Paper

Multi-operator Differential Evolution with MOEA/D for Solving Multi-objective Optimization Problems

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Abstract — In this paper, we propose a multi-operator differential evolution variant that incorporates three diverse mutation strategies in MOEA/D. Instead of exploiting the local region, the proposed approach continues to search for optimal solutions in the entire objective space. It explicitly maintains diversity of the population by relying on the benefit of clustering. To promote convergence, the solutions close to the ideal position, in the objective space are given preference in the evolutionary process. The core idea is to ensure diversity of the population by applying multiple mutation schemes and a faster convergence rate, giving preference to solutions based on their proximity to the ideal position in the MOEA/D paradigm. The performance of the proposed algorithm is evaluated by two popular test suites. The experimental results demonstrate that the proposed approach outperforms other MOEA/D algorithms.

Keywords — differential evolution, multi-objective, mutation-operators, weighted-aggregation

1. Introduction

Multi-objective evolutionary algorithms (MOEAs) are applied for decoding various multi-objective optimization problems (MOP) [1]–[3]. To develop an effective and efficient MOEA, one cannot overlook some serious concerns such as the selection of solution for the offspring in order to evolve the population. Another concern is related to how diversity of the population may be maintained while choosing the solutions for the successive generations. And finally, it is very hard to balance the diversification-intensification relationship in MOP, since the objectives might be conflicting in nature. Depending upon the selection criteria for new solutions, MOEAs are broadly classified into three categories: Pareto-dominance-based MOEAs [4]–[7], performance indicator-based approaches [8]–[11], and decomposition-based algorithms [12]–[15]. However, a general approach is to transform the MOP into multiple single-objective problems, i.e. to transform a decision-space into an objective space for developing MOP frameworks.

In recent years, the decomposition-based MOEA technique (MOEA/D) has gained attention for solving MOP [16]. The popular examples are MOEA/D-DE [17] and MOEA/D-CMA [18], utilizing the single-search mutation operator of differential evolution (DE) to converge the entire population towards the Pareto front. Likewise, MOEA/D with a distance update strategy (MOEA/D-DU) [19] motivates researchers to measure the distance between the value of weighted-aggregation function and its corresponding vector in MOEA/D. Despite their valuable results, the aforementioned frameworks suffer from the following disadvantages:

– the solutions are selected either randomly or from the local region. In MOEA/D-DE, the parent vector is either chosen from the neighbor or randomly from the entire population. This type of selection is likely to mislead the search process and confine it to a certain area of the Pareto front;
– similarly, in MOEA/D-CMA, few solutions are mutated through CMA-ES [20]–[22] and most of them are expected to converge through DE. This study may enhance diversity of the population, but lacks in faster convergence towards the Pareto front;
– in the existing studies, offspring is generated by means of conventional approaches (either by DE or GA [19]). These are not capable of producing reliable results for all the sub-problems and, hence, may be stuck in the local minima.

To cope with this, we propose a multi-operator based differential evolution with MOEA/D (MOEA/D-MODE) that alleviates, to certain extent, the shortcomings in the area of diversity-preservation and convergence rate.

This paper relies on the clustering-based MOEA/D that has the advantages of multiple-mutation strategies of the established evolutionary approach (DE), to ensure the equilibrium between exploration and exploitation. Furthermore, the difficulty to pull the solutions to the Pareto front is taken care of to some extent as well. Our contribution is described below:

– a novel multi-operator DE variant (MOEA/D-MODE) is proposed for ensuring a better trade-off between diversity and convergence in the MOEA/D multi-objective optimizer. To implement this idea, three diverse mutation strategies of DE are employed;
– clustering-based evolution is emphasized which can explicitly facilitate better diversification. The clusters are of varying sizes and each cluster is operated with a distinct mutation operator;
– contemporary ideas are combined in order to select the solution vector for the generating mutant solution;

– to ensure maximum diversity, we have incorporated polynomial mutation followed by standard crossover techniques to yield novel solutions in the sub-population. Then, we compared the proposed algorithm with three existing solutions: MOEA/D-CMA, MOEA/D-DE, and MOEA/D-DU, and also discussed potential reasons behind the failure of these methods proposed in MOP.

The remaining sections of the paper are organized as follows. Section 2 illustrates the fundamentals of MOP and is followed by a presentation of the related work in Section 3. In Section 4, the crucial components of MOEA/D-MODE are discussed. In Section 5 comprehensive implementation of the proposed algorithm with the aim to solve MOP is presented. Section 6 describes the experimental studies in terms of benchmark functions, parameter settings, and evaluation metrics for comparison purposes. In Section 7, performance of the improved MOEA/D-MODE algorithm is analyzed (with respect to two aspects) and statistical results comparing the solution with, other algorithms are verified. Finally, the paper is concluded in Section 8.

2. Background

Any MOP can be defined as:

$$\text{minimize: } F(x) = f_1(x), \ldots, f_m(x) \quad \text{subject to : } x \in \Omega, \quad (1)$$

where Ω represents the decision (solution) space of a d-dimensional vector $x = (x_1, x_2, \ldots, x_d)$, and $F : \Omega \rightarrow \mathbb{R}^m$ contains m continuous objective values in the $\mathbb{R}^m$ objective space. Moreover, if Ω is a connected and closed region in the objective space $\mathbb{R}^m$ and the corresponding objective solutions are continuous of x, Eq. (1) is referred to as a continuous MOP.

Let $u = (u_1, \ldots, u_m)$ and $v = (v_1, \ldots, v_m) \in \mathbb{R}^m$ be two solutions, u dominates v if and only if:

$$u_i \leqslant v_i, \text{ for all } i = 1, \ldots, m,$$
$$u_i < v_i, \text{ for any } i \in 1, \ldots, m.$$

Solution $x^* \in \Omega$ is said to be a Pareto optimal solution if there does not exist $x \in \Omega$ such that $f(x)$ dominates $f(x^*)$. All the reliable Pareto solutions together form a set, known as a Pareto set (PS):

$$PS = \{x \in \Omega | x \text{ is Pareto optimal}\}. \quad (2)$$

The set of all the Pareto objective vectors, known as the Pareto front (PF), is given as:

$$PF = \{f(x) \in \mathbb{R}^* m, x \in PS\}. \quad (3)$$

For a given MOP, the ideal solution z^* is the best solution vector $z^* = z_1^*, z_2^*, \ldots, z_m^*$, where z_i^* represents the best solution (here the minimum value) of f_i, for every $i = 1, 2, \ldots, m$.

The prime objective of any MOP technique is to guide the population of worthwhile solutions toward the PF, ensuring convergence and, simultaneously, maximum distribution over the PF for diversity related purposes.

3. Related Works

Three categories of MOEAs may by used in MOP. So, this section is devoted to discussing the literature based on the aforementioned categories. In the majority of literature focusing on MOEAs assistance of the Pareto dominance is relied upon [4]–[7], [23], [24]. In these studies, the effectiveness of a solution is measured by the Pareto dominance relations with the remaining solutions encountered in the last search. It is an iterative process that runs for each individual element in the objective-space. Since the dominance feature alone could hamper the diversity of the solutions, some alternatives may be combined in MOEAs, such as crowding and fitness sharing [24], [25]. One of the most popular Pareto-dominance MOAE schemes is NSGA-II [6]. The crucial characteristic of NSGA-II is its rapid nondominated sorting to rank the solutions for further selection.

Indicator-driven MOEAs are another category, as they endeavor to optimize performance metrics as an indicator [8], [9]. They ensure the desired ordering sequence of the optimal sets that will be used to approximate the Pareto front. The most widely adopted performance indicator is hypervolume (HV), which possess significant theoretical characteristics. In the literature, we have few canonical performance indicator-based MOEAs [8], [9] that disguise HV as the selection factor. One of the suggestions is to rank the solutions yielded by the HV indicator rather than estimating their exact values [9]. Another alternative strategy is to find other indicators that are computationally less expensive and offer fair theoretical characteristics, e.g. Λ_p [28]. Such an approach has been embraced in a few MOEAs.

The category of decomposition-based MOEAs exploits the aggregation function in which the objectives of a MOP are aggregated using randomly distributed weight-vectors. This set of weight-vectors will eventually create multiple weighted-aggregation functions, each of them representing a single-objective problem. Diversity of the population is maintained by ensuring fair distribution of the weight-vectors in the objective space. MOEA based on the decomposition (MOEA/D) [16] is a scheme that is most widely adopted in the domain of multi-objective optimization. New frameworks based on MOEA/D and relevant to the study performed in this paper are reviewed in the following subsections.

3.1. MOEA/D with DE

The general practice in MOEA/D is to decompose PF approximations of a problem (1) into several scalar-optimization functions. Li and Zhang in [17] extended the work by implementing DE and polynomial mutation for maintaining the diversity of the population in MOEA/D. In such an approach, three parent solutions are selected having a low probability of $1 - \delta$. In such a way, a wide range of offspring could be produced and, thus, the exploration capability was enhanced. Furthermore, there is a restriction on replacing the maximum number of solutions with a new child solution. Instead of relying upon the neighborhood of size T, parameter

n_r is introduced. It limits the size of the solution-vector to be replaced by the new offspring.

The differences between MOEA/D-MODE and MOEA/D-DE can be summarized in the following manner:

– in MOEA/D-DE, a single mutation strategy is incorporated that utilizes three parent solutions only. The standard DE technique is used to produce new offspring. Multi-operator DE often outperforms single mutation DE in the case of single-objective problems. However, to enhance the search capabilities of MOP, multiple mutation strategies are ensembled in MOEA/D-MODE;

– the extra measure taken in MOEA/D-MODE is the implementation of the crossover technique after the polynomial mutation. The crossover technique is useful for exploiting regions formed by mutant vectors. This allows to strengthen the trade-off between exploration and exploitation.

3.2. MOEA/D with CMA-ES

Working on MOEA/D frameworks, Li *et al.* [18] introduced the covariance matrix adaptation evolution strategy (CMA-ES) into MOP in order to balance CMA-ES and DE efficiently. CMA-ES is an evolutionary approach which allows to generate novel solutions using the Gaussian distribution model. To lower the cost of computation, the problem domain is organized into a group of sub-problems where only one sub-problem is optimized through CMA-ES and others are evolved by applying the DE approach. The best solutions optimized by CMA-ES are always carried forward in the distribution mean update. This leads to faster convergence.

The differences between MOEA/D-MODE and MOEA/D-CMA are such that MOEA/D-CMA involves clustering of sub-problems, with only a few of them being optimized by the Gaussian distribution model of CMA-ES. It seems the algorithm is more focused on DE, as the majority of sub-problems are evolved by means of the DE mutation strategy. Unlike MOEA/D-CMA, MOEA/D-MODE allows different mutation strategies to be applied in the clusters of the sub-problems, thus maintaining diversity and working in a single flow.

3.3. MOEA/D with Distance Update Strategy

Another MOEA variant based on the decomposition technique, as proposed by Yuan *et al.* in [19], uses the aggregation function to speed up the convergence in multiple-objective optimization. As the number of objectives increases exponentially, it becomes difficult to maintain diversity and to approach the PF uniformly. To cope with this challenge, researchers have performed extensive analyses on the aggregation functions by estimating the perpendicular distance from the weight-vector of the solution in the high-dimension objective space. The performance of such an approach in the case of a 2-objective optimization problem, (and with more than 2 objectives) has been analyzed as well. The differences between our approach and MOEA/D with the distance update strategy include the following:

– in MOEA/D-MODE, the worst neighbor is used as the solution according to its distance from the weight-vector and the best solution according to the better aggregation function value corresponding to the sub-population;

– DE generally offers better results compared with the genetic operators in the case of single-optimization problem. The Cr parameter sets the number of new solutions to be exploited. With the low value of Cr, a wide range of child solutions will by covered, while a high value of Cr is focused on the parent vector only. Due to the above-mentioned reasons, DE search operators are incorporated in MOEA/D-MODE to solve MOP.

4. Pivotal Components of MOEA/D-MODE

The single-mutation strategy is incorporated into decomposition-based multi-objective optimization for population evolution-related purposes. In the proposed algorithm, we adopt a novel approach involving multiple-mutation operators. Each of them is applied uniquely to evolve the sub-populations, leading to stronger exploration and better convergence. As illustrated in Fig. 1, the dashed lines represent the contour of the sub-problems decomposed by Eq. (5). The clusters are organized based on the weight vectors $\lambda_1, \lambda_2, \ldots, \lambda_8$. In each generation, one solution at a time is taken from the cluster and the assigned mutation strategy is applied.

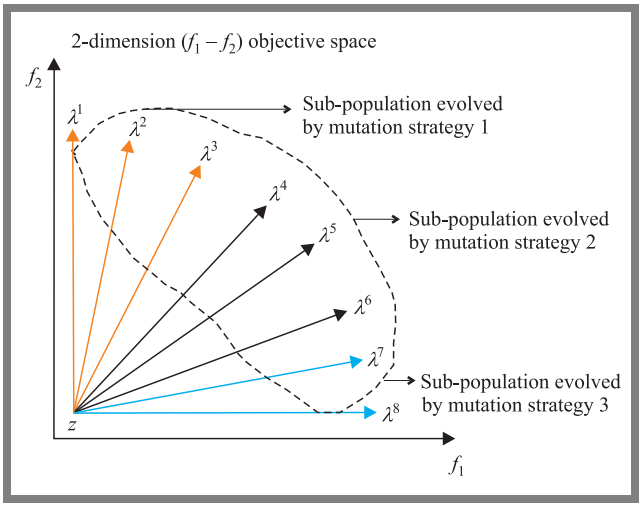

Fig. 1. Illustration on the clusters of the sub-problems decomposed by Eq. (5). Each cluster is assigned a unique mutation operator for generating novel solutions. Here, there are three clusters: $1 = \{1, 2, 3\}$, $2 = \{4, 5, 6\}$ and $3 = \{7, 8\}$.

4.1. Neighbor Selection and Clustering

The common practice in decomposition-based MOEAs is to transform the MOP into many single-objective problems, with each objective being a weighted combination of different objectives. This is achieved by initializing the weight-vectors in the objective-space.

Let $\lambda_i = (\lambda_{i,1}, \lambda_{i,2}, \ldots, \lambda_{i,m})^T$, for $i = 1, 2, \ldots, N$, be uniformly distributed weight-vectors for N solutions, such that $\sum_{j=1}^{m} \lambda_{i,j} = 1$. Under such an assumption, the neighbors of each unique solution are identified according to their similarity. This is achieved by computing the $N \times N$ Euclidean distance metric:

$$\text{dist}(u, v) = \sqrt{\sum_{i=1}^{m} (\lambda_{u_i} - \lambda_{v_i})^2}, \tag{4}$$

where $\text{dist}(u, v)$ represents the Euclidean distance between two solutions u and v. The closer the distance, the higher the neighborhood relationship. Therefore, in MOEA/D-MODE, we construct a best-neighbors vector B of size T for further processing such that $B = \{x_1, x_2, \ldots, x_T\}$.

To achieve maximum diversity even in the later stages of the population, the objective is to initially disintegrate the entire population and cluster the solutions based on the assigned weight vectors. All sub-populations have different sizes. In conjunction, multiple-mutation techniques of DE have been applied that ensure better coverage of the search space. This practice is likely to explicitly maintain the diversity of solution during evolution of the population.

To accomplish the task k-means clustering [29] is applied with $k = 3$, since three diverse mutant operators are considered in this algorithm to process three sub-populations.

4.2. Parent Selection and Offspring Generation

Another major concern is the selection of parent solutions for offspring generation. It is important that the selection criteria be driven not only by the distant vector λ_i but also by proximity to the ideal position in the objective space, i.e. using the aggregation-function value $G_i(x)$ given in Eq. (5). Such an approach is driven by the likelihood that, a solution which is inferior in terms of the λ_j may contribute to a better $G_i(x)$ value.

Therefore, in this paper, we consider the weighted-aggregation function value $G_i(x)$ which underlines the best solution in the sub-population, while selecting the parent solutions for the respective mutation operators.

In the proposed algorithm, three diverse mutation operators are applied to turn on the novel solutions. Additionally, polynomial mutation and crossover techniques have been incorporated that are rarely applied in existing MOEA/D variants. The crossover techniques employ either a binomial crossover or an exponential crossover for the new solution u.

4.3. Updating Solutions in the Sub-population

The most common scheme for using aggregation functions in updating neighbors of the solution is the Tchebycheff function [30]. In this function, the scalar optimization sub-problem is given by:

$$G_i(x) = \max_{1 \leqslant i \leqslant m} \{\lambda_i | f_i(x) - z_i^*|\} \quad \text{subject to } x \in \Omega, \tag{5}$$

where m denotes the number of objectives, λ is a uniformly distributed weight vector across each objective, and z_i^* represents best the solution found so far for each objective i.

The problem of converging the entire solution-set towards PF is remodeled into N scalar sub-problems requiring optimization. Eventually, the spread of the final solutions could be evenly distributed if $G(x)$ and λ are appropriately determined. Once the new offspring u is achieved, the solutions in the sub-population get updated if:

$$G(x) > G(u), \tag{6}$$

where x denotes the solution-vector in the cluster (i.e. sub-population). Otherwise, the same parent solution will be carried forward to the next generation. Table 1 summarizes the concepts exploited in the proposed MOEA/D-MODE algorithm.

Tab. 1. MOEA/D-MODE concept.

No.	Stages	Technique used in MOEA/D-MODE
1	Selection of neighbors for each solution	Led by the distant vector λ
2	Solution clustering	3-means clustering based on factor λ
3	Parent solutions are selected for offspring	According to the best $G_i(x)$ Eq. (5) in the sub-population
4	Offspring generation	Three diverse mutation strategies have been incorporated
5	Maximal diversity	Enhanced by polynomial mutation and crossover techniques
6	Update solutions in the sub-population	Using Eq. (6)

5. MOEA/D-MODE Algorithm

This section focuses on the mathematical model of multi-operator DE for solving MOP. Additionally, we describe our approach consisting in exploiting three diverse mutation operators along with a polynomial mutation and standard crossover techniques.

Initially, the population is initialized randomly with N number of candidate solutions as:

$$x_{i,j} = x_{i,j}^{\text{lower}} + (x_{i,j}^{\text{upper}} - x_{i,j}^{\text{lower}}) \times \text{rand} \quad i \in N \text{ and } j = 1, 2, \ldots, D, \tag{7}$$

where `rand` is a function that generates random numbers between $[0 \ldots 1]$ [31]. The terms lower and upper represent lower and upper boundaries of variable x in the D-dimension.

After generating the sub-populations, solutions in the sub-populations are evolved via multi-mutation operators, where each sub-population is assigned a unique mutation operator. This allows to maintain diversity of the internal population.

Algorithm 1. MOEA/D-MODE

Parameter initialization:
MAX_{FES}, N, K, T, $FES \leftarrow 0$
Controlling parameters initialization:
F, p_m, η
Weight vectors Λ:
initialize a set of weight vectors $\Lambda = \{\lambda_1, \lambda_2, \ldots, \lambda_N\}$
Population initialization:
random population X of size N as $\{x_1, x_2, \ldots, x_N\}$
instantiate a ideal point $z^* = z_1^*, z_2^*, \ldots, z_m^*$
T-neighbors initialization:
for $i \leftarrow 1$ to N **do**
  $B(i) \leftarrow \{i_1, i_2, \ldots, i_T\}$
end
Clustering:
$C \leftarrow k$-means(Λ, K)
while $FES \leqslant MAX_{FES}$ **do**
  for $s \leftarrow 1$ to N **do**
    $P \leftarrow B(s)$
    if $s \in$ any C **then**
      Generate mutant vector from three defined mutation operators as:

$$\overline{y}_s = \begin{cases} Eq.\ (8) & \text{if } s \in C(1) \\ Eq.\ (9) & \text{if } s \in C(2) \\ Eq.\ (10) & \text{if } s \in C(3) \end{cases}$$

      $y_s \leftarrow$ PolynomialMutation$(x^s, \overline{y}_s)$
      $u_s \leftarrow$ Crossover(x^s, y_s)
      $z^* = \min(z^*, z(u_s)^*)$
      UpdateSubPopulation(u_s, z^*, C)
    end
  end
  $FES \leftarrow FES + N$
end

For each unique solution x^s chosen from the respective sub-population, a mutant vector $\overline{y}_s$ is generated as follows:

– sub-population 1: DE/parent-to-worst/1

$$\overline{y}_s = x^s + F \times (x_{p1} - x^s + x_{r1} - x_{\text{worst}}), \tag{8}$$

– sub-population 2: DE/parent-to-worst/1

$$\overline{y}_s = x^s + F \times (x_{p2} - x^s + x_{r2} - x_{\text{worst}}), \tag{9}$$

– sub-population 3: DE/weighted-rand-to-worst/1

$$\overline{y}_s = x^s + F \times (x_{r3} + x_{p3} - x_{\text{worst}}). \tag{10}$$

x^s denotes the target vector, x_{p1}, x_{p2}, and x_{p3} are 40%, 16% and 25% of the best solutions chosen from sub-populations 1, 2, and 3, respectively. Additionally, the topmost solutions are extracted from the respective clusters and marked as x_{r1}, x_{r2}, and x_{r3}, respectively. In the propounded multi-operator DE for MOP, the objective is to filter the solutions that cannot be converged to PF and, hence, maintain the maximum distance from the Pareto optimal solutions. This is implemented as $x_{\text{worst}} \in B$ which is the worst neighbor of x^s since their distance $\lambda_s - \lambda_{x_{\text{worst}}}$ differs significantly.

The three mutation strategies presented above have their own advantages, such as:

– sub-population-based evolution is used where each of them holds a variable number of solutions. This practice is likely to explicitly maintain population diversity throughout the evolution;
– the solutions that achieve significantly close proximity to the ideal position are exploited to improve the selection procedures not only from the neighborhood, but that paves the way for the maximum space coverage;
– each mutant operator tries to maintain the maximum distance from the solution that seems less promising at the time. Hence, it brings all the solutions close to the PF.

Polynomial mutation is adopted widely in evolutionary approaches in order to allow variation in the solutions. The above mutation strategies are followed by polynomial mutation in which y is generated from $\overline{y}$ in the following manner:

$$y_k = \begin{cases} \overline{y_k} + \sigma_k \times (\text{upper}_k - \text{lower}_k) & p_m \\ \overline{y_k} & 1 - p_m \end{cases}, \tag{11}$$

where

$$\sigma_k = \begin{cases} (2 \times \text{rand})^{\frac{1}{\eta+1}} - 1 & \text{if rand} < 0.5 \\ 1 - (2 - 2 \times \text{rand})^{\frac{1}{\eta+1}} & \text{otherwise} \end{cases}. \tag{12}$$

The `rand` function produces a random number between $[0 \ldots 1]$. There are two controlling parameters: p_m which defines the expectation of the number of mutated variables and η representing the distribution index of the polynomial mutation. The terms upper_k and lower_k are the upper and lower boundaries of the k-th decision variable of solution s, respectively.

In order to find Pareto optimal solutions, a crossover technique is employed. In this approach, maximum exploitation could be maintained along with the evolution of new solutions u yielded from y. Either binomial or exponential crossover is applied according to:

$$u_k = \begin{cases} \text{if rand} < 0.4 \\ \quad x_k^s \\ \text{otherwise} \\ \quad \begin{cases} y_k & \text{for } k = \langle l \rangle_D, \langle l+1 \rangle_D, \ldots, \langle l+L-1 \rangle_D, \\ x_k^s & \text{for rest of } k \in [1, D] \end{cases} \end{cases} \tag{13}$$

where $\langle\rangle$ is a modulo operator in the exponential crossover.

After evolving the solutions in a sub-population, the ideal position is changed. Therefore, we get a new $z^* = \min[z(x)^*, z(u)^*]$. Subsequently, $G(u)$ is computed as:

$$G_i(u) = \max_{1 \leqslant i \leqslant m} \{\lambda_i | f_i(u) - z_i^*|\}. \tag{14}$$

Once the weighted function $G(u)$ has been obtained, next generation solutions are decided. To ensure the better solutions, the solutions in the sub-population are updated as:

$$x_k = \begin{cases} u_k & \text{if } G(x) > G(u) \\ x_k & \text{otherwise} \end{cases}. \tag{15}$$

where x is the target solution in the sub-population, u depicts a new solution corresponding to x, for each component $k \in 1, \ldots, D$. Similarly, the entire mechanism is implemented for the solutions in the remaining sub-populations. Algorithm 2 shows the sub-population updating criteria in the propounded variant of DE for multi-objective optimization.

Algorithm 2. UpdateSubPopulation(u_s, z^*, C)

Compute $G(x_C)$ according to Eq. (5)
Compute $G(u_s)$ according to Eq. (14)
if $G(x_C) > G(u_s)$ **then**
  Update the solutions x of cluster C where $s \in C$
end

6. Experimental Setup

The implementation of MOEA/D-MODE is executed with Matlab R using the PlatEMO framework [32]. Its performance is evaluated with the use of two test suites, with respect to three well-known decomposition-based MOEAs frameworks for solving MOP: MOEA/D-CMA, MOEA/D-DE, and MOEA/D-DU.

First, MOP benchmark functions are tethered with the bias difficulties as well as BT1-BT9 instances [18] included. For BT1 to BT8, there are two objectives, whereas BT9 alone is a many-objective problem defined with the use of three objectives.

In the second step, the behavior of MOEA/D-MODE on the ZDT series [33] is evaluated. Such a method is conceived purely for two-objective test problems. However, ZDT5 is excluded from the experimental study, since it involves binary computations. Both of the test suites having diverse function problems of dimension $D \in \{10, 30\}$ and objectives $M \in \{2, 3\}$.

The control parameters and other relevant data of proposed algorithm MOEA/D-MODE are provided in Table 2. The other common parameters are:

- **number of runs and MAX$_{FES}$.** MOEA/D-MODE and the remaining competing algorithms participating in the comparison are run 30 times, independently in each of the test suites. The termination criterion for all the algorithms is set to 10,000 for all test problems;
- **weight-vector Λ.** Weight-vector $\Lambda = \{\lambda_1, \ldots, \lambda_N\}$ is a set of uniformly distributed random values and has the size of $N \times M$, where N shows the population size and M denotes the total number of objectives;
- **population size N.** To promote a fair comparison, the MOEA/D-MODE framework, and other algorithms assume the population size to be 100 for each test problem;
- **neighborhood size T.** In the proposed MOEA/D-MODE framework, and in other algorithms (MOEA/D-CMA, MOEA/D-DE, and MOEA/D-DU), T is initialized to 10;
- **mutation parameters (p_m and η).** All respective algorithms rely on polynomial mutations for introducing new solutions. Mutation probability p_m is set to 1 with a large distribution index (with its value equaling 20) is used for mutation η.

Tab. 2. Parameters settings of MOEA/D-MODE.

Parameter	Symbol	Value
Maximum function evaluations	MAX$_{FES}$	10,000
Population size	N	100
Neighbors size	T	10
Number of clusters	C	3
Scaling factor	F	0.5
Crossover probability	Cr	1
Expectation of the mutated variables	p_m	1
Distribution index	η	20

Some algorithms are characterized by particular parameter settings. In MOEA/D-DE and MOEA/D-DU, δ is the probability of selecting parents from local regions, and is set to 0.9. n_r is used by MOEA/D-DE to determine the maximum number of solutions replaced by the offspring. The value chosen is 2. On the other hand, parameter K holds different meanings in MOEA/D-DU and MOEA/D-CMA respectively. In MOEA/D-DU, K denotes the number of the nearest weight vectors, whereas in MOEA/D-CMA, K represents the number of groups. Both algorithms assume that this value equals 5.

6.1. Evaluation Metrics

Inverted generational distance (IGD) [34] is used as a performance evaluation metric. IGD is a metric that is widely adopted in the multi-objective domain and it allows to obtain collective information on the convergence and distribution of solutions. In the objective-space, we need a significant number of uniformly distributed variables that converge to PF in order to efficiently estimate IGD.

Along with IGD, we incorporate another well-known metric, namely hyper-volume (HV) [11], as the predominant comparison factor. HV is crucially cooperative to PF, and its encouraging theoretical characteristics turn it into a fair metric [35]. It can represent both convergence and distribution of the solutions. The larger the HV value, the better the level fo quality.

Selection of the reference point is the main concern encountered while computing HV. In this paper, following the recommendation from [36] and [37], we assumed the reference point to be $1.1z^{\text{nad}}$, where z^{nad} is analytically computed against each function instance. Besides, according to the setup used in [38] and [39], the solutions that do not converge to the reference point are ignored for HV computation.

To understand the difference for statistical significance of function instances, we performed the Wilcoxon Rank-Sum test [40] with normal approximation, tie-breaking, and with the significance level set to 1%. It was performed on the HV metric scores yielded by algorithms other than the proposed solution.

Tab. 3. Comparison of algorithms based on HV results, for an average and standard deviation (in brackets). The best results are highlighted in bold print.

Function	M	D	MOEA/D-MODE	MOEA/D-CMA	MOEA/D-DE	MOEA/D-DU
BT1	2	30	**0** (**0**)	**0** (**0**)	**0** (**0**)	**0** (**0**)
BT2	2	30	**0** (**0**)	**0** (**0**)	**0** (**0**)	**0** (**0**)
BT3	2	30	**0** (**0**)	**0** (**0**)	**0** (**0**)	**0** (**0**)
BT4	2	30	**0** (**0**)	**0** (**0**)	**0** (**0**)	**0** (**0**)
BT5	2	30	**0** (**0**)	**0** (**0**)	**0** (**0**)	**0** (**0**)
BT6	2	30	**0.121** (**0.040**)	0 (0)	0 (0)	0 (0)
BT7	2	30	**0.092** (**7.25** $\times$ **10**$^{-3}$)	0 (0)	0.013 (0.027)	6.659 10^{-3} (0.015)
BT8	2	30	**0.096** (**0.021**)	0 (0)	0 (0)	0 (0)
BT9	3	30	**0** (**0**)	**0** (**0**)	**0** (**0**)	**0** (**0**)
ZDT1	2	30	**0.532** (**0.062**)	0.516 (0.03)	0.26 (0.075)	0.264 (0.103)
ZDT2	2	30	**0.238** (**0.075**)	0.224 (0.034)	0.017 (0.06)	0 (0)
ZDT3	2	30	**0.605** (**0.127**)	0.423 (0.057)	0.288 (0.091)	0.462 (0.052)
ZDT4	2	10	**0.38** (**0.106**)	0 (0)	0 (0)	0 (0)
ZDT6	2	10	0.271 (0.033)	**0.371** (**0.048**)	3.595 (0.053)	0 (0)

7. Result Analysis

First the convergence and distribution of MOEA/D-MODE solutions obtained with the use of the two test suites, i.e. BT and ZDT, are analyzed (Table 3). The set of non-dominated solutions found by the proposed algorithm in 30 independent runs is depicted in Fig. 2 and 3. Based on these illustrations, the following observations may be made.

In the BT test suite, BT6-BT8, Fig. 2f–h, shows the convergence of the solutions to the PF across the objective space. Only a few of the candidate solutions try to reach the PF. This indicates that the embedded mutation strategy requires a greater ability to deal with the variations in MOP.

From BT1-BT8 (Fig. 2a–g), one may conclude that the solution set is distributed in the objective space, but does not converge to the optimal PF. This may be due to the early termination of the algorithm. Further iterations are needed for the evolution, so that it may converge very well, since the optimization problem involves tough biases.

The result of the only function problem based on 3-objectives is depicted in Fig. 2i. It illustrates the distribution of the solutions along the PF but the results shown are not encouraging. It seems that the normal population size, taken for MOP, e.g. 100, is not suitable for a problem that involves more than 2-objectives.

As far as the analysis of the ZDT series (Fig. 3) is concerned, the proposed algorithm shows far better results. It is clearly seen that the solution-set becomes converged to the PF (Fig. 3a–e). MOEA/D-MODE shows a better convergence rate in ZDT3 (Fig. 3c). However, there is still some room for improvement in the convergence rate in order to optimize different classes of problems.

7.1. Statistical Analysis

Table 4 shows a statistical comparison between MOEA/D-MODE and of other algorithms. Table 5, in turn, contains the IGD results. W^+ stands for the number of test instances in the case of which MOEA/D-MODE is significantly superior. $W^=$ means there are no significant differences between the obtained scores, and W^- is the number of instances for which existing solutions perform significantly better than MOEA/D-MODE.

The comments concerning MOEA/D-MODE and covering all 14 test instances are as follows:
In the BT test suite, MOEA/D-MODE shows a certain advantage over MOEA/D variants, i.e. MOEA/DE, MOEA/D-CMA and MOEA/D-DU. In the majority of test problems, MOEA/D-MODE achieves results that are comparable with those of the three remaining algorithms. However, it also shows an improvement in three function instances that are overlooked by the other alternatives.

When comparing results for the ZDT test series, one may clearly observe that MOEA/D-MODE remains competitive in the majority of test instances. It has shown that the multi-

Tab. 4. Summary of statistical results on HV metrics between MOEA/D-MODE and the rival algorithms.

Test suite	Algorithm	W^+	$W^=$	W^-
BT	MOEA/D-CMA	3	6	0
	MOEA/D-DE	3	6	0
	MOEA/D-DU	3	6	0
ZDT	MOEA/D-CMA	4	0	1
	MOEA/D-DE	4	0	1
	MOEA/D-DU	5	0	0

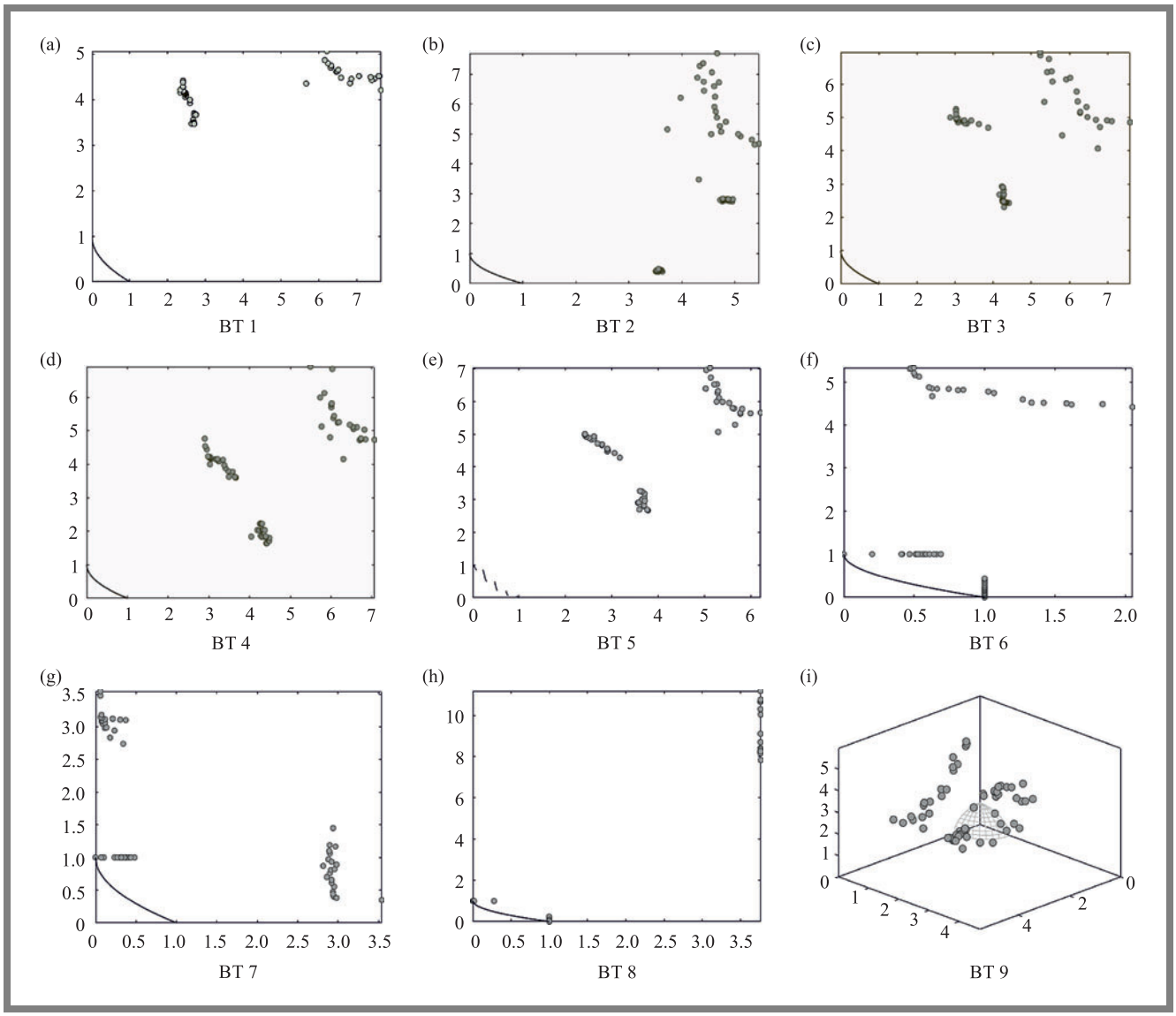

Fig. 2. Pareto front of BT-test suite. The axes are the objective values for BT1-BT8 test problems that are defined based on 2-objectives. Since BT9 is a 3-objective problem, the Pareto front has a 3-dimensional geometry. The solid curve represents the Pareto optimal front whereas the solid points depict the regions estimated by MOEA/D-MODE.

operator procedure in MOEA/D-MODE is superior or equivalent to state-of-the-art MOEA/D methods.

The proposed MOEA/D-MODE is specifically competitive when compared with two MOEA/D variants, i.e. MOEA/D-CMA and MOEA/D-DE. Test results verify that the crucial components of MOEA/D-MODE, i.e. multi-operator DE and parent selection schemes, facilitate reliable results to a greater extent than in other DE variants. However, the proposed algorithm has some room for improvement in handling functions with bias difficulties.

7.2. Further Discussion

The first concern is why the existing algorithms, i.e. MOEA/D-DE and MOEA/D-CMA are outperformed by MOEA/D-MODE. In fact, they fail to exhibit performance that would be on par with the proposed MOEA/D variant. We suspect two potential reasons. Firstly, both state-of-the-art methods overly, emphasize the weight vectors that may be confined by only one solution or particular region. So it is likely to mislead from the corresponding area of PF and fail to preserve diversity. Secondly, normal parent selection criteria are applied. The procedures are biased towards preferring solutions from the local area in order to produce offspring. It is more likely that other regions in the objective-space may by overlooked. On the other hand, MOEA/D-MODE achieves better results in terms of selecting those solutions that have a fair aggregation score, but may be far from the weight vector. This has been even experimentally verified by using multiple mutation strategies during the evolutionary task.

The second concern is why MOEA/D-MODE fails to be better than the other solution when dealing with 3-objective optimization. Population size may be one of the critical reasons here. In the analysis, a normal population size of 100 is

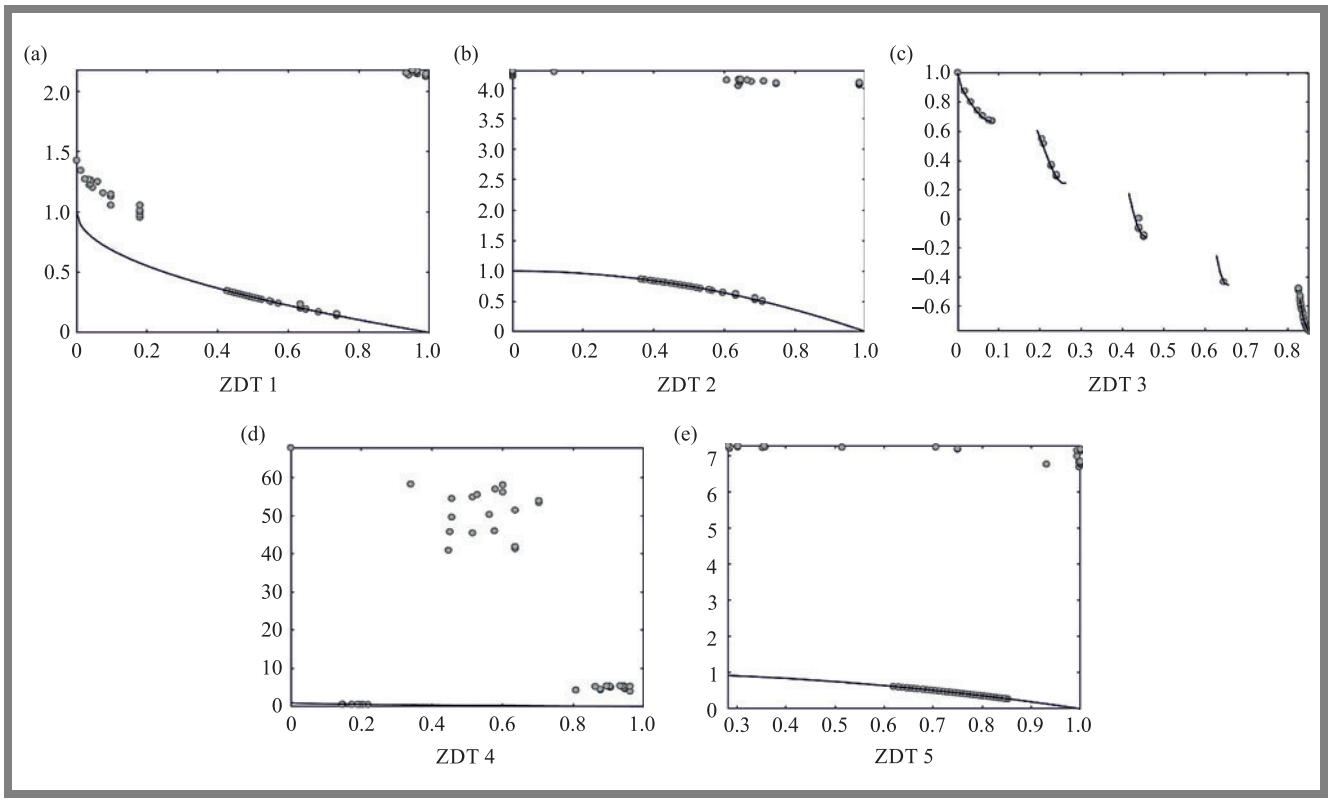

Fig. 3. Pareto front of ZDT-test suite. The axes are the objective values for test problems confined to a 2-dimensional space. The solid curve represents the Pareto optimal front whereas the solid points depict the regions estimated by MOEA/D-MODE.

Tab. 5. Summary of average IGD results and standard deviation (in brackets) compared between the MOEA/D-MODE algorithm and other algorithms. The best results are highlighted in bold print.

Function	M	D	MOEA/D-MODE	MOEA/D-CMA	MOEA/D-DE	MOEA/D-DU
BT1	2	30	3.955 (0.158)	**3.851** (**0.023**)	3.894 (0.052)	3.996 (0.128)
BT2	2	30	2.262 (0.603)	1.61 (0.05)	1.73 (0.127)	**1.405** (**0.097**)
BT3	2	30	**3.844** (**0.342**)	3.939 (0.064)	3.957 (0.091)	3.947 (0.136)
BT4	2	30	3.913 (0.225)	3.796 (0.08)	3.845 (0.108)	**3.715** (**0.137**)
BT5	2	30	3.929 (0.15)	**3.87** (**0.042**)	3.929 (0.069)	3.97 (0.13)
BT6	2	30	**0.676** (**0.219**)	2.341 (0.372)	1.844 (0.174)	2.041 (0.386)
BT7	2	30	**0.819** (**0.056**)	1.555 (0.262)	1.023 (0.24)	1.323 (0.464)
BT8	2	30	**0.81** (**0.115**)	5.254 (0.457)	4.32 (0.378)	3.834 (0.413)
BT9	3	30	3.711 (0.292)	**3.085** (**0.074**)	3.43 (0.162)	3.206 (0.074)
ZDT1	2	30	0.226 (0.112)	**0.152** (**0.024**)	0.419 (0.088)	0.405 (0.133)
ZDT2	2	30	0.274 (0.128)	**0.173** (**0.035**)	0.697 (0.181)	1.087 (0.172)
ZDT3	2	30	**0.194** (**0.119**)	0.299 (0.055)	0.459 (0.097)	0.219 (0.048)
ZDT4	2	10	**0.513** (**0.196**)	6.035 (1.94)	3.941 (1.33)	42.143 (13.0)
ZDT6	2	10	0.161 (0.071)	**0.023** (**0.056**)	0.029 (0.052)	3.7 (0.739)

used to converge the solutions to the PF. Perceptively, more solutions are required to bring the entire population to the PF in a higher-dimensional space. A smaller population size distributes the solutions sparsely in a high-order objective space. Thus, the sparse solutions fail to capture some areas from the entire PF, and this leads to a slow population convergence rate.

Poor performance of MOEA/D-MODE in ensuring faster convergence in the case of biased optimization problems, i.e. in the BT test suite, is the third concern. Despite its encouraging results concerning the evaluation of metrics (HV and IGD), it fails to show any superiority in terms of the convergence rate in BT test functions. We suspect that an

early termination of the algorithm is the reason here. As biases may cause large-scale changes objective vectors, the search operators need to remain strong. To achieve this, MAX_{FES} must be greater than 10,000, so that enough time is ensured for better exploitation of the regions. Apart from this, normal function problems, such as ZDT, have shown successful convergence with the standard procedures, as shown in Fig. 3.

8. Conclusion and Future Work

In this paper, a MOEA/D-MODE algorithm is proposed for solving multi-objective optimization problems and for improving the exploration-exploitation equilibrium. The concept is to put forth a multi-operator DE variant with complicated MOEA/D that ensures the distribution of the solutions throughout the evolutionary process. Specifically, in MOEA/D-MODE, the entire population is divided into multiple sub-populations, which are thereafter evolved by the assigned mutant operators of DE. In MOEA/D-MODE, we argue that the solution involves in the preference with respect to the proximity to the ideal position in the objective-space could improve the optimal results rather than relying upon the weight-vectors only.

We have analyzed the influence of multiple operators on the quality of MOEA/D-MODE, and several discussions have been conducted. We have shown that MOEA/D-MODE outperforms MOEA/D alternatives in terms of maintaining the convergence rate and distribution of solutions while solving MOP. Well-known test suites (BT and ZDT) with a total of 14 function instances have been employed to evaluate the algorithm's superiority. The results show that multiple mutation may achieve unprecedented results when coupled with MOEA/D.

In the future, we would extend our work to the high-dimension objective space. It would be interesting to address the problem of multiple-objective optimization with the concern of multi-operator evolutionary approach. We also would like to improve the outcomes of studies concerned with optimization problems involving bias difficulties.

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