Paper

Cooperative and Non-cooperative, Integrative and Distributive Market Games with Antagonistic and Altruistic, Malicious and Kind Ways of Playing

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Abstract—The article illustrates distinctions between important concepts of game theory, which support understanding the relation between subjects on competitive and regulated telecommunications services market. Especially it shows that often used distinction between retail and wholesale market that treat them respectively as competitive and cooperative can be misleading or even wrong.

Keywords— *antagonism and altruism, competition, cooperative games, integrative and distributive processes, non-cooperative games.*

1. Introduction

Liberalization of the telecommunications services market transformed so far monopolistic market into competitive one. However it is a specific competition because market players are also forced (by the telecommunications low and decisions of the regulators) into cooperation: networks of the operators ought to be interconnected. For these reasons telecommunications market is not only competitive, but also cooperative.

Cursory analysis leads us to conclusion, that the boundary between the issues of competition and cooperation runs the same way as the boundary between the retail and the wholesale market. However more careful analysis shows that it should not be true. In fact competition is not an opposing part to the cooperation: these concepts comes from different "layers" of interaction between players.

The article discusses three layers, that defines the complexity of interaction between players in market games: possibility of concluding enforceable agreements outside the formal rules of the game, the structure of the payoff matrix, the goals of the players, and explains the essence of the important phenomenon that occurs on each of them.

2. Simple Theoretical Model of a Market Game

Let us describe the market game in the concepts of game theory. Every market participator can be treated as a player, which has his own strategy of playing (e.g., prices on the retail market, interconnection fees on the wholesale market, etc.). The players evaluate their decisions (set strategies) by the single-criteria or aggregated, multiple-criteria goal function, which can be called as their payoff function. The value of the payoff function depend on the strategies set by each player in the game.

Table 1
Relationship between concepts of strategy and outcomes of payoff function

Strategies	b_1	b_2	$\boldsymbol{b_3}$	b_4
a_1			$\vdots$	
$\boldsymbol{a_2}$	......	......	$[\boldsymbol{V_3^A(a_2), V_2^B(b_3)}]$	......
a_3			$\vdots$	
a_4			$\vdots$	

For the case of two players it is useful to illustrate relation between strategies and payoff functions in the form of the so called payoff matrix. Table 1 illustrates a simple payoff matrix for two market players – A and B. Player A chooses one of four strategies: a_1, a_2, a_3, a_4, and player B one of b_1, b_2, b_3, b_4. Choosing the strategy a_i by player A and b_j by player B results in obtaining $V_j^A(a_i)$ by player A and $V_i^B(b_j)$ by player B.

3. Distinction for the sake of the Way of Setting a Solution

From the early work of John Nash [1] differentiation between *cooperative* and *non-cooperative* games starts in game theoretical analysis. In *non-cooperative games* players are unable to conclude enforceable agreements outside the formal rules of the game. *Cooperative games* allow such agreements[1]. Actually, Nash also assumed that in a non-cooperative game, the players will be unable to com-

[1]Nash suggests that non-cooperative games are more basic, that cooperative games may fruitfully be analyzed by reformulating them as non-cooperative ones and by solving for the Nash equilibria [2]. This approach has come to be known as the Nash program [1]. It allows unification of the theory and enables better understanding of the different solution concepts that have been proposed in cooperative theory.

municate with each other. Yet, as it was noticed by Harold W. Kuhn [1], this would be a needlessly restrictive assumption. For if the players cannot enter into enforceable agreements, then their ability to communicate will be of no real help toward a cooperative outcome.

Such differentiation – differentiation on the level of possibility to conclude enforceable agreements – explains us most of all the *way* of setting the result of the game. Such two different ways – with and without agreements – have found reflection into two different theoretical methods of solving games which are now called Nash solution (for cooperative games) [2] and Nash equilibrium (for non-cooperative ones) [3], [4].

Competition is the relation between two or more subjects (players) which arises when such players strives for the same and limited goods. So in terms of game theory such relation is well described by model of zero-sum (or constant-sum) game [2]: the more one player gets the more the other (others) should loose. An example of such game we have in Table 2.

Table 2
Competitive, zero-sum game

Strategies	b_1	b_2
a_1	[–1, 1]	[4, –4]
a_2	[2, –2]	[–3, 3]

So using the above mentioned concepts we should say that popular distinction: *cooperative – wholesale market* and *competitive – retail market* is not too precise. Rather we should say: *cooperative – wholesale market* and *non-cooperative retail market*[2].

4. Distinction for the sake of the Structure of the Payoff Matrix

Another concepts useful for understanding discussed issues comes from the theory of negotiations [4]. Negotiations are in fact *cooperative game* (as on the wholesale telecommunications services market). In every negotiations, where are discussed at least two different issues, and where preferences of the parts are not strictly the same, it is possible to engage the negotiators into a process which is called *integration*. During it parts tries to find such correlation between their preferences which enables *increasing* the size of the "cake" before *dividing* it. In fact such process bases on mutual exchange less preferable issues (or their parts) on more preferable ones. Process of dividing such "cake" (occurring during the integrative process or not) is called *distributive*, and so in fact it is exact *competitive process*.

Saying in terms of game theory or decision support *integrative* process means seeking for effective, Pareto-optimal solutions [5], whilst *distributive* process means making actions for choosing one of two different (and differently preferable by players) solutions (effective or not). It is interesting to notice, that zero-sum game is a game, where every result is Pareto-optimal, and so there exist a place only for *distributive* process.

In negotiations *distributive* process is the only if negotiations concern only one issue [4]. In such situations there is no place for any *integration*, for increasing the size of the "cake". Such "cake" can be only divided and the more one part gets the more the other looses. So every solution, every division is Pareto-optimal. However it is true only if the whole "cake" was divided, if "no gold was left on the table" [6], [7] (strictly speaking only such situations can be modeled as a zero-sum games). If it is possible to exclude some part of the "cake" from the division (from the *distributive* process), then we have place for something like *de-integrative* process which is strictly opposite to the *integrative* one. An example of such game we have in Table 3.

Let us assume, that the values of outcomes corresponds to the share of the divided (distributed) object. In this game there are three effective solutions, that "divide the whole cake": [0.3, 0.7], [0.7, 0.3] and [0.5, 0.5]. The result [0.4, 0.4] is not effective, and means "leaving on the table some gold" (0.2 part of the object), and so choosing it (if players does know the effective results) can be treated as a result of a *de-integrative* process.

Table 3
Game with possibility of de-integration

Strategies	b_1	b_2
a_1	[0.4, 0.4]	[0.5, 0.5]
a_2	[0.3, 0.7]	[0.7, 0.3]

Is there any opposite process to the *distribution* in such games? Generally we say that this is *concentration*, but in a game it means no decision, giving up any solution, and so, from our point of view it is not an interesting case. However as we have games where there is only place for *distributive* process (zero-sum games), so we have games where there is no place for any *distribution*, only an *integration* (and *de-integration*) can take place. An example of such game we have in Table 4, where – assuming that both players aim only at maximizing their own payoffs – preferences of both players between individual solutions are exactly the same. If we assume that players aims at choosing an effective result, and are able (during the *integrative* process) to doing so, there is no place for *distributive* process, because there

Table 4
Game with no place for distributive process

Strategies	b_1	b_2
a_1	[1, 1]	[4, 4]
a_2	[2, 2]	[3, 3]

[2]If really decisions on the retail market are made by players without making any (public or tacit) agreements.

is only one effective result in this game. More over making a comparison between any two different solutions we have that always one of them is betters for both players.
In some cases the place for *distributive* process can arise if *integrative* process finishes without finding the only effective solution. An example of such game we have in Table 5.

Table 5
Game with a place for distributive process only if integrative process finishes without finding the only effective result

Strategies	b_1	b_2
a_1	[1, 1]	[4, 4]
a_2	[3, 2]	[2, 3]

If for example during a negotiations, assuming that players aims at maximizing their own payoffs players didn't find (during an *integrative* process) a solution [4, 4], than a place for *distributive* process would arise: players would bargain on selections one of two, differently preferable solutions: [3, 2] and [2, 3].
Analogically it is also possible that there would be no place for a *distributive* process because of the same reason. We have this in a game as in Table 6.

Table 6
Game without a place for distributive process if players didn't find all effective solutions

Strategies	b_1	b_2
a_1	[1, 1]	[2, 2]
a_2	[3, 4]	[4, 3]

If, for example during the negotiations players found only two or three results (but only one effective: [3, 4] or [4, 3]), then there would be no place for a *distributive* process.
It seems to be useful to define a single type of integrative-distributive games, encompassing three before mentioned cases:

– game with possibility of de-integration,

– game with a place for distribution only if integrative process finishes without finding the only effective result,

– game without a place for distribution if players didn't find all effective results.

In fact an *integrative* process can occur if there exists a place for improving a given solution for every of the players at a given stage of a game. Starting from any ineffective result always such possibility exists. In every above mentioned cases there exists at least one ineffective result, so such situation occurs. Also in every cases there are at least two incomparable solutions – solutions, that any of them is better for every of the players – so there exists a place for a *distributive process* (it's true even if there exists only one effective result in a game, because players should not know, which result is effective, an can decide to finish integrative process after finding two different and incomparable ineffective results).
So, respectively to the structure of the payoff matrix of a given game we can distinguish the following different types of games:

– *distributive games*: games with no place for *integrative* process (strictly competitive games, zero-sum or constant-sum games),

– *integrative games*: games with no place for *distributive* process (not competitive games),

– *integrative-distributive games*.

Having this we can say that *wholesale* telecommunications services market, respectively to structure of the payoff matrix can be treated as *distributive*, *integrative* or *integrative-distributive cooperative game*. Analogically *retail* telecommunications services market, can be treated as *distributive*, *integrative* or *integrative-distributive non-cooperative game*. So we see that as on the retail (*non-cooperative*) so on the wholesale (*cooperative*) market a place for a *distributive* process – a real *competition* – can exist. Also we see, that on the retail (*non-cooperative*) market there can exist a place for something like *integration*, for increasing the size of the "cake", for finding such solution which would be better for every of the players than another accessible solution.
However there is a difference between *integrative* and *distributive* processes on retail and wholesale markets. Such difference comes from the *way* of setting a result: *cooperative* on the wholesale markets and *non-cooperative* on the retail markets. On the wholesale markets *integrative* and *distributive* process can proceed during one, single game (during a one round of the negotiations by making temporary decisions). On the retail markets such process proceeds only if a game is repeated (by making real decisions). Moreover, on the wholesale markets *integrative* and *distributive* processes can proceed independently. On the retail markets such *integration* and *distribution* are realized simultaneously: by making a decision by the last mover in the game[3].

5. Distinction for the sake of the Aims of the Players

Until to the first works of John C. Harsanyi on the games with incomplete information (so called *I-games* [8]–[11]) it was generally assumed, that in any games players have all information, necessary to define the strategic form of a given game (its *basic mathematical structure*).

[3]Partially *integration* and *distribution* are realized also by a decisions of the first (and eventually next, but not last) mover, whose decisions creates the finale alternatives to the last mover.

Harsanyi showed that in many real situations this is too hard assumption. Players often does not know: the form of their own or the other player's payoff function, the set of the accessible strategies, the scope of information that the other player possess, etc.

In real situations there exists one thing, that really can be interpreted as a part of a strategic form of a game, yet seems to be simple to pass over. General assumption in game theory is that players aims at maximizing their own payoff functions. These functions – interpreted as utility functions – are formulated in such way, that their maximization leads to obtaining the appointed goal. Such utility function describes how good for a given player (under his subjective preferences) is the obtained objective state. Here arises very subtle problem.

Let us consider simple example. In a given game there are only two different results [3, 4] and [1, 3]. The values reflects the profits in money of players A and B. Let us assume, that both players prefer to get more money than less, and that their utility is proportional to the amount of gotten many. So such results expressed in terms of utility have the same form: [3, 4] and [1, 3]. The answer, which result should be chosen by players seems to by simple: effective [3, 4]. However it is true only, if – as it is usually assumed in game theory analysis, and us during formulation of utility function for these results – players evaluate the results only by the value of money, they get themselves.

This assumption can be called as assumption of *neutral* way of playing, by players: players are interesting only in evaluation the values gotten by themselves.

In market games such assumption is too hard. Evaluation solely the values gotten by itself is a good approach only in short term. In long term players should take into consideration relative values, because after crossing a certain distance between the positions of the players on the market, such distance can increase very quickly: strong player becomes stronger, weak becomes weaker. So in our example we could assume, that players evaluate the obtained results not by the values of money obtained by themselves but as a difference between the values gotten by both players. So for player A the result [3, 4] may have utility $3-4=-1$, and for result [1, 3]: $1-3=-2$. For player B the utilities would be exactly opposite: for [3, 4]: $4-3=1$, and for [1, 3]: $3-1=2$. These new utility function defines in fact different solutions (in terms of utility): for the values [3, 4] now the result in terms of utility is $[-1, 1]$ and for [1, 3] – $[-2, 2]$. Both of them are effective.

Such aspirations of players can be called as *antagonistic*. Generally, when we say antagonism of the player, we mean of the situation, when the player aims not only in maximization of his own payoff function (defined as an evaluation only his own vale), but also in minimization of the other player's payoff function (defined in the same way). As an opposition we can formulate an aspiration which can be called as *altruistic*. In such a case a given player would aspire to maximize the payoff function of the other player. Now we formulate in mathematical form some examples of the *antagonistic* and *altruistic* way of playing, which can we called antagonistic and altruistic aims. For the simplicity we formulate them only for the player B.

5.1. Examples of Antagonistic Aims

Antagonistic aim of player B reflects his approach to his own payoff function and to player's A payoff function. There could be many of such aims. Below we will present some of them.

Let's $\breve{b}_k$ be the (kth) antagonistic strategy (move) of player B. The most antagonistic move of player B is such, that B aims first of all at minimization of the A's payoff function, and he considers his own payoff function only in a case of ambiguity (two or more different strategies give the same and the smallest outcome to player A). This can be expressed as follows:

$$\breve{b}_k(a_i) = \arg\operatorname{lex}\min_j \left\{ V_j^A(a_i), -V_i^B(b_j) \right\}. \qquad (1)$$

The least antagonistic move of player B is such, that B aims first of all at maximization of his own payoff function and in the case of ambiguity (two or more different strategies give the same – and the highest – outcome to him) he chooses this, that gives the smallest outcome to player A. This can be expressed as follows:

$$\breve{b}_k(a_i) = \arg\operatorname{lex}\max_j \left\{ V_i^B(b_j), -V_j^A(a_i) \right\}. \qquad (2)$$

Strategies (1) and (2) determine (for a given strategy of player A) the range of outcomes that player A can obtain in a situation that player B plays in an antagonistic way. Below some other antagonistic aims of player B are described.

Player B can aim at maximizing of his own payoff function and at minimizing of player's B payoff function with different power to both of them expressed by a weight coefficient α. In such a way a general form of a formula (1) can be obtained[4]:

$$\breve{b}_k(a_i) = \arg\max_j \left\{ \alpha \cdot V_i^B(b_j) - (1-\alpha) \cdot V_j^A(a_i) \right\}. \qquad (3)$$

Strategy (3) can be interpreted as aiming at maximizing the difference between the outcomes of player B and A.

Player B can also aim at obtaining assumed value of the difference – δ between the outcomes of the players, and after that at maximizing of his own payoff function. This can be expressed as the following lexicographic optimization task:

$$\breve{b}_k(a_i) = \arg\operatorname{lex}\max_j \left\{ \Delta_{ij}, V_i^B(b_j) \right\}, \qquad (4)$$

where:

$$\Delta_{ij} = \min \left\{ \delta, \alpha \cdot V_i^B(b_j) - (1-\alpha) \cdot V_j^A(a_i) \right\}.$$

Another kind of antagonistic strategy can be expressed as aiming at maximization of an own payoff function with simultaneous aiming at ensuring that the other player's payoff

[4]The formula (1) can be generalized to (3) by assumption $\alpha \gg (1-\alpha)$.

function does not exceed assumed threshold value ν. This can be expressed as the following optimization task:

$$\breve{b}_k(a_i) = \arg\max_j \left\{ V_i^B(b_j) \right\}, \tag{5}$$

under constraint:

$$V_j^A(a_i) \le \nu.$$

There is a possibility to make an opposite approach: minimization of the player's A payoff function, under assumption that the outcome of player B would not be smaller then the threshold value ν:

$$\breve{b}_k(a_i) = \arg\min_j \left\{ V_j^A(a_i) \right\}, \tag{6}$$

under constraint:

$$V_i^B(b_j) \ge \nu.$$

In the case of using strategy (5) or (6) it is important to asses correctly the value of the threshold ν in order to assure that the appropriate optimization problems will have a solution.

It is possible to express the antagonistic approach of the player B with using the concepts of reference point method [5], [12] by introducing reservation and aspiration point for the payoff functions of the player A and B. Payoff function of the player A will be treated here as the minimized criterion and the player's B as the maximized criterion. Partial achievement function for player B is then expressed as follows:

$$\eta_B\left(V_i^B(b_j)\right) = \begin{cases} \frac{\beta(V_i^B(b_j) - \underline{V}^B)}{\overline{V}^B - \underline{V}^B} & \text{for} \quad V_i^B(b_j) < \underline{V}^B \\ \frac{V_i^B(b_j) - \underline{V}^B}{\overline{V}^B - \underline{V}^B} & \text{for} \quad \underline{V}^B \le V_i^B(b_j) \le \overline{V}^B \\ 1 + \frac{\alpha(V_i^B(b_j) - \overline{V}^B)}{\overline{V}^B - \underline{V}^B} & \text{for} \quad \overline{V}^B < V_i^B(b_j), \end{cases} \tag{7}$$

where $\underline{V}^B$ represents reservation point, and $\overline{V}^B$ represents aspiration point for the payoff function $V_i^B(b_j)$ of player B.

Partial achievement function for player A is expressed as

$$\eta_A\left(V_j^A(a_i)\right) = \begin{cases} 1 + \frac{\alpha(V_j^A(a_i) - \overline{V}^A)}{\overline{V}^A - \underline{V}^A} & \text{for} \quad V_j^A(a_i) < \overline{V}^A \\ \frac{V_j^A(a_i) - \underline{V}^A}{\overline{V}^A - \underline{V}^A} & \text{for} \quad \overline{V}^A \le V_j^A(a_i) \le \underline{V}^A \\ \frac{\beta(V_j^A(a_i) - \underline{V}^A)}{\overline{V}^A - \underline{V}^A} & \text{for} \quad \underline{V}^A < V_j^A(a_i). \end{cases} \tag{8}$$

In such a case antagonistic response (antagonistic strategy) of player B can be defined as the following formulae:

$$\begin{aligned} \breve{b}_k(a_i) = \quad & \arg\max_j \left\{ \min\left\{ \eta_A\left(V_j^A(a_i)\right), \eta_B\left(V_i^B(b_j)\right) \right\} \right. \\ & \left. + \rho \cdot \left(\eta_A\left(V_j^A(a_i)\right) + \eta_B\left(V_i^B(b_j)\right) \right) \right\}. \end{aligned} \tag{9}$$

5.2. Examples of Altruistic Aims

Let's $\widehat{b}_k$ be the (kth) altruistic strategy (move) of player B. The most altruistic move of player B is such, that B aims first of all at maximization of the A's payoff function, and he considers his own payoff function only in a case of ambiguity (two or more different strategies give the same and the highest outcome to player A). This can be expressed as

$$\widehat{b}_k(a_i) = \arg\operatorname{lex}\max_j \left\{ V_j^A(a_i), V_i^B(b_j) \right\}. \tag{10}$$

The least altruistic move of player B is such, that B aims first of all at maximization of his own payoff function and in the case of ambiguity (two or more different strategies give the same – and the highest – outcome to him) he chooses this, that gives the highest outcome to player A. This can be expressed as follows:

$$\widehat{b}_k(a_i) = \arg\operatorname{lex}\max_j \left\{ V_i^B(b_j), V_j^A(a_i) \right\}. \tag{11}$$

Strategies (10) and (11) determine (for a given strategy of player A) the range of outcomes that player A can obtain in a situation that player B plays in an altruistic way. Below some other altruistic moves of player B are described.

Player B can aim at maximizing of his own payoff function and at maximizing of player's B payoff function with different power to both of them expressed by a weight coefficient α. In such a way a general form of a formula (10) can be obtained[5]:

$$\widehat{b}_k(a_i) = \arg\max_j \left\{ \alpha \cdot V_i^B(b_j) + (1-\alpha) \cdot V_j^A(a_i) \right\}. \tag{12}$$

Strategy (12) can be interpreted as aiming at maximizing the sum of the outcomes of player B and A.

Another kind of altruistic strategy can be expressed as aiming at maximization of an own payoff function with simultaneous aiming at ensuring that the other player's payoff function will be not smaller than the assumed threshold value ν. This can be expressed as the following optimization task:

$$\widehat{b}_k(a_i) = \arg\max_j \left\{ V_i^B(b_j) \right\}, \tag{13}$$

under constraint:

$$V_j^A(a_i) \ge \nu.$$

There is a possibility to make an opposite approach: maximization of the player's A payoff function, under assuming that the outcome of player B would not be smaller then the threshold value ν:

$$\widehat{b}_k(a_i) = \arg\max_j \left\{ V_j^A(a_i) \right\}, \tag{14}$$

under constraint:

$$V_i^B(b_j) \ge \nu.$$

[5]The formula (10) can be generalized to (12) by assumption $\alpha \gg (1\text{–}\alpha)$.

In the case of using strategy (13) or (14) it is important to correctly asses the value of the threshold ν in order to assure that the appropriate optimization problems will have a solution.

It is possible to express the altruistic approach of the player B with using the concepts of reference point method by introducing reservation and aspiration point for the payoff functions of the player A and B. Payoff functions of the player A and B are treated here as the maximized criterion. Partial achievement function for player B is then expressed as follows:

$$\eta_B\left(V_i^B(b_j)\right)=\begin{cases}\frac{\beta(V_i^B(b_j)-\underline{V}^B)}{\overline{V}^B-\underline{V}^B} & \text{for} \quad V_i^B(b_j)<\underline{V}^B\\ \frac{V_i^B(b_j)-\underline{V}^B}{\overline{V}^B-\underline{V}^B} & \text{for} \quad \underline{V}^B\le V_i^B(b_j)\le\overline{V}^B\\ 1+\frac{\alpha(V_i^B(b_j)-\overline{V}^B)}{\overline{V}^B-\underline{V}^B} & \text{for} \quad \overline{V}^B<V_i^B(b_j)\,,\end{cases}\tag{15}$$

where $\underline{V}^B$ represents reservation point, and $\overline{V}^B$ represents aspiration point for the payoff function $V_i^B(b_j)$ of player B.

Partial achievement function for player A is expressed as

$$\eta_A\left(V_j^A(a_i)\right)=\begin{cases}\frac{\beta(V_j^A(a_i)-\underline{V}^A)}{\overline{V}^A-\underline{V}^A} & \text{for} \quad V_j^A(a_i)<\underline{V}^A\\ \frac{V_j^A(a_i)-\underline{V}^A}{\overline{V}^A-\underline{V}^A} & \text{for} \quad \underline{V}^A\le V_j^A(a_i)\le\overline{V}^A\\ 1+\frac{\alpha(V_j^A(a_i)-\overline{V}^A)}{\overline{V}^A-\underline{V}^A} & \text{for} \quad \overline{V}^A<V_j^A(a_i)\,.\end{cases}\tag{16}$$

In such a case altruistic move of player B can be defined as the following formulae:

$$\begin{aligned}\widehat{b}_k(a_i) \quad = \quad & \arg\max_j\Bigg\{\min\Big\{\eta_A\Big(V_j^A(a_i)\Big),\eta_B\Big(V_i^B(b_j)\Big)\Big\}\\ & +\rho\cdot\Big(\eta_A\Big(V_j^A(a_i)\Big)+\eta_B\Big(V_i^B(b_j)\Big)\Big)\Bigg\}.\end{aligned}\tag{17}$$

5.3. Examples of Irrational Aims

As an irrational way of playing we mean such, that a given player aims most of all at minimizing his own payoff function. It should be stressed that as in antagonistic so in altruistic ways of playing there is a place for deteriorating of the own payoff. However it is rather a consequence of the main goal: decreasing (in antagonistic) or increasing (in altruistic) the payoff of the other player. If such deteriorating of the own payoff couldn't find any justification in such mainly antagonistic or altruistic aims, than we should treat it as irrational.

Here we present some examples of irrational aims:

$$\tilde{b}_k(a_i)=\arg\operatorname{lex}\min_j\Big\{V_i^B(b_j),V_j^A(a_i)\Big\}\,,\tag{18}$$

$$\tilde{b}_k(a_i)=\arg\operatorname{lex}\min_j\Big\{V_j^A(a_i),V_i^B(b_j)\Big\}\,,\tag{19}$$

$$\tilde{b}_k(a_i)=\arg\operatorname{lex}\max_j\Big\{V_j^A(a_i),-V_i^B(b_j)\Big\}\,,\tag{20}$$

$$\tilde{b}_k(a_i)=\arg\operatorname{lex}\max_j\Big\{-V_i^B(b_j),V_j^A(a_i)\Big\}\,,\tag{21}$$

$$\tilde{b}_k(a_i)=\arg\min_j\Big\{V_j^A(a_i)\Big\}\,,\tag{22}$$

under constraint:

$$V_i^B(b_j)\le\nu\,,$$

$$\tilde{b}_k(a_i)=\arg\max_j\Big\{V_j^A(a_i)\Big\}\,,\tag{23}$$

under constraint:

$$V_i^B(b_j)\le\nu.$$

5.4. Context Relative Aim

Let us look once again on two before defined strategies:

$$\check{b}_k(a_i)=\arg\max_j\Big\{\alpha\cdot V_i^B(b_j)-(1-\alpha)\cdot V_j^A(a_i)\Big\}\tag{24}$$

and

$$\widehat{b}_k(a_i)=\arg\max_j\Big\{\alpha\cdot V_i^B(b_j)+(1-\alpha)\cdot V_j^A(a_i)\Big\}.\tag{25}$$

It was said that antagonistic strategy (24) can be interpreted as aiming at maximizing the difference between the outcomes of player B and A. Altruistic strategy (25) can be interpreted as aiming at maximizing the sum of the outcomes of player B and A.

Looking on these we can simply formulate a strategy which in fact can't be unambiguously classified as antagonistic or altruistic, and which is not a irrational one. We mean of a strategy defined as minimization of the difference between payoffs:

$$b_k(a_i)=\arg\min_j\Big\{\alpha\cdot V_i^B(b_j)-(1-\alpha)\cdot V_j^A(a_i)\Big\}.\tag{26}$$

Minimization of the difference between payoffs seems to be an example of the altruistic strategy: a given player (B) is willing to decrease his own outcome and at the same time to increase the outcome of the other player, in order to ensure the smallest difference between them. However

Table 7
A game where minimization of the difference between the outcome of player B and A can't be interpreted as an altruistic aim

Strategies	b_1	b_2
a_1	[1, 1]	[2, 3]

in some cases such aim gives a solution which should be interpreted as a result of antagonistic or irrational aim. Let us consider a game with payoff matrix like in Table 7.

For the simplicity we assumed that there is only one strategy of player A. Aim defined by strategy (26) leads to the result: [1, 1], which minimizes the difference between the payoffs. However this solution can't be interpreted as a result of altruism of the player B. In fact the payoff of player A is worse than it would be for a [2, 3]. If however a payoff matrix would be like in Table 8, then aim defined by strategy (26) leads to [3, 3], which can be interpreted as a result of altruistic move of player B.

Table 8
A game where minimization of the difference between the outcome of player B and A can be interpreted as an altruistic aim

Strategies	b_1	b_2
a_1	[3, 3]	[2, 3]

So we see, that interpretation of strategy (26) depends on the form of the payoff matrix, and so is *context relative*. This can also lead to misleading the real motives of choosing given strategies by players.

5.5. Outside the Mathematical Structure of a Game: Malicious and Kind Aims

Now we ask an important question: are aims of the players (*antagonistic* or *altruistic*) a part of a *basic mathematical structure* of a game or are they outside it? Or in different way: can we transform games with such aims of players into games with new payoff function and *neutral* aims of the players? The answer seems to be ambiguous.

From one point of view we can say like that: the aim of a player can be simply expressed in values of a utility. In fact the above mentioned *antagonistic* and *altruistic* aims have defined real utility of any solutions, described in terms of utility with *neutral* aim.

Let us remind the before considered example. In a given game there are only two different results: [3, 4] and [1, 3]. The values reflect the profits in money of players A and B. Both players prefer to get more money than less, and their utility is proportional to the amount of gotten money (*neutral aim*). So such results expressed in terms of utility have the same form: [3, 4] and [1, 3]. However if for example player A aims into an *antagonistic* aim defined as aiming at maximization the difference between the outcomes of the players, than in fact he has different utility function (payoff function), defined as a difference between the utility values expressed with assumption of a *neutral* aim. So we can incorporate an *antagonistic* aim of player A into his payoff function, and treat this new situation as a game with *neutral* aims of the players. In such a game the solutions will be expressed in form $[3-4,\, 4]=[-1,\, 4]$ and $[1-3,\, 3]=[-2,\, 3]$. So we see that different than *neutral* aims of the players can be simply incorporated into a payoff function of a player and so can be treated as a part of a *basic mathematical structure* of a game.

However there are two aims of the players, which may cause a problem with incorporation them into a payoff function of a players, and so which seems to be outside the *basic mathematical structure* of a game. We call them: *malicious* and *kind* way of playing.

A *malicious* way of playing means that a given player *defines* his own aim as the opposite of the aim of the other player. A *kind* way of playing means that a given player *defines* his own aim as an exact realization of the other's player aim. Of special importance is here the word *defines*. Such word justifies why such way of playing are not called merely as *opposing* and *convergent*. For example, if both players are going to play in the least antagonistic way, and so aims at choosing the following strategies:

$$\breve{b}_k(a_i) = \arg\operatorname{lex}\max_j \left\{ V_i^B(b_j), -V_j^A(a_i) \right\}, \qquad (27)$$

$$\breve{a}_k(b_j) = \arg\operatorname{lex}\max_i \left\{ V_j^A(a_i), -V_i^B(b_j) \right\}, \qquad (28)$$

the aims:

$$\mathrm{aim}_B = \operatorname{lex}\max_j \left\{ V_i^B(b_j), -V_j^A(a_i) \right\},$$

$$\mathrm{aim}_A = \operatorname{lex}\max_i \left\{ V_j^A(a_i), -V_i^B(b_j) \right\}$$

are really *opposing*, but we can say that (for example) player A plays in a *malicious* way if he explicitly defines his aim as

$$\mathrm{aim}_A = \sim \mathrm{aim}_B\,,$$

where $\sim$ means *opposing of* (independently of in which way aim_B would be defined). Analogically we could say that player A plays in a *kind* way only if he explicitly defines his aim as

$$\mathrm{aim}_A = \mathrm{aim}_B\,.$$

Malicious and *kind* aims can be also incorporated into payoff functions of players. If for example player A aims at a *neutral* aim defined as maximizing of his own payoff function V^A, then a *malicious* aim of player B can be incorporated into his payoff function by putting $V^B = -V^A$, and treated this function as a function in game with a *neutral aim*. Analogically we can express a *kind* aim of player B by putting $V^B = V^A$. So we see that games with *malicious* aims can be treated as a *distributive* games (zero-sum, strictly competitive) with *neutral* aims, and games with *kind* aims as an *integrative* game (with no place for any *distribution*).

However a problem arises when both players would like to play in a *malicious* (or *kind*) way. How to define their aims in terms of the value of the payoff function? What is a *basic mathematical structure* of such a game, under assumption that players play in a *neutral* way? We cant find a satisfying answer on these questions.

6. Opposition and Convergence of Aims and Problems with Cooperation

Our earlier analysis appointed three important aspects that define the relation between players, and in fact define the real game:

1. The way of setting a final solution, which can be:
   - *cooperative*: players are able to conclude enforceable agreements outside the formal rules of the game,
   - *non-cooperative*: players are unable to conclude enforceable agreements outside the formal rules of the game.
2. The structure of the payoff matrix, that define the game as:
   - *distributive*: game with no place for integrative process (strictly competitive game, zero-sum or constant-sum game),
   - *integrative*: game with no place for distributive process (not competitive game),
   - *integrative-distributive*: there exists a place as for integration so for distribution.
3. The aims of the players, which can be:
   - *neutral*: a given player is interesting only in his own payoffs and aims at maximizing it,
   - *antagonistic*: a given player aims at minimizing the payoff function of the other player,
   - *altruistic*: a given player aims at maximizing the payoff function of the other player,
   - *irrational*: a given player aims at minimizing of his own payoff function,
   - *context-relative*: a given player aims at minimization of the difference between payoffs of the players,
   - *malicious*: a given player tries to thwart another player's plans,
   - *kind*: a given player tries to help in realizing another player's plans.

Now we will analyze the relation between such aspects.

Players decide to play or are forced (more precisely: should be forced) into playing in *cooperative* way only if it leads to more effective or more fair solution than gotten during a *non-cooperative* playing. Increasing effectiveness of the solution comes from *integrative* process. Increasing fairness of the solution is related with the *distributive* process. *Integration* as a process of increasing the "size of a cake" is profitable for both players. *Distribution*, as a process of dividing such "cake" always mean that the more one player gets the more the other should loose. So increasing the fairness of the solution always mean that during it one player will get more and the other will lose. So we can say, that *cooperation* would be always more simple and more natural in *integrative* games than in *distributive* games. In a *integrative-distributive* games *cooperation* will be desirable by a given player only if he hoped that he got more during an *integrative* process than he could probably lose during a *distributive* one. Of course a player will desire a *cooperation* if he hoped that he increases his payoffs also during a *cooperative-distributive* process.

So we can say, the more *integrative* structure of the payoff matrix the simpler *cooperation* between players. Analogically the more *distributive* structure of the payoff matrix the more difficult *cooperation*, the strongest incentive for one of the players to play in a *non-cooperative* way.

Interesting relation occurs between the aims of the players and the process of *cooperation*. Generally we can say: the more *convergent* aims of the players the simpler *cooperation*, the more *opposing* aims – the more difficult *cooperation*. So we should ask: which aims are convergent, and which are opposing?

It is obvious that if one player played in a *malicious* way then real *cooperation* would be impossible (one player would like to get exactly opposing solution then the other). If one player played in a *kind* way then *cooperation* would be very simple (both players would like to get exactly the same solution). Paradoxically when both players played in *kind* way then *cooperation* may be difficult because of problems with definition of real aim.

It is interesting that also *antagonistic* aims of the players can make *cooperation* simple. In fact *antagonistic* move of one player can be the most desirable from the other player point of view, so antagonistic aim can be treated not as an *opposing* but *convergent*.

Let us consider the following example.

The pay off matrix is like in Table 9.

Table 9
Convergent antagonistic aims

Strategies	b_1	b_2
a_1	[1, 3]	[3, 4]
a_2	[2, 4]	[4, 5]

Let players aim at realizing the following antagonistic goals:

- Player A: maximization of his own payoff function under constraint that the payoff of the player B will be not higher than 4.
- Player B: maximization of his own payoff function under constraint that the difference between the payoffs of the players will be not smaller than 2 (with advantage of the payoff of player A).

Under such assumption, both players would like to set a solution [2, 4]. Under such aims of the players it is the best result in this game for both players, so *cooperation* in this case would be very simple. Obviously *cooperation* would be also simple if both players played in neutral or altruistic way. In such cases both players would like to set a solution [4, 5]. However if player A played in antagonistic way and player B in neutral or altruistic, then *cooperation* could be difficult, because both players would like to set different solution.

This example shows also that in the case of antagonistic aims of both players we have something like *changing the meaning of effectiveness*: both players prefer [2, 4] over [4, 5].

The above discussed example shows us, that in some cases antagonism of the players can make *cooperation* simpler.

It is interesting, that in some cases cooperation with antagonistic aims of both players can be simpler even than in the case of altruistic aims of both of them (not only one of them). Let us consider the following example. The payoff matrix in a game is like in Table 10.

Table 10
Difficult cooperation in the case of altruism of the players

Strategies	b_1	b_2
a_1	[6, 5]	[2, 5]
a_2	[2, 4]	[5, 6]

The players can aim at antagonistic goals:

- Player A: maximization of his own payoff function under constraint that the payoff of the player B will be not higher than 4.
- Player B: maximization of his own payoff function under constraint that the difference between the payoffs of the players will be not smaller than 2 (with advantage of the payoff of player B).

If they are not afraid to disclose them, then they simply find a solution [2, 4], as a satisfying one.

However if the players aims at altruistic aims defined as maximizing the own payoff function under assumption that the other player's payoff would be not smaller than 5, then they would have a problem, which solution should be chose. Player A prefers [6, 5] and player B prefers [5, 6], and if they would not change theirs aims the negotiations may be very strong. In fact above defined antagonistic aims (in this game) were here more convergent than such altruistic aims. So we see that in some cases cooperation among antagonistic players may be simpler than between altruistic ones.

Our conclusion can be justified also in different way. If we transform a game with antagonistic or altruistic aims into a game with new payoff function (which reflects such aims) and neutral aims, then we find that cooperation was simple there where was only one effective solution (where there was no place for distribution). Analogically cooperation was difficult where in such new game (with neutral aims) there was more then one effective result.

7. Summary and Final Conclusions

As it was said in the introduction, cursory analysis of the telecommunications services market leads to conclusion, that the boundary between the issues of *competition* and *cooperation* runs the same way as the boundary between the retail and the wholesale market. Now we see that competition is not an opposing part to the cooperation: in fact these concepts comes from different "layers" of interaction between players. The concept of *cooperation* explains *the way of setting a final result*, while *competition* is in fact a *distribution* process and its existence depends on the *structure of the payoff matrix* and the *aims* of the players. So it is possible, that under some kinds of payoff functions and aims of the players real *competition* can take place as on the retail so on the wholesale markets. It is also possible (even if only theoretically), that competition take place *only* on (*cooperative*) wholesale market, because on (*non-cooperative*) retail market the structure of the payoff matrix and the aims of the players may make a place only for *integrative* process.

On the wholesale telecommunications services market *cooperation* – negotiations on the conditions of the interconnection – is necessary (network ought to be interconnected) and due to unequal distributed negotiations power – forced by the regulator. Our analysis of convergence and opposition of aims of the players shows that such cooperation may be – respectively to the form of payoff matrix and aims of the players – more or less "natural", simple to introducing. Intervention of regulator often stops on the level of *the way of setting a final solution*: players may and ought to negotiate a final solution. Sometimes such intervention changes also the form of the payoff function (e.g., by changing the structure of the cost function, or setting a limitations on the prices). However probably newer such intervention changes the aims of the players. So finally as course of *cooperation* so final result of a game stays difficult to predict.

Probably the most unexpected conclusion of our analysis is that in some cases cooperation between two players which aims at antagonistic goals may be simpler then between players which would like to play in an altruistic way. In fact, under our definition antagonism does not should mean malice of the players – though it might mean. Paradoxically, it is possible that some altruistic aims may express the most malicious way of playing.

Generally as *antagonism* so *altruism* mean that the main player's aim is not objective, but relative: the player evaluates obtained outcome of his payoff function not as an independent single criterion but in comparison to the other player's payoff function (double criteria of evaluation). In this sens, a player which aims at realizing of an antagonistic or altruistic goal doesn't have to know the other player's aim (possibly also antagonistic or altruistic), what should be necessary if he really aims at realizing malice (trying to thwart another player's plans) or kind (trying to help in realizing another player's plans) objective.

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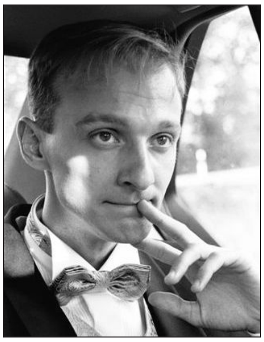

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