Paper

Design Low Complexity SCMA Codebook Using Arnold's Cat Map

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Abstract — In 5G wireless communications, sparse code multiple access (SCMA) – a multi-dimensional codebook based on a specific category of the non-orthogonal multiple access (NOMA) technique - enables many users to share non-orthogonal resource components with a low level of detection complexity. The multi-dimensional SCMA (MD-SCMA) codebook design presented in this study is based on the constellation rotation and interleaving method. Initially, a subset of the lattice Z^2 is used to form the mother constellation's initial dimension. The first dimension is then rotated to produce other dimensions. Additionally, interleaving is employed for even dimensions to enhance fading channel performance. Arnold's chaotic cat map is proposed as the interleaving method to reduce computational complexity. Performance of the SCMA codebook based on interleaving is evaluated by comparing it with selected codebooks for SCMA multiplexing. The metrics used for performance evaluation purposes include bit error rate (BER), peak to average power ratio (PAPR), and minimum Euclidean distance (MED), as well as complexity. The results demonstrate that the suggested codebook with chaotic interleaving offers performance that is equivalent to that of the conventional codebook based on interleaving. It is characterized by lower MED and higher BER compared to computer-generated and 16-star QAM codebook design approaches, but its complexity is lower than that of the conventional codebook based on interleaving.

Keywords — chaotic interleaving, codebook design, dimension rotation, Euclidean distance, sparse code multiple access

1. Introduction

Non-orthogonal multiple access (NOMA) is a technology that may potentially help achieve high spectral efficiency and massive connectivity in 5G systems [1]. In a NOMA system, two or more users are placed on top of a single physical resource (such as power, frequency, time, or code) to offer an overloading ratio that is greater than one [2]. Unlike conventional orthogonal multiple access technologies, NOMA is capable of supporting numerous users. Power-domain multiplexing and code-domain multiplexing are the two categories into existing, dominant NOMA schemes may be divided. The equivalent schemes are power-domain NOMA and multiple access with low-density spreading [1]. NOMA is based on its ability to support many users with a limited quantity of resources. Frequency-division multiple access (FDMA), time-division multiple access (TDMA) for 2G, code-division multiple access (CDMA) for 3G, and orthogonal frequency-division multiple access (OFDMA) for 4G are all orthogonal multiple access (OMA) techniques that are most commonly used in contemporary wireless communication systems [3]. NOMA supports high connectivity rates, as it allows more people to communicate simultaneously than other OMA methods. Multiple users can share the same frequency resources by utilizing the near-far effect and by relying on cutting-edge uplink NOMA techniques [2], [3].

A non-orthogonal technique known as sparse code multiple access (SCMA) is principally based on the multi-dimensional (MD) codebook. Different bitstreams are directly mapped by SCMA to various sparse codewords, as illustrated in Fig. 1, where each user has a predefined codebook (there are 6 users). The placements of zeros in various codebooks are distinct to help users avoid colliding, and all codewords in the same codebook have zeros in the same two dimensions. Two bits are assigned to a complicated codeword for each user. Passwords of all users are multiplexed over four communal orthogonal resources [4].

2. Related Work

An effective codebook for the SCMA system has been widely researched in recent years. Cai *et al.* [5] developed a codebook for the optimization of the MD constellation using rotation and interleaving. A lattice that was originally intended for creating multi-dimensions for codebooks is rotated in the process. Interleaving is additionally used to improve communication over a fading channel. Performance showed that such a method offers only minor improvements compared to low-density signature (LDS). It achieves 10^{-4} BER at 15 dB SNR for a fading channel with reduced PAPR, while for uplink resource allocation it achieves 10^{-4} BER at 21 dB. In paper [8], Bonilla uses a similar approach as [6], [7], but in this case it relies on a minimum Euclidean distance (MED)-based detector at the receiver, with interleaving and phase rotation. It achieves 10^{-4} BER for codebooks with 4 arrays, 8 arrays, and 16 arrays with E_b/N_o 5 dB, 6 dB, and 6.5 dB, respectively. In [9], Liu recommended segmentation and combining for MED enhancement based on the factor graph. He showed that irregular LDS are quite similar to the recommended approach at higher SNR, emphasizing the significance of an irregular factor graph of LDS. It achieves 10^{-4} BER at 10 dB SNR over a fading channel with LDPC coding. Yu [10] suggested a star-QAM-based codebook design to improve MED. Four constellation point vectors that represent the elements of

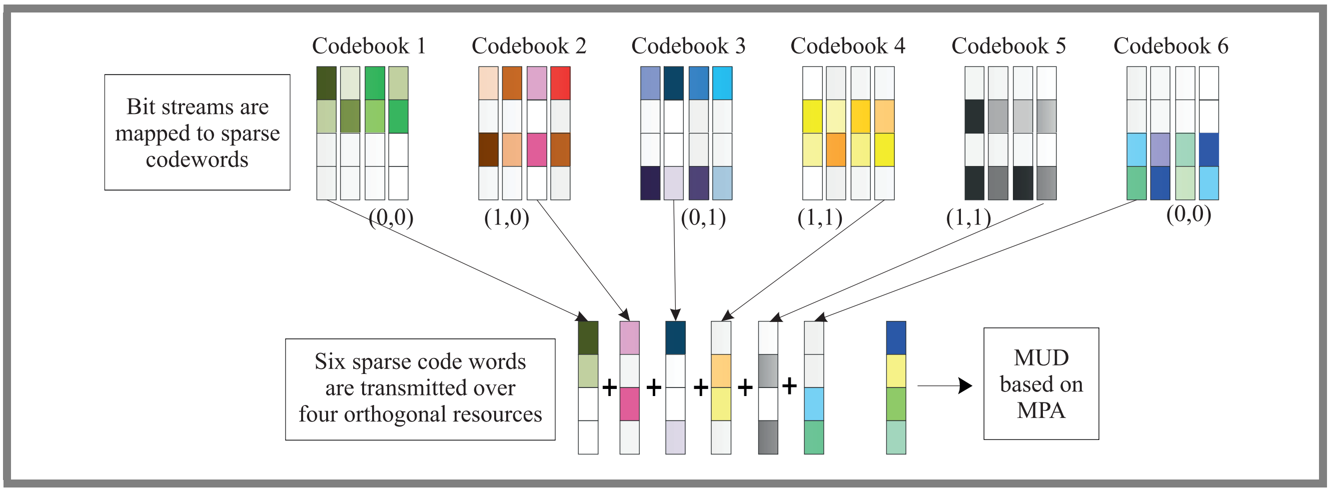

Fig. 1. Encoding and multiplexing in SCMA.

an n-dimensional star constellation is created as part of the process.

Yasmine *et al.* [11] utilized trellis coded modulation (TCM) to improve MED values and star-QAM constellations. Using this technique, they were able to obtain 10^{-4} BER at 8–10 dB SNR across an AWGN channel. In article [12], codebooks are designed using golden angle modulation (GAM) constellations which provide outstanding error rate performance in both uplink and downlink Rayleigh fading channels. Paper [13] illustrates development of a low complexity codebooks algorithm for almost optimum codebooks. In [14], researchers suggested to use partial Gaussian approximation, threshold-based edge selection and Gaussian approximation (ESGA) to significantly reduce receiver complexity. In paper [15], the rotation angle research zone is reduced as much as possible using a posteriori analysis and a second-order rational polynomial is then used to find a nearly optimal angle using the Levenberg-Marquardt algorithm. Such an approach is useful when fitting non-linear least squares curves. Compared to an exhaustive search, this technique is substantially simpler.

The main contributions of this work can be summarized as follows:

- In contrast to existing interleaving techniques, this work uses chaotic interleaving relying on Arnold's cat map;
- A detailed SCMA codebook design with 8 users using the proposed interleaving method is presented;
- An interleaving formula alternative to the one presented in [7] is derived;
- A formula for complexity reduction using the proposed method is given.

The structure of this paper is as follows. Section 2 describes the SCMA codebook design steps using the interleaving approach. Section 3 explains the proposed method and presents an illustrative example codebook design. Section 4 covers the simulation results, while Section 5 presents conclusions and future outlooks.

3. SCMA Codebook Design Using Interleaving

The SCMA system's codebook is a crucial factor for allowing the users to be multiplexed over the same resource. Therefore, it is essential in determining how users encode a bitstream using the codebook structure. The following summarizes the encoding procedure which results in the creation of the codebook. Mother constellations (MC) are produced using gray mapping coding vectors, constellation rotation and interleaving design. Then, in order to form a multidimensional base constellation, such a structure is rotated. The base constellation is rotated once more to obtain a collection of codebooks for further users, after the entire structure has been obtained for one user. This process is performed in the following manner [7]:

Creation of the mother constellation

1) Let S_1 be a subset of lattice Z^2 defined as:

$$S_1 = \{Qm(1+i)|Qm = 2m-1-M, \; m = 1,\dots,M\}, \quad (1)$$

where Z is the set of integers and M is the modulation order.

2) Set vector S_1 from Eq. (1) based on gray mapping and assuming $M = 4$. Then the S_1 vector which is the starting point for constructing the MC is represented as:

$$S_{11} = -3 - 3\text{i}, \qquad S_{12} = -1 - 1\text{i},$$
$$S_{13} = 1 + 1\text{i}, \qquad S_{14} = 3 + 3\text{i}.$$

3) S_N is generated from the preceding step and defined as $S_N = S_1 U_N$, where U_N is a multidimensional phase rotation matrix:

$$U_N = \text{diag}(1\text{e}^{1\theta_{l-1}}) \subset C^{MN}. \quad (2)$$

4) θ_{l-1} is the factor that rotates vector S_1 to achieve the multi-dimension in the MC, the base vector is S_1, and N stands for dimensions after rotating the vector. The rotating factor is defined as:

$$\theta_{l-1} = (l-1)\frac{\pi}{MN}, \quad l = 1,\dots,N. \quad (3)$$

Setting $N = 3$ and $M = 4$ allows the angle of rotation phases in the $\mathbf{MC}$ to be calculated as:

For $l = 1$, we have $\theta_{l-1} = \theta_0 = (1-1) \times \frac{\pi}{(4)(3)} = 0$.

For $l = 2$, we have $\theta_{l-1} = \theta_0 = (2-1) \times \frac{\pi}{(4)(3)} = \frac{\pi}{12}$.

For $l = 3$, we have $\theta_{l-1} = \theta_0 = (3-1) \times \frac{\pi}{(4)(3)} = \frac{\pi}{6}$.

For $\theta_0 = 0$:

$$S_{11} = -3 - 3\mathrm{i}, \quad S_{12} = -1 - 1\mathrm{i}, \\ S_{13} = 1 + 1\mathrm{i}, \quad S_{14} = 3 + 3\mathrm{i}.$$

When $\theta_1 = \frac{\pi}{12}$, we have:

$$S_{21} = -2.1213 - 3.6742\mathrm{i}, \quad S_{22} = -0.7071 - 1.2247\mathrm{i}, \\ S_{23} = 0.7071 + 1.2247\mathrm{i}, \quad S_{24} = 2.1213 + 3.6742\mathrm{i}.$$

When $\theta_2 = \frac{\pi}{6}$, we have:

$$S_{31} = -1.0981 - 4.0981\mathrm{i}, \quad S_{32} = -0.3660 - 1.3660\mathrm{i}, \\ S_{33} = 0.3660 + 1.3660\mathrm{i}, \quad S_{34} = 1.0981 + 4.0981\mathrm{i}.$$

5) The N-dimensional matrix $\mathbf{MC}$ with M points can be expressed by rotating the S_N vector as:

$$\mathbf{MC} = (S_1, S_2, \ldots, S_N)^{\mathrm{T}} = \begin{bmatrix} S_{11} & S_{12} & \cdots & S_{1N} \\ \vdots & \ddots & \vdots & \vdots \\ S_{N1} & \cdots & \cdots & S_{MN} \end{bmatrix}.$$

6) To enhance the performance of the codewords in the presence of fading, the vector components must be interleaved and rearranged as follows [5], [7]:

After the S vectors have been obtained, the interleaving of the even-numbered S vectors is completed (S_2 is symbolized by S_e). For interleaving, only even dimensions (rows) of the $\mathbf{MC}$ are reordered. For example, after interleaving, the S_e vector is S_e^*, where e is an even number in N and is:

$$S_e^* = \Big\{ -S_e, \frac{M}{2+1}, \ldots, -S_e, \frac{3M}{4}, S_e, \frac{3M}{4+1}, \ldots, -S_e, \\ M, S_e, M, \ldots, -S_e, \frac{3M}{4}, S_e, \frac{3M}{4}, \ldots, S_e, \frac{M}{2+1} \Big\}, \quad (4)$$

or in an easier form:

$$S_e^* = \Big\{ -S_e \mathrm{Rem}\left(\frac{\frac{M}{2}}{M+1}\right) + \cdots + S_e \mathrm{Rem}\left(\frac{M}{M+1}\right) \\ -S_e \mathrm{Rem}\left(\frac{4M}{3(M+1)}\right) + \cdots + S_e \mathrm{Rem}\left(\frac{2M}{M+1}\right) \Big\}. \quad (5)$$

For $M = 4$, the interleaved vector becomes:

$$S_2^* = [-0.707 - 1.225\mathrm{i} \quad 2.121 + 3.674\mathrm{i} \\ - 2.121 - 3.6742\mathrm{i} \quad 0.707 + 1.225\mathrm{i}],$$

In main constellation, we place S_1, S_2^* and S_3:

$$\mathbf{MC} = \begin{bmatrix} -3.0000 - 3.0000\mathrm{i} & -1.0000 - 1.0000\mathrm{i} & 1.0000 + 1.0000\mathrm{i} & 3.0000 + 3.0000\mathrm{i} \\ -0.7071 - 1.2247\mathrm{i} & -1.0981 - 4.0981\mathrm{i} & 2.1213 + 3.6742\mathrm{i} & -0.3660 - 1.3660\mathrm{i} \\ -2.1213 - 3.6742\mathrm{i} & 0.3660 + 1.3660\mathrm{i} & 0.7071 + 1.2247\mathrm{i} & 1.0981 + 4.0981\mathrm{i} \end{bmatrix}.$$

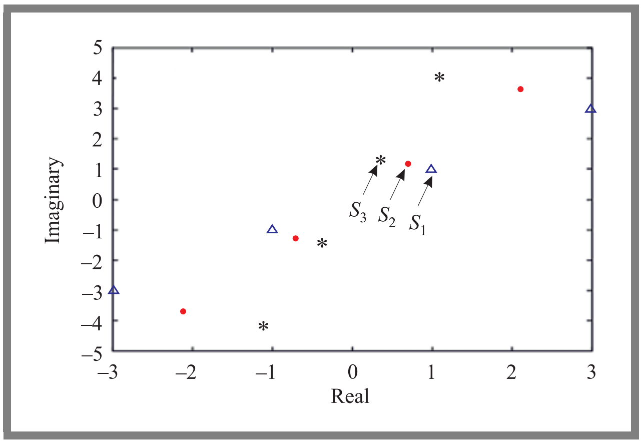

Fig. 2. Main constellation and rotation.

Codebook design

After the complex MC has been obtained, its components are added to the sparse code words. The codewords for the various symbols transmitted by user j are concatenated to create codebook $\mathbf{X}_j$ for that single user, where each symbol is represented by a column vector with K rows. Two of the rows have complex numbers in them, and the value of the other two equals 0. Consequently, the SCMA codebook $\mathbf{X}_j$, often known as for j-th user, is generated as follows [5], [7]:

$$\mathbf{X}_j = V_j \mathbf{\Delta}_j \mathbf{MC} \qquad j = 1, 2, \ldots, J. \quad (6)$$

A sparsely populated V_j represents the dispersion matrix for each user, where $V \in B^{K \times N}$, mapping the K-dimensional codeword $\mathbf{X}$ onto the N-dimensional complex constellation point. In this state, if $K = 6$, $N = 3$ and $J = 8$, matrices V_j, $j = 1, 2, \ldots, J$ are:

$$\mathbf{V}_1 = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} \quad \mathbf{V}_2 = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \end{bmatrix}$$

$$\mathbf{V}_3 = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix} \quad \mathbf{V}_4 = \begin{bmatrix} 0 & 0 & 0 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \end{bmatrix}$$

$$\mathbf{V}_5 = \begin{bmatrix} 0 & 0 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix} \quad \mathbf{V}_6 = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$\mathbf{V}_7 = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \end{bmatrix} \quad \mathbf{V}_8 = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix}.$$

Rotation of the corresponding $\mathbf{MC}$ is required by the codebook design for different users. The phase rotation angles [5], [7] are:

$$\varphi_u = (u-1)\frac{2\pi}{Mdf} + e_u\frac{2\pi}{M} \quad u = 1, \ldots, df, \tag{7}$$

where e_u is an integer representation of Z and the overloading factor or the number of elements present simultaneously in a given subcarrier is represented by df. The user's dimension on the interfering layers must remain at least as far apart from other layers' dimensions, thus φ_u is the minimum ideal rotation phase angle.

4. Proposed Method

A new even dimension interleaving method using Arnold's cat map is proposed here. The Arnold's cat map is a square matrix expressed as [16]:

$$\mathbf{A} = \begin{bmatrix} 2 & 1 \\ 1 & 1 \end{bmatrix} \bmod N. \tag{8}$$

where N is the maximum index of the dimensional matrix (i.e. $N \times N$ elements matrix) to be interleaved using Arnold's cat map. The coordinates of each element which the ordered pair represents (X, Y) are in the range of $[0 \ldots 1]$ when modulo 2 is applied. Such a transformation is mainly used for interleaving pixels in image processing applications. It creates a discrete-time dynamic system in which the $\mathbf{\Gamma}_{\text{cat}}$ mapping iterations control the evolution as:

$$\mathbf{\Gamma}_{\text{cat}} = \begin{bmatrix} X_{n+1} \\ Y_{n+1} \end{bmatrix} = \begin{bmatrix} 2 & 1 \\ 1 & 1 \end{bmatrix} \begin{bmatrix} X_n \\ Y_n \end{bmatrix} \bmod N, \tag{9}$$

where $N = \sqrt{M}$.

To improve performance of SCMA and reduce computational complexity, we propose using Arnold's cat map for interleaving the mother constellation using Eq. (9) instead of the interleaving process expressed by Eq. (5). To perform the interleaving of vector S_2 using Arnold's cat map, $\mathbf{S}_2$ should be converted into a $N \times N$ square matrix and processed. When N is 2, S_2 would be a 2×2 matrix as:

$$\mathbf{S}_2 = \begin{bmatrix} -2.1213 - 3.6742\text{i} & 0.7071 + 1.2247\text{i} \\ -0.7071 - 1.2247\text{i} & 2.1213 + 3.6742\text{i} \end{bmatrix}.$$

Then, the locations of each element in the $\mathbf{S}_2$ matrix are changed using Arnold's cat map, as given in Eq. (9), where $\begin{bmatrix} X_n \\ Y_n \end{bmatrix}$ is the location of elements before interleaving and $\begin{bmatrix} X_{n+1} \\ Y_{n+1} \end{bmatrix}$ is the corresponding location after interleaving. Hence, $\mathbf{S}_2$ after interleaving would be:

$$\mathbf{S}_2 = [2.1213 + 3.6742\text{i} \quad -2.1213 - 3.6742\text{i} \quad 0.7071 + 1.2247\text{i} \quad -0.7071 - 1.2247\text{i}].$$

$\mathbf{MC}$ after the proposed interleaving becomes:

$$\mathbf{MC} = \begin{bmatrix} -3.000 - 3.000\text{i} & 1.000 - 1.000\text{i} & 1.000 + 1.000\text{i} & 3.000 + 3.000\text{i} \\ 2.121 + 3.674\text{i} & -1.098 - 4.098\text{i} & -2.121 - 3.674\text{i} & -0.366 - 1.366\text{i} \\ 0.707 + 1.224\text{i} & 0.366 + 1.366\text{i} & -0.707 - 1.225\text{i} & 1.098 + 4.098\text{i} \end{bmatrix}.$$

The main constellation and rotation using the proposed chaotic interleaving can be explained using Algorithm 1.

Algorithm 1. Mother constellation creation.

Input: $k, j, df, \text{dv}, M, N$. j is the number of users, k is the number of resources or subcarriers, df is the number of users on one resource, dv is the number of resources for one user.
Result: creation of main constellation $\mathbf{MC}$
Begin
1: For $m = 1$ to M
2: $Q_m = 2m - 1 - M : S_1 = Q_m(1 + \text{i})$
3: End for M
4: For $r = 1$ to N
5: $\theta_{r-1} = (r-1) \cdot \frac{\pi}{MN} : \text{UN} = \text{diag}(1\text{e}^{1\theta r - 1}) :$
$S_{1r} = Un(r, 1) \cdot S_1$
6: Select even elements of S_{1r} only
7: Perform interleaving using Arnold's cat map
8: End for N
9: Construct main constellation: $\mathbf{MC} = [S_1, S_{2'}, \ldots, S_N]$
10: End

The values calculated from Eq. (7) if $M = 4$ and $df = 4$ are: $\varphi_1 = 0, \varphi_2 = \frac{\pi}{8}$ and $\varphi_4 = \frac{3\pi}{8}$. These are the ideal parameters that maintain the constant Euclidean distance between the codewords and are based on Latin squares and the structure of the φ_1 mother codebook [5], [7]. The rotation angles factor graph that follows this principle is:

$$\mathbf{F}_\varphi = \begin{bmatrix} \varphi_1 & \varphi_2 & \varphi_3 & 0 & 0 & 0 & \varphi_4 & 0 \\ \varphi_2 & 0 & 0 & \varphi_3 & \varphi_4 & 0 & \varphi_1 & 0 \\ 0 & \varphi_1 & 0 & \varphi_2 & 0 & \varphi_3 & 0 & \varphi_4 \\ \varphi_3 & 0 & \varphi_4 & 0 & \varphi_1 & 0 & 0 & \varphi_2 \\ 0 & \varphi_4 & 0 & \varphi_1 & 0 & \varphi_2 & \varphi_3 & 0 \\ 0 & 0 & \varphi_2 & 0 & \varphi_3 & \varphi_4 & 0 & \varphi_1 \end{bmatrix}. \tag{10}$$

As long as the minimal dimensional distance between the layers is maintained, there are several alternatives for how to arrange the phase rotation, depending on the codebook structure. The angles are formulated in matrix $\mathbf{F}_{\varphi j}$ as:

$$\mathbf{F}_{\varphi j} = \begin{bmatrix} \varphi_1 & \varphi_2 & \varphi_3 & \varphi_3 & \varphi_4 & \varphi_3 & \varphi_4 & \varphi_4 \\ \varphi_2 & \varphi_1 & \varphi_4 & \varphi_2 & \varphi_1 & \varphi_2 & \varphi_1 & \varphi_2 \\ \varphi_3 & \varphi_4 & \varphi_2 & \varphi_1 & \varphi_3 & \varphi_4 & \varphi_3 & \varphi_1 \end{bmatrix}, \quad (11)$$

From the $\mathbf{F}_{\varphi j}$, the rotation operator $\mathbf{\Delta}_j$ is produced as follows:

$$\mathbf{\Delta}_j = \text{diag}(F_{\varphi j}), \quad \forall j = 1, 2, \ldots, J \quad (12)$$

In this case, when $J = 8$, we have:

$$\mathbf{\Delta}_1 = \begin{bmatrix} \varphi_1 & 0 & 0 \\ 0 & \varphi_2 & 0 \\ 0 & 0 & \varphi_3 \end{bmatrix} \quad \mathbf{\Delta}_2 = \begin{bmatrix} \varphi_2 & 0 & 0 \\ 0 & \varphi_1 & 0 \\ 0 & 0 & \varphi_4 \end{bmatrix}$$

$$\mathbf{\Delta}_3 = \begin{bmatrix} \varphi_3 & 0 & 0 \\ 0 & \varphi_4 & 0 \\ 0 & 0 & \varphi_2 \end{bmatrix} \quad \mathbf{\Delta}_4 = \begin{bmatrix} \varphi_3 & 0 & 0 \\ 0 & \varphi_2 & 0 \\ 0 & 0 & \varphi_1 \end{bmatrix},$$

$$\mathbf{\Delta}_5 = \begin{bmatrix} \varphi_4 & 0 & 0 \\ 0 & \varphi_1 & 0 \\ 0 & 0 & \varphi_3 \end{bmatrix} \quad \mathbf{\Delta}_6 = \begin{bmatrix} \varphi_3 & 0 & 0 \\ 0 & \varphi_2 & 0 \\ 0 & 0 & \varphi_4 \end{bmatrix}$$

$$\mathbf{\Delta}_7 = \begin{bmatrix} \varphi_4 & 0 & 0 \\ 0 & \varphi_1 & 0 \\ 0 & 0 & \varphi_3 \end{bmatrix} \quad \mathbf{\Delta}_8 = \begin{bmatrix} \varphi_4 & 0 & 0 \\ 0 & \varphi_2 & 0 \\ 0 & 0 & \varphi_1 \end{bmatrix}.$$

SCMA codebooks $\mathbf{X}_j$ for the j-th user using Eq. (6) and after normalizing the values are:

$$\mathbf{X}_1 = \begin{bmatrix} 0.322 - 0.322i & -0.108 - 0.108i & -0.113 - 0.195i & 0.338 + 0.585i \\ 0.108 + 0.108i & 0.322 + 0.322i & -0.338 - 0.585i & 0.113 + 0.195i \\ 0.000 + 0.000i & 0.000 + 0.000i & -0.259 - 0.966i & -0.086 - 0.322i \\ 0.000 + 0.000i & 0.000 + 0.000i & 0.000 + 0.000i & 0.000 + 0.000i \\ 0.000 + 0.000i & 0.000 + 0.000i & 0.086 + 0.322i & 0.259 + 0.966i \\ 0.000 + 0.000i & 0.000 + 0.000i & 0.000 + 0.000i & 0.000 + 0.000i \end{bmatrix},$$

$$\mathbf{X}_2 = \begin{bmatrix} -0.322 - 0.322i & -0.108 - 0.108i & 0.000 + 0.000i & 0.000 + 0.000i \\ 0.106 + 0.108i & 0.322 + 0.322i & 0.000 + 0.000i & 0.000 + 0.000i \\ -0.051 - 0.089i & 0.154 + 0.267i & 0.000 + 0.000i & 0.000 + 0.000i \\ -0.259 - 0.966i & -0.086 - 0.322i & 0.000 + 0.000i & 0.000 + 0.000i \\ -0.154 - 0.267i & 0.051 + 0.089i & 0.000 + 0.000i & 0.000 + 0.000i \\ 0.086 + 0.322i & 0.259 + 0.966i & 0.000 + 0.000i & 0.000 + 0.000i \end{bmatrix},$$

$$\mathbf{X}_3 = \begin{bmatrix} -0.478 - 0.478i & -0.159 - 0.159i & 0.000 + 0.000i & 0.000 + 0.000i \\ 0.159 + 0.159i & 0.478 + 0.478i & 0.000 + 0.000i & 0.000 + 0.000i \\ 0.000 + 0.000i & 0.000 + 0.000i & -0.167 - 0.289i & 0.500 + 0.866i \\ 0.000 + 0.000i & 0.000 + 0.000i & -0.054 - 0.201i & -0.018 - 0.067i \\ 0.000 + 0.000i & 0.000 + 0.000i & -0.500 - 0.866i & 0.167 + 0.289i \\ 0.000 + 0.000i & 0.000 + 0.000i & 0.018 + 0.067i & 0.054 + 0.201i \end{bmatrix},$$

$$\mathbf{X}_4 = \begin{bmatrix} 0.000 + 0.000i & -0.000 + 0.000i & -0.707 - 0.707i & -0.236 - 0.236i \\ 0.000 + 0.000i & 0.000 + 0.000i & 0.236 + 0.236i & 0.707 + 0.707i \\ -0.113 - 0.195i & 0.338 + 0.585i & 0.000 + 0.000i & 0.000 + 0.000i \\ -0.054 - 0.201i & -0.018 - 0.067i & 0.000 + 0.000i & 0.000 + 0.000i \\ -0.338 - 0.585i & 0.113 + 0.195i & 0.000 + 0.000i & 0.000 + 0.000i \\ 0.018 + 0.067i & 0.054 + 0.201i & 0.000 + 0.000i & 0.000 + 0.000i \end{bmatrix},$$

$$\mathbf{X}_5 = \begin{bmatrix} 0.000 + 0.000i & 0.000 + 0.000i & -0.707 - 0.707i & -0.236 - 0.236i \\ 0.000 + 0.000i & 0.000 + 0.000i & 0.236 + 0.236i & 0.707 + 0.707i \\ 0.000 + 0.000i & 0.000 + 0.000i & -0.051 - 0.089i & 0.154 + 0.267i \\ 0.000 + 0.000i & 0.000 + 0.000i & -0.118 - 0.440i & -0.039 - 0.147i \\ 0.000 + 0.000i & 0.000 + 0.000i & -0.154 - 0.267i & 0.051 + 0.089i \\ 0.000 + 0.000i & 0.000 + 0.000i & 0.039 + 0.147i & 0.118 + 0.440i \end{bmatrix},$$

$$\mathbf{X}_6 = \begin{bmatrix} 0.000+0.000\text{i} & 0.000+0.000\text{i} & 0.000+0.000\text{i} & 0.000+0.000\text{i} \\ 0.000+0.000\text{i} & 0.000+0.000\text{i} & 0.000+0.000\text{i} & 0.000+0.000\text{i} \\ -0.478-0.479\text{i} & -0.159-0.159\text{i} & 0.000+0.000\text{i} & 0.000+0.000\text{i} \\ 0.104+0.180\text{i} & -0.104-0.180\text{i} & -0.258-0.966\text{i} & -0.086-0.322\text{i} \\ 0.159+0.159\text{i} & 0.476+0.478\text{i} & 0.000+0.000\text{i} & 0.000+0.000\text{i} \\ 0.035+0.060\text{i} & -0.035-0.060\text{i} & 0.086+0.322\text{i} & 0.259+0.966\text{i} \end{bmatrix},$$

$$\mathbf{X}_7 = \begin{bmatrix} -0.147-0.147\text{i} & -0.049-0.059\text{i} & 0.047+0.082\text{i} & -0.047-0.082\text{i} \\ 0.049+0.049\text{i} & 0.147+0.157\text{i} & 0.016+0.027\text{i} & -0.016-0.027\text{i} \\ 0.000+0.000\text{i} & 0.000+0.000\text{i} & -0.259-0.966\text{i} & -0.086-0.322\text{i} \\ 0.000+0.000\text{i} & 0.000+0.000\text{i} & 0.000+0.000\text{i} & 0.000+0.000\text{i} \\ 0.000+0.000\text{i} & 0.000+0.000\text{i} & 0.086+0.322\text{i} & 0.259+0.966\text{i} \\ 0.000+0.000\text{i} & 0.000+0.000\text{i} & 0.000+0.000\text{i} & 0.000+0.000\text{i} \end{bmatrix},$$

$$\mathbf{X}_8 = \begin{bmatrix} 0.000+0.000\text{i} & 0.000+0.000\text{i} & 0.000+0.000\text{i} & 0.000+0.000\text{i} \\ 0.000+0.000\text{i} & 0.000+0.000\text{i} & 0.000+0.000\text{i} & 0.000+0.000\text{i} \\ -0.707-0.707\text{i} & -0.236-0.236\text{i} & 0.104+0.180\text{i} & -0.104-0.180\text{i} \\ 0.000+0.000\text{i} & 0.000+0.000\text{i} & 0.025-0.092\text{i} & -0.008-0.031\text{i} \\ 0.236+0.236\text{i} & 0.707+0.707\text{i} & 0.035+0.060\text{i} & -0.035-0.060\text{i} \\ 0.000+0.000\text{i} & 0.000+0.000\text{i} & 0.008+0.031\text{i} & 0.025+0.092\text{i} \end{bmatrix},$$

The codebook for each user from the main constellation can be calculated using Algorithm 2, and the codebook of the first user is shown in Fig. 3.

Algorithm 2. Codebook design from main constellation

Input: Set $k, j, df, \text{dv}, M, N$

Result: Creation of codebook for each user from main constellation $\mathbf{MC}$

Begin

1: For $j = 1$ to J

2: Compute the dispersion matrix for each user V_j

3: For $u = 1$ to df

4: Compute the phase rotation angle $\varphi_u = (u-1)\frac{2\pi}{M\,df} + eu\frac{2\pi}{M}$.

5: Using factor graph of rotation angles matrix F_φ in Eq. (10), obtain rotation operator $\mathbf{\Delta}_j = \text{diag}(F_{\varphi j})$

6: Construct the codebook of each user $\mathbf{X}_j = V_j \mathbf{\Delta}_j \mathbf{MC}$

7: End for u

8: End for j

9: End

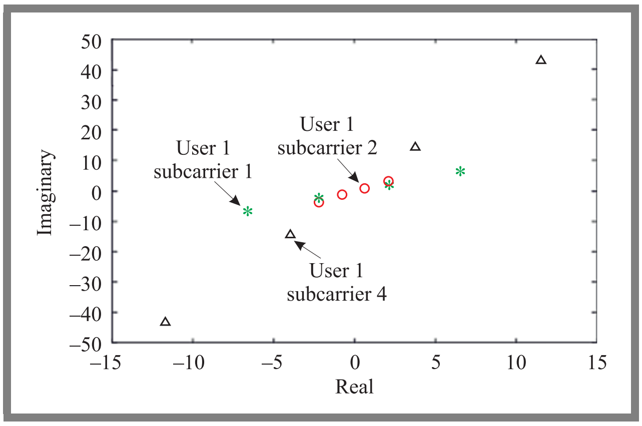

Fig. 3. Codebook of the first user.

5. Simulation Results

Matlab was used to evaluate the effectiveness of the designed chaotic interleaving-based SCMA codebooks. Performance of the designed SCMA system was measured in terms of MED, PAPR, computational complexity, and BER over a noisy channel. The minimum square Euclidean distance of the codeword from the mother codebook was computed using the following formula [4]:

$$d_{\min}^2 = \begin{cases} 8[N_0^{\sim} + 4N_e^{\sim}] & M = 4 \\ 8N & M > 4 \end{cases}, \tag{13}$$

where $N_0^{\sim}$ and $N_e^{\sim}$ are the numbers of odd and even dimensions. respectively. On the other hand, PAPR was computed using [8]:

$$\text{PAPR}(\mathbf{MC}) = 10\log\left(\frac{3[No(M-1)^2 + Ne]}{N(M^2-1)}\right). \tag{14}$$

Comparisons with various methods from the literature were conducted to assess the effectiveness of the proposed chaotic interleaving-based SCMA codebook. [5], [7], [9], and [11] have been selected for comparison – see Tab. 1. Fig. 4 shows a comparison of BER for different codebook design methods, while Fig. 5 illustrates the comparison of BER for different interleaving methods. It can be seen from these figures that MED is maximized when $M = 4$ and the proposed method achieves almost the same performance as in [5], [7] in terms of MED, PAPR, and obtainable SNR at $\text{BER} = 10^{-4}$. The

Tab. 1. Comparison of different SCMA methods.

Method	J	K	M	N	df	SNR at BER $= 10^{-4}$	PAPR	MED
MDconst [5], [7]	6	4	4	2	3	21 dB	0	2
Proposed	6	4	4	2	3	22 dB	0	2
MDconst [5], [7]	8	6	4	3	4	23 dB	1.96	1.64
Proposed	8	6	4	3	4	24 dB	1.96	1.64
MDconst [5], [7]	6	4	16	2	3	27 dB	0	0.88
Proposed	6	4	16	2	3	28 dB	0	0.88
16-star QAM [9]	6	4	4	2	3	10 dB	0	2.11
CG SCMA [11]	6	4	4	2	3	8-9 dB	0	2.16

results also show that the 16-star QAM constellation, as well as the segmentation and CG-SCMA methods achieve better BER than other methods due to the fact that coding associated with modulation is used.

Computational complexity is an important performance indicator that should be taken into account while designing codebooks. Low memory usage, as well as high speed of encoding and decoding activities guarantee good performance metrics. The described interleaving method is simpler in terms of computational cost, as it needs fewer mathematical operations.

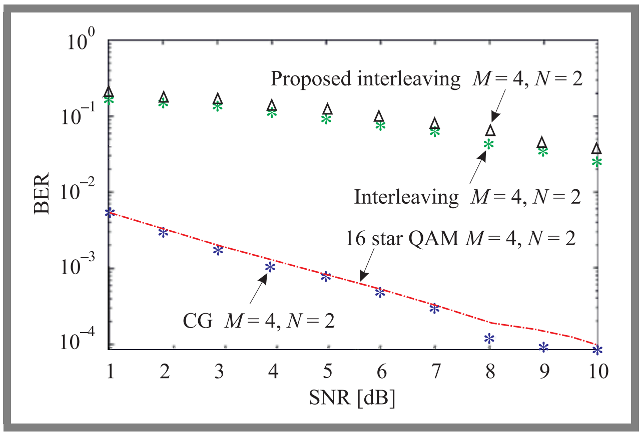

Fig. 4. Comparison of BER of different SCMA methods.

From Eq. (5), one may notice that 11 operations, such as division and multiplication, are required to perform interleaving for $M = 4$, while 23 operations are required for $M = 8$. Therefore, the number of operations required in the interleaving method presented in [5], [7] is $3 \times M - 1$. On the other hand, in the proposed interleaving method, according to Eq. (9), the required number of operations is 8 for $M = 4$, and 16 for $M = 8$ $(2 \times M)$. The percentage reduction can be expressed as:

$$\text{Reduction} = \frac{M-1}{3M-1} \cdot 100[\%]. \tag{15}$$

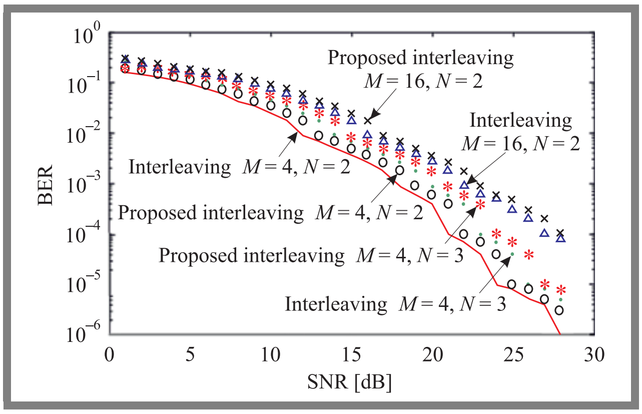

Fig. 5. BER of interleaving methods.

Tab. 2. Comparison of different SCMA interleaving methods in terms of complexity.

Number of codewords M	Number of multiplications and divisions		Percentage reduction
	Interleaving method [5], [7]	Proposed chaotic interleaving	
4	11	8	27%
8	23	16	30%
16	47	32	32%

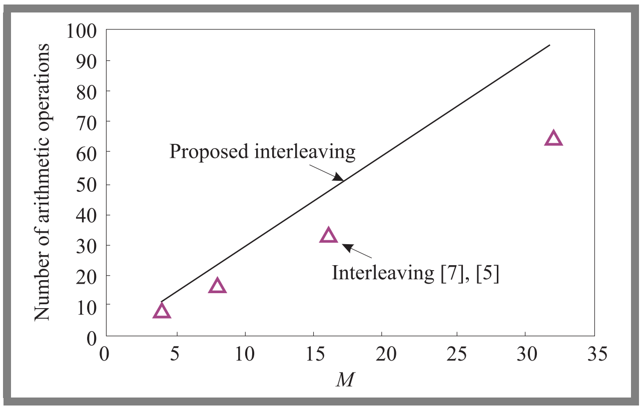

Fig. 6. Complexity of the proposed interleaving scheme compared with papers [5] and [7].

6. Conclusions

The paper presents an efficient SCMA codebook design based on interleaving. It covers several codebook design techniques that have been documented in the literature. The proposed method reduces computational complexity and offers, by relying on the Arnold's cat map for interleaving, better performance than other existing interleaving approaches. Future research should concentrate on improving MD constellations by including sophisticated models or machine learning techniques, such as neural networks.

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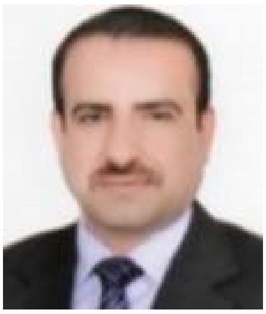

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