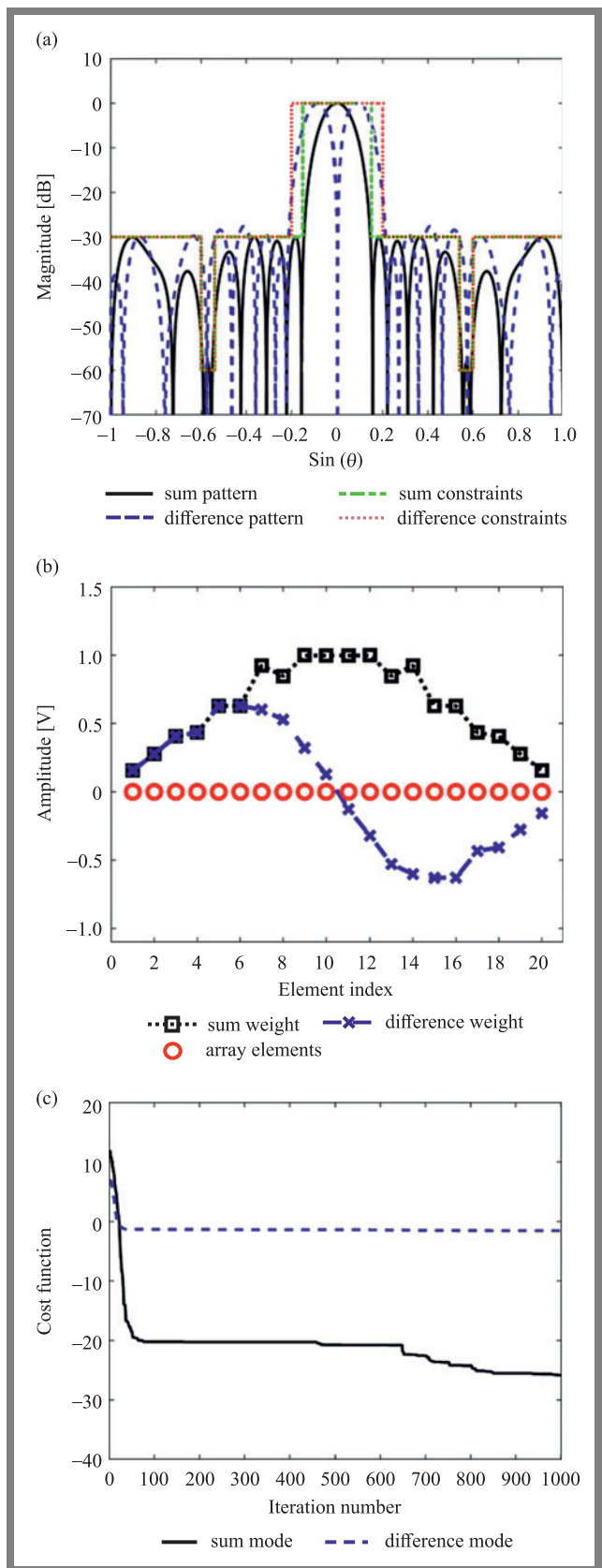

Fig. 3. Sum and difference patterns (a), its corresponding amplitude weights (b), and the cost function (c) for $N = 20$ and $N_c = 60\%$ (i.e. partially common weights).

Tab. 1. Performance of the optimized sum and difference patterns.

Performance	Optimized sum pattern	Optimized difference pattern										
		Common weight percentages N_c [%]										
		0	10	20	30	40	50	60	70	80	90	100
Taper efficiency	0.82	0.53	0.51	0.51	0.52	0.53	0.53	0.52	0.47	0.45	0.41	0.35
K_r [1/rad]		0.78	0.73	0.73	0.76	0.77	0.78	0.77	0.70	0.67	0.64	0.58
Directivity [dB]	10.25	8.41	8.24	8.21	8.35	8.40	8.41	8.37	7.92	7.74	7.47	6.94
Peak SLL [dB]	-30	-30	-30	-30	-30	-30	-29	-28	-25	-28	-22	-15
Average SLL [dB]	-35.86	-33.57	-34.37	-35.31	-35.14	-32.06	-32.50	-32.74	-27.82	-27.25	-27.50	-18.46
FNBW [deg]	0.29	0.41	0.43	0.44	0.42	0.41	0.41	0.42	0.71	0.74	0.84	0.84
Weights	$a_n^S = a_{-n}^S$	$a_n^d = -a_{-n}^d$										
	0.15	0.23	**0.15**	**0.15**	**0.15**	**0.15**	**0.15**	**0.15**	**0.15**	**0.15**	**0.15**	**0.15**
	0.27	0.28	0.24	**0.27**	**0.27**	**0.27**	**0.27**	**0.27**	**0.27**	**0.27**	**0.27**	**0.27**
	0.40	0.56	0.49	0.46	**0.40**	**0.40**	**0.40**	**0.40**	**0.40**	**0.40**	**0.40**	**0.40**
	0.43	0.63	0.54	0.68	0.64	**0.43**	**0.43**	**0.43**	**0.43**	**0.43**	**0.43**	**0.43**
	0.62	0.75	0.71	0.70	0.66	0.65	**0.62**	**0.62**	**0.62**	**0.62**	**0.62**	**0.62**
	0.62	0.81	0.70	0.86	0.87	0.57	0.56	**0.62**	**0.62**	**0.62**	**0.62**	**0.62**
	0.92	0.82	0.77	0.77	0.76	0.62	0.61	0.60	**0.92**	**0.92**	**0.92**	**0.92**
	0.84	0.62	0.59	0.70	0.66	0.48	0.46	0.52	0.70	**0.84**	**0.84**	**0.84**
	1.00	0.45	0.40	0.41	0.37	0.32	0.32	0.32	0.56	0.63	**1.00**	**1.00**
	0.99	0.12	0.13	0.21	0.18	0.11	0.11	0.12	0.17	0.24	0.15	**0.99**

Tab. 2. Comparison with other papers.

Method	Complexity reduction	Sharing percentage	Peak SLL for sum pattern	Peak SLL for difference pattern	Pre-assumption	Optimum solution
Independent weight vector method [12] and [13]	0%	0%	-30 dB	-30 dB	Two separate weight vectors	Yes
Alvarez method [15]	25%	50%	-30 dB	-23.8 dB	It uses pre-fixed Taylor and Bayliss distributions	Yes
Morabito and Rocca method [17]	30%	60%	-28 dB	-24 dB	The problem should be convex	Not always
Mohammed method [22]	30%	60%	-24 dB	-15.32 dB	Uses FFT and the inter-element spacing is uniform	No
Proposed	30%	60%	-30 dB	-28 dB	It doesn't need any assumption	Yes

is higher than, the prescribed mask limit of -30 dB, the average sidelobes of the resulting difference pattern amount to -32.74 dB, i.e. are lower than the mask limit. Moreover, complexity of the feeding network is reduced by more than half with respect to the first case (i.e. $N_c = 0\%$) with separate weights. In addition, the cost functions of this case were found to be satisfactory (Fig. 3c).

In the third example, the results for the case of $N_c = 100\%$ and for a total number of array elements equal to $N = 20$ (i.e. fully common weight vectors) are shown in Fig. 4. In this case, all the weights of the difference mode are enforced to be the same as those of the sum mode with a phase shift

equal to π. For such a specific case, there was a sudden change in the amplitude weights of the difference mode at the central elements of the array. This sudden change causes relatively high sidelobes in the difference pattern (Fig. 4a) and an unsatisfactory cost function (Fig. 4c).

Next, a larger array composed of $N = 100$ elements is considered. $N_c = 60\%$ and the levels of the UB_{Sum} and UB_{Dif} are set at the same levels as in the previous examples. The results are shown in Fig. 5 and match the observations from Fig. 3. This proves the generality of the proposed idea.

In the last example, the proposed method is compared with other published papers in terms of complexity reduction and
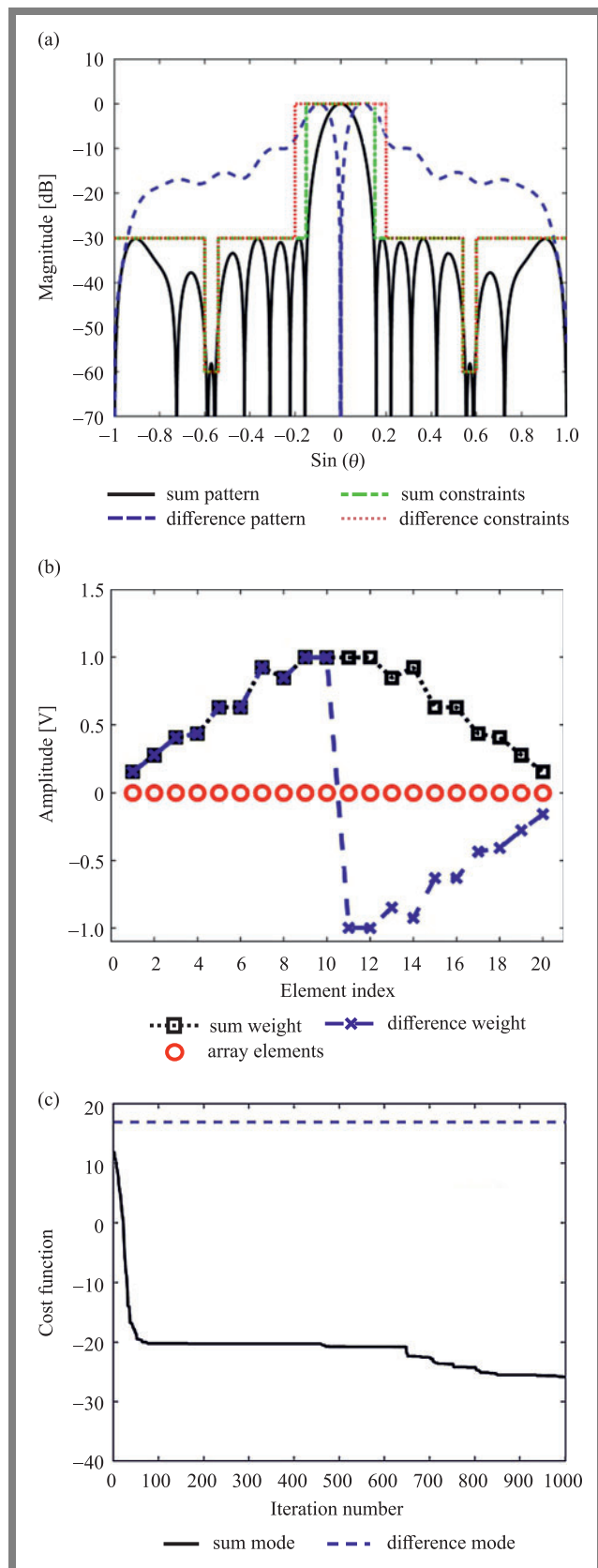
Fig. 4. Sum and difference patterns (a), its corresponding amplitude weights (b) and the cost function (c) for $N = 20$ and $N_c = 100\%$.

peak sidelobe level in the obtained difference pattern. The comparison is shown in Table 2.
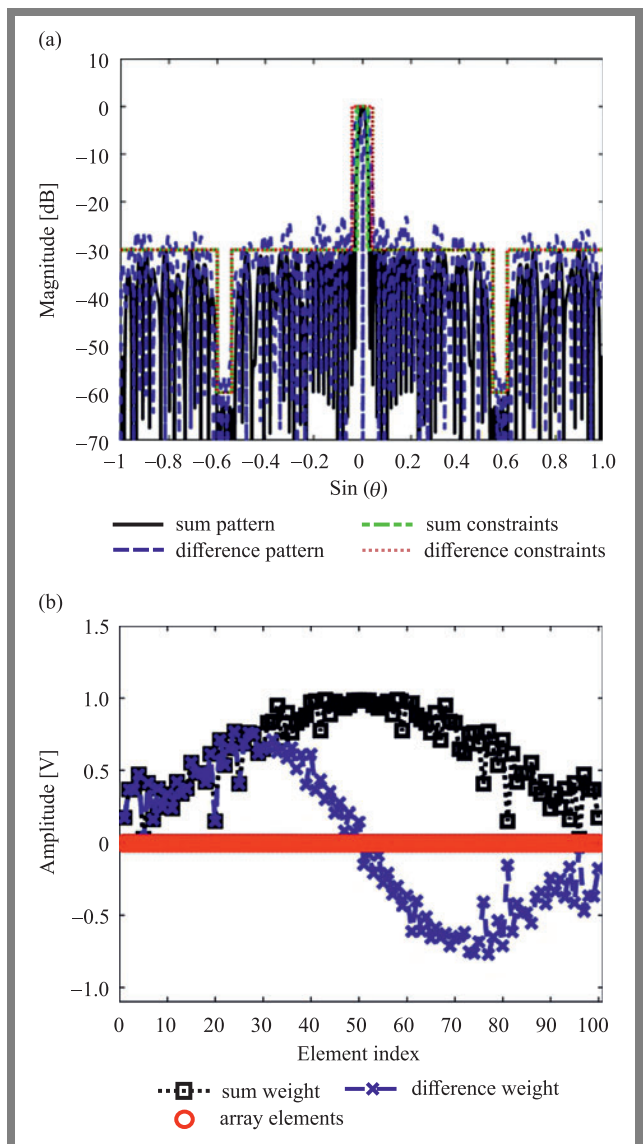

Fig. 5. Sum and difference patterns (a), and its corresponding amplitude weights (b) for $N = 100$ and $N_c = 60$.

4. Conclusions

In large tracking radar antenna arrays, complexity of the feeding networks is a major challenge. Thus, it is highly desired to simplify the feeding network as much as possible while generating the required sum and difference patterns. In a fully common weight vector case, where a single common attenuator and phase shifter is attached to each element for both sum and difference patterns, a significant reduction in the feeding network's complexity may be obtained by efficiently adjusting its amplitude and phase. However, this advantage comes at the cost of higher sidelobe levels in the difference pattern.

The problem of the high sidelobe level in the difference pattern was solved by using a partially common weight vector instead of its full counterpart and the complexity was found to be acceptable. It is found from the simulation that the sidelobe level of the difference pattern was reduced from -15 dB to more than -28 dB when switching from the fully

common weight vector to the partial one, with a sharing percentage of 60%. Also, it is found that the performance metrics of the difference pattern in terms of taper efficiency, angle sensitivity, directivity, average sidelobe, and beam width were reduced with an increase in the sharing percentage. The partially common weight vector of up to 60% was found to be an excellent choice for practical implementations.

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Jafar Ramadhan Mohammed received his B.Sc. and M.Sc. degrees in Electronics and Communication Engineering from Mosul University, Iraq in 1998, and 2001, respectively, and a Ph.D. degree in Digital Communication Engineering from Panjab University, India in 2009. He is currently a Professor and Vice Chancellor for Scientific Affairs at Ninevah University, Mosul. His main research interests are in the area of digital signal processing and its applications, antennas and adaptive arrays.
E-mail: jafar.mohammed@uoninevah.edu.iq
Ninevah University, Mosul, Iraq

Duaa Alyas Aljaf received her B.Sc. degree in Electrical Engineering from the College of Engineering, Mosul University, in 2016 and M.Sc. in Communication Engineering from the College of Engineering, Al-Nahrain University in 2019. Currently, she works at the Autonomous Control Department at the Mosul Dam. She is currently pursuing her Ph.D. at the College of Engineering, Mosul University. Her research interests include antenna arrays, array pattern optimization for 5G systems and beyond.
E-mail: doaa.engeneering@gmail.com
Mosul University, Mosul, Iraq