The New Analytical Model of Free Resources in the Limited Availability Resources

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Abstract — The article presents a new analytical model for determining the probability of availability of a certain amount of well-defined free resources (e.g. a link) in a group of dedicated resources (e.g. a group of links) jointly serving a mixture of different classes of multiservice traffic. The presented method can be used to model access to resources in data centers, in particular in the software-defined networks, assuming – for reliability reasons – that the user should have access to a certain minimum number of specific separated resources. The proposed analytical model was verified by appropriate simulation experiments, which confirmed the satisfactory accuracy of the results obtained.

Keywords — analytical modeling, limited-availability group, multiservice system, networks

1. Introduction

Design process of today's ICT networks should take into account the diversity of services supported and their quality requirements [1]. Managing the allocation of resources in network systems and mitigating the risks associated with fluctuations in network traffic are critical challenges for operators operating in dynamic and demanding environments [2]–[3]. Load balancing, the art of distributing network traffic between multiple resources, plays a key role in ensuring optimal utilization of network systems, maintaining adequate service responsiveness and delivering a service with guaranteed quality parameters to the end user [4]–[6]. Similarly, the identification of potential risks and the implementation of appropriate mitigation measures are important for the smooth delivery of services and minimizing the impact of unpredictable and sudden spikes in resource access requests [7]–[11].

In the process of network design, it is important to be able to quantitatively assess the traffic that can be served by the network with given quality parameters [12]–[13]. For this purpose, appropriately developed analytical models of network systems (e.g. nodes, links, groups of links) can be used. This paper proposes a new analytical model of a group of separated resources, allowing to determine the distribution of well-defined free separated resources in a group of separated resources serving multiservice traffic.

In the notation adopted in the article, separated resources or full resources are understood as resources that can handle a new request as long as they have free throughput. An example of such a system could be a single link. The notion of a group of separated resources characterizes a system, consisting of e.g. several links, in which it is assumed that a new request can be serviced only if the conditions for its servicing exist in one of the component links (i.e. single separated resources) of the group. This means that, in such a system, the throughput required by a request cannot be "distributed" between resources. In the literature, such a system is referred to as limited availability resources (LAR), or limited availability group (LAG) [14]–[16].

The article is divided into two main parts. The first part (Section 1) presents a description of multiservice traffic and an analytical model of full-availability resources and limited-availability resources serving a mixture of multiservice traffic. This is followed by the presentation of a new analytical model to assess the availability of selected resources in limited-availability resources. The second part (Section 2) presents the results of a comparison between the proposed analytical model and the results of simulation experiments. The article concludes with a brief summary in Section 4.

2. The Analytical Model

This section presents the basic analytical relationships used in the proposed constrained resource model. First, how to model multiservice traffic is presented, followed by the basic resource models and constrained resources in a multiservice network. Later, a new distribution in constrained resources, i.e. the distribution of strictly free resources, is presented.

2.1. Traffic Offered in a Multiservice Network

We assume that the system under consideration supports a mixture of Engset, Erlang, and Pascal traffic, referred to as BPP traffic (the abbreviation comes from the streams of requests specific to the traffic type: Binomial-Poisson-Pascal), which consists of a set of M_{En} Engset traffic classes, set of M_{Er} Erlang traffic classes and set of M_{Pa} Pascal traffic classes. We also assume that the set $M = M_{En} \cup M_{Er} \cup M_{Pa}$ denotes the set of all traffic classes served by the network. The intensities of the individual traffic types included in the BPP mix are variously dependent on the occupancy status of

the system. The busy state is determined by the number of busy allocation units (AUs). The AU is expressed in Kbps and is calculated on the basis of the greatest common divisor of the throughput of all traffic classes served in a given system.

For Erlang traffic of class i, the average traffic offered $A_{\mathrm{Er}}(i,n)$ in the occupied state n AUs is state-independent, therefore:

$$\underset{i\in\mathbf{M}_{\mathrm{Er}}}{\forall}\ \underset{1\leqslant n\leqslant f}{\forall}\ A_{\mathrm{Er}}(i,n)=A_{\mathrm{Er}}(i)\,, \qquad (1)$$

where f is the capacity of the resources under consideration. For Engset classes, the traffic volume $A_{\mathrm{En}}(j,n)$ decreases, while for Pascal traffic classes the volume $A_{\mathrm{Pa}}(l,n)$ increases as the occupancy state of the system n increases:

$$\underset{j\in\mathbf{M}_{\mathrm{En}}}{\forall}\ \underset{1\leqslant n\leqslant f}{\forall}\ A_{\mathrm{En}}(j,n)=\alpha_{\mathrm{En}}(j)\big[S_{\mathrm{En}}(j)-y_{\mathrm{En}}(j,n)\big]\,, \quad (2)$$

$$\underset{l\in\mathbf{M}_{\mathrm{Pa}}}{\forall}\ \underset{1\leqslant n\leqslant f}{\forall}\ A_{\mathrm{Pa}}(l,n)=\alpha_{\mathrm{Pa}}(l)\big[S_{\mathrm{Pa}}(l)+y_{\mathrm{Pa}}(l,n)\big]\,, \quad (3)$$

where $S_{\mathrm{X}}(c)$ denotes the number of traffic sources of class c ($c\in\mathbf{M}$) of type X ($X\in\mathrm{En},\mathrm{Er},\mathrm{Pa}$).

The parameter $\alpha_{\mathrm{X}}(c)$ specifies the average traffic volume generated by one free traffic source of class c of type X. The parameter $y_{\mathrm{X}}(c,n)$ in turn determines the average number of class c requests of type X that are served in a state of n busy AUs.

The average offered traffic volume of class j ($j\in\mathbf{M}_{\mathrm{En}}$) of type Engset $A_{\mathrm{En}}(j)$ and the average offered traffic volume of class l ($l\in\mathbf{M}_{\mathrm{Pa}}$) of type Pascal $A_{\mathrm{Pa}}(l)$ are defined by:

$$\underset{j\in\mathbf{M}_{\mathrm{En}}}{\forall}\ A_{\mathrm{En}}(j)=\frac{\alpha_{\mathrm{En}}(j)\,S_{\mathrm{En}}(j)}{1+\alpha_{\mathrm{En}}(j)}\,, \qquad (4)$$

$$\underset{j\in\mathbf{M}_{\mathrm{Pa}}}{\forall}\ A_{\mathrm{Pa}}(l)=\frac{\alpha_{\mathrm{Pa}}(l)\,S_{\mathrm{Pa}}(l)}{1-\alpha_{\mathrm{Pa}}(l)}\,. \qquad (5)$$

2.2. Separated Resources in a Multiservice Network

Separated resources (e.g. a single link) serving multiservice traffic can be described on the basis of the full availability resources (FAR) model, also often referred to as FAG. The model assumes that all free allocation units are available for incoming new requests. This means that a resource will serve a new request whenever it has a sufficient number of free AUs in a given busy state necessary to serve a given request [17]. Assume that the resource handles a mixture of BPP traffic. For such a system, the following occupancy distribution has been proposed in [16]:

$$
\begin{aligned}
\big[P(n)\big]_f=\frac{1}{n}\Bigg\{ &\sum_{\mathbf{M}_{\mathrm{Er}}} A_{\mathrm{Er}}(i)\,t_{\mathrm{Er}}(i)\,\big[P\big(n-t_{\mathrm{Er}}(i)\big)\big]_f \\
&+\sum_{\mathbf{M}_{\mathrm{En}}}\alpha_{\mathrm{En}}(j)\big[S_{\mathrm{En}}(j)-y_{\mathrm{En}}\big(j,n-t_{\mathrm{En}}(j)\big)\big] \\
&\qquad\cdot t_{\mathrm{En}}(j)\,\big[P\big(n-t_{\mathrm{En}}(j)\big)\big]_f \\
&+\sum_{\mathbf{M}_{\mathrm{Pa}}}\alpha_{\mathrm{Pa}}(l)\big[S_{\mathrm{Pa}}(l)+y_{\mathrm{Pa}}\big(l,n-t_{\mathrm{Pa}}(l)\big)\big] \\
&\qquad\cdot t_{\mathrm{Pa}}(l)\,\big[P\big(n-t_{\mathrm{Pa}}(l)\big)\big]_f\Bigg\}. \quad (6)
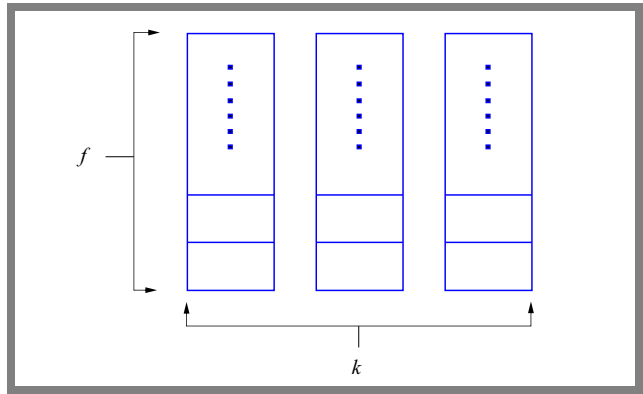\end{aligned}
$$

Fig. 1. Conceptual diagram of LAR consisting of $k=3$ links of f AUs capacity.

In describing the relevant parameters of the distribution (6), the notation used in Subsection 2.2 has been adopted, which will also be used consistently in the following sections, i.e., $c\in\mathbf{M}$, $i\in\mathbf{M}_{\mathrm{Er}}$, $j\in\mathbf{M}_{\mathrm{En}}$, $l\in\mathbf{M}_{\mathrm{Pa}}$, $X\in\{\mathrm{En},\mathrm{Er},\mathrm{Pa}\}$. The parameter $\big[P(n)\big]_f$ denotes the probability of occupying n AUs in a resource of f AUs. The parameter $t_{\mathrm{X}}(c)$ specifies the number of AUs requested to process a class c request of type X, while $y_{\mathrm{X}}(c,n)$ specifies the average number of class c requests of type X that are served in a state of n busy AUs:

$$
\begin{aligned}
y_{\mathrm{En}}\big(j,n\big)= & \\
\frac{\alpha_{\mathrm{En}}(j)\big[S_{\mathrm{En}}(j)-y_{\mathrm{En}}\big(j,n-t_{\mathrm{En}}(j)\big)\big]\big[P\big(n-t_{\mathrm{En}}(j)\big)\big]_f}{\big[P(n)\big]_f}\,,
\end{aligned} \quad (7)
$$

$$
\begin{aligned}
y_{\mathrm{Pa}}\big(l,n\big)= & \\
\frac{\alpha_{\mathrm{Pa}}(l)\big[S_{\mathrm{Pa}}(l)+y_{\mathrm{Pa}}\big(l,n-t_{\mathrm{Pa}}(l)\big)\big]\big[P\big(n-t_{\mathrm{Pa}}(l)\big)\big]_f}{\big[P(n)\big]_f}\,.
\end{aligned} \quad (8)
$$

The blocking probability for requests of class c of type X is due to the lack of a sufficient number of free allocation units in the resource. It can be expressed by the following formula:

$$E(c)=\sum_{n=f-t_{\mathrm{X}}(c)+1}^{f}\big[P(n)\big]_f\,. \qquad (9)$$

The occupancy distribution (6) includes the parameters $y_{\mathrm{En}}(j,n)$ and $y_{\mathrm{Pa}}(l,n)$ defined by the formulas (7) and (8). They are also determined by the occupancy distribution (6). Thus, an iterative algorithm is required to determine the distribution (6). In each iteration step, the algorithm determines an approximation of the distribution (6) based on the mean values of the number of supported notifications of the Engset classes $y_{\mathrm{En}}(j,n)$ and Pascal's $y_{\mathrm{Pa}}(l,n)$, which in turn were determined in the previous iteration step. A detailed description of the algorithm for the computation of the occupancy distribution of (6) can be found in the paper [16].

2.3. Resources with Limited Availability

A limited-access resource LAR is a model of k full-access, separated resources (e.g. links), each with a capacity of f AUs. Figure 1 shows a conceptual diagram of a LAR. The model assumes that a new request can only be accepted for service if there is at least one separated resource that can

completely service the request. The algorithm for accepting new requests for service excludes the possibility of splitting the requested allocation units into several constituent, separated resources [14]–[15].

The occupancy distribution of LARs can be determined by the following formula [16]:

$$
\begin{aligned}
\big[P(n)\big]_f = \\
\frac{1}{n}\Bigg\{ \sum_{\mathbf{M}_{\text{Er}}} A_{\text{Er}}(i) t_{\text{Er}}(i) \sigma_{\text{Er}}\big(i, n - t_{\text{Er}}(i)\big)\big[P\big(n - t_{\text{Er}}(i)\big)\big]_{kf} \\
+ \sum_{\mathbf{M}_{\text{En}}} \alpha_{\text{En}}(j)\big[S_{\text{En}}(j) - y_{\text{En}}\big(j, n - t_{\text{En}}(j)\big)\big] t_{\text{En}}(j) \\
\cdot \sigma_{\text{En}}\big(j, n - t_{\text{En}}(j)\big)\big[P\big(n - t_{\text{En}}(j)\big)\big]_{kf} \\
+ \sum_{\mathbf{M}_{\text{Pa}}} \alpha_{\text{Pa}}(l)\big[S_{\text{Pa}}(l) + y_{\text{Pa}}\big(l, n - t_{\text{Pa}}(l)\big)\big] t_{\text{Pa}}(l) \\
\cdot \sigma_{\text{Pa}}\big(l, n - t_{\text{Pa}}(l)\big)\big[P\big(n - t_{\text{Pa}}(l)\big)\big]_{kf} \Bigg\},
\end{aligned}
\tag{10}
$$

where $[P(n)]_{kf}$ is the probability of occupancy of n AUs in a LAR with a capacity of kf AUs.

The parameters $\alpha_X(c), t_X(c), S_X(c)$ have the same interpretation as in the distribution (6). In turn, $y_X(c, n)$ is the average number of requests, of class c of type X, served in state n in the LAR:

$$
\begin{aligned}
y_{\text{En}}(j, n) = \frac{\alpha_{\text{En}}(j)\big[S_{\text{En}}(j) - y_{\text{En}}\big(j, n - t_{\text{En}}(j)\big)\big]}{\sqrt{\big[P(n)\big]_{kf}}} \\
\cdot \frac{\sigma_{\text{En}}\big(j, n - t_{\text{En}}(j)\big)\big[P\big(n - t_{\text{En}}(j)\big)\big]_{kf}}{\sqrt{\big[P(n)\big]_{kf}}},
\end{aligned}
\tag{11}
$$

$$
\begin{aligned}
y_{\text{Pa}}(l, n) = \frac{\alpha_{\text{Pa}}(l)\big[S_{\text{Pa}}(l) + y_{\text{Pa}}\big(l, n - t_{\text{Pa}}(l)\big)\big]}{\sqrt{\big[P(n)\big]_{kf}}} \\
\cdot \frac{\sigma_{\text{Pa}}\big(l, n - t_{\text{Pa}}(l)\big)\big[P\big(n - t_{\text{Pa}}(l)\big)\big]_{kf}}{\sqrt{\big[P(n)\big]_{kf}}}.
\end{aligned}
\tag{12}
$$

As in the case of FAR with multiservice traffic [12], [17]–[18], the LAR occupancy distribution (10) includes the parameters $y_{\text{En}}(j, n)$ and $y_{\text{Pa}}(l, n)$ defined by the formulae (11) and (12). These parameters are also determined from the occupancy distribution (10). Therefore, an iterative algorithm is required to determine the distribution (10). In each iteration step of the algorithm, an approximation of the distribution (10) is determined based on the average values of the number of served requests of the Engset and Pascal classes, which in turn were determined in the previous iteration step. A detailed description of the algorithm can be found in [16].

The parameter $\sigma_X(c, n)$, occurring in (10), (11) and (12), is the so-called conditional transition probability between adjacent states n and $n + t_X(c)$ of a process for handling requests of class c of type X. This parameter, for a state n of busy AUs, determines the probability of such a distribution of free AUs in the LAR that allows a new class c [14] request

to be served:

$$
\sigma_X(c, n) = \frac{F\big(kf - n, k, f, 0\big) - F\big(kf - n, k, t_X(c) - 1, 0\big)}{F\big(kf - n, k, f, 0\big)}.
\tag{13}
$$

The function $F(x, k, f, h)$ determines combinatorially the number of distributions of x free AUs in k separated component resources, each with a capacity of f AUs. In addition, it is assumed that there are h free AUs in each component resource:

$$
\begin{aligned}
F(x, k, f, h) = \\
\sum_{z=0}^{\left\lfloor \frac{x - kh}{f - h + 1} \right\rfloor} (-1)^z \binom{k}{z}\binom{x - k(h - 1) - 1 - z(f - h + 1)}{k - 1}.
\end{aligned}
\tag{14}
$$

2.4. Distribution of Strictly Defined Free Resources in LAR

The distribution presented above (10) is the basis for defining the distribution $[W(c, s)]_{kf}$, i.e. the probability that s of the separated component resources of the LAR can support a new class submission c. This probability will also be referred to hereafter as the probability of the availability of the selected s component resources for class submissions.

The distribution of $[W(c, s)]_{kf}$ can be determined by the following reasoning: let the distribution of $[H(c, s|n)]_{kf}$ determine the conditional probability that s of the selected, separated component resources of the LAR can handle class c requests, provided that there are occupied n AUs in the LAR. Assume that x AUs, out of n belong to s selected component resources. These resources will remain available for c class requests until their maximum occupancy exceeds $(f - t_c)$ AUs. Thus, the number of possible favorable distributions x of AUs in the s selected component resources can be given by the function $F(x, s, f - t_X(c), 0)$. These distributions correspond to arbitrary distributions of the remaining $(n - x)$ AUs in the remaining $(k - s)$ separated resources: $F(n - x, k - s, f, 0)$. The total number of favorable distributions of n AUs in the LAR is thus given by the product $F(x, s, f - t_X(c), 0)F(n - x, k - s, f, 0)$. In turn, the total number of all possible distributions of n AUs in all k component resources of the LAR is $F(n, k, f, 0)$.

Thus, given all possible values of the parameter x, it is possible to determine the probability $[H(c, s|n)]_{kf}$ as follows:

$$
\begin{aligned}
\big[H(c, s|n)\big]_{kf} = \\
\sum_{x=0}^{min\{s(f - t_X(c)), n)\}} \frac{F\big(x, s, f - t_X(c), 0\big)F\big(n - x, k - s, f, 0\big)}{F\big(n, k, f, 0\big)}.
\end{aligned}
\tag{15}
$$

Now, taking into account all possible occupancy states n $(0 \leqslant n \leqslant kf)$, it is possible to determine the probability $[W(c, s)]_{kf}$ of availability s of selected component resources in the LAR for class c submissions based on the formula:

$$
\big[W(c, s)\big]_{kf} = \sum_{n=0}^{kf} \big[H(c, s|n)\big]_{kf}\big[P(n)\big]_{kf}.
\tag{16}
$$

In summary, the following calculation method can be used to determine the probability of availability s of selected component resources in the LAR for class c requests (of Erlang, Engset, or Pascal type):

1) Introduction of data on the traffic offered: sets of classes of individual traffic types $\mathbf{M}_{En}$, $\mathbf{M}_{Er}$, $\mathbf{M}_{Pa}$ with the relevant parameters: $\alpha_X(c)$, $S_X(c)$, $t_X(c)$ ($X \in \{En, Er, Pa\}$) for Engset traffic ($c \in \mathbf{M}_{En}$), and Pascal traffic ($c \in \mathbf{M}_{Pa}$) and $A_{Er}(c)$, $t_{Er}(c)$ for Erlang traffic ($c \in \mathbf{M}_{Er}$);

2) Introduction of LAR structural parameters, i.e. the number of constituent resources k and their capacity f;

3) Determination of the occupancy distribution of the LAR based on the formula (10);

4) Determination, based on the relation (15), of the conditional probability $[H(c, s|n)]_{kf}$ that s of selected separated LAR resources are available for class c requests, provided that the total number of occupied AUs in the LAR is equal to n;

5) Determination, based on the formula (16), of the unconditional probability $[W(c, s)]_{kf}$ that s of selected separated LAR resources can support a class c request.

3. Numerical example

This section presents the results obtained from the proposed analytical model and the results of the corresponding simulation experiments. The objective of the research undertaken was to verify the accuracy of the analytical model in the context of the offered motion and structural parameters of the LAR.

The results of the study are shown in the following graphs showing the probability of availability $[W(c, s)]_{kf}$ – formula (16) – as a function of the traffic volume a offered per allocation unit of system:

$$a = \sum_{c \in \mathbf{M}} \frac{A_X(c)\, t_X(c)}{kf}, \tag{17}$$

where $X \in \{En, Er, Pa\}$ and $\mathbf{M} = \mathbf{M}_{En} \cup \mathbf{M}_{Er} \cup \mathbf{M}_{Pa}$.

To evaluate the performance of the proposed analytical model, a series of simulation experiments were conducted, each consisting of seven independent runs. The length of each simulation run was set at $100,000$ system time units, and 500 time units were taken as the system stabilization time. The results of the simulations were statistically analyzed to determine 95% confidence intervals. The values of the determined confidence intervals are always smaller than the symbols determining the simulation results.

The results of the study are presented in the form of plots comparing the calculations with the simulation method depending on the average traffic volume per allocation unit of the system, determined on the basis of Eq. (17). The following designations have been adopted for the graphs: Cm for the results of analytical calculations and Sm for the corresponding simulation results for calls of class m. The study was carried out for five selected LAR systems, which differ in capacity and in the structure and type of traffic offered:
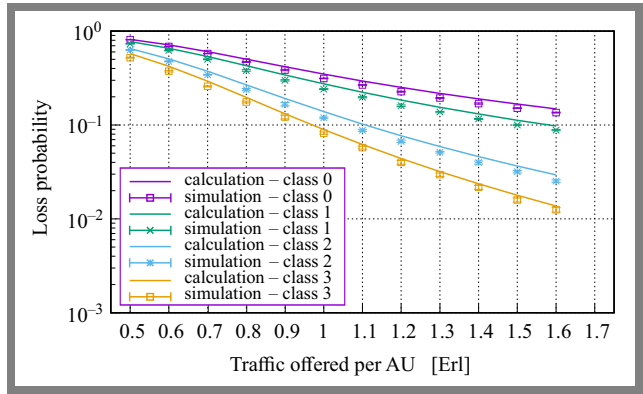

Fig. 2. Probability of availability $s = 2$ of specific component resources in System 1.
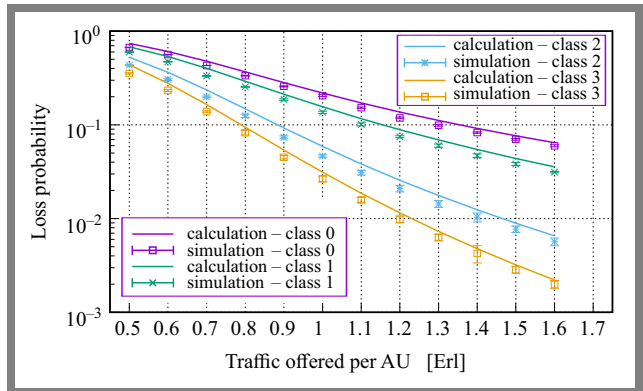

Fig. 3. Probability of availability $s = 3$ of specific component resources in System 2.

- System 1: $f = 40$ AUs, $k = 4$, $s = 2$, $t_{Er\,(class\,0)} = 4$ AUs, $t_{Er\,(class\,1)} = 5$ AUs, $t_{Er\,(class\,2)} = 8$ AUs, $t_{Er(class\,3)} = 10$ AUs.

- System 2: $f = 40$ AUs, $k = 4$, $s = 3$, $t_{Er\,(class\,0)} = 4$ AUs, $t_{Er\,(class\,1)} = 5$ AUs, $t_{Er\,(class\,2)} = 8$ AUs, $t_{Er\,(class\,3)} = 10$ AUs.

- System 3: $f = 20$ AUs, $k = 4$, $s = 2$, $t_{Er\,(class\,0)} = 4$ AUs, $t_{Er\,(class\,1)} = 5$ AUs, $t_{Er\,(class\,2)} = 8$ AUs, $t_{Er\,(class\,3)} = 10$ AUs.

- System 4: $f = 40$ AUs, $k = 4$, $s = 2$, $t_{Er\,(class\,0)} = 5$ AUs, $t_{Er\,(class\,1)} = 8$ AUs, $t_{Er\,(class\,2)} = 10$ AUs, $t_{Er\,(class\,3)} = 20$ AUs.

- System 5: $f = 40$ AUs, $k = 4$, $s = 2$, $t_{Er\,(class\,0)} = 4$ AUs, $t_{En\,(class\,1)} = 5$ AUs, $Sources_{En\,(class\,1)} = 60$, $t_{Pa\,(class\,2)} = 8$ AUs, $Sources_{Pa} = 60$, $t_{Er\,(class\,3)} = 10$ AUs.

An equal number of constituent resources forming the LAR was assumed in all systems ($k = 4$). It was also assumed that traffic is offered to each system in the following proportion:

$$A_X(1)\, t_X(1) : A_X(2)\, t_X(2) : A_X(3)\, t_X(3) : A_X(4)\, t_X(4)$$
$$= 1 : 1 : 1 : 1. \tag{18}$$

The analyzed probabilities of availability s of selected LAR component resources for class-specific requests in System 1, are shown in Fig. 2. As expected, the probability of availability $s = 2$ of selected component resources decreases as the
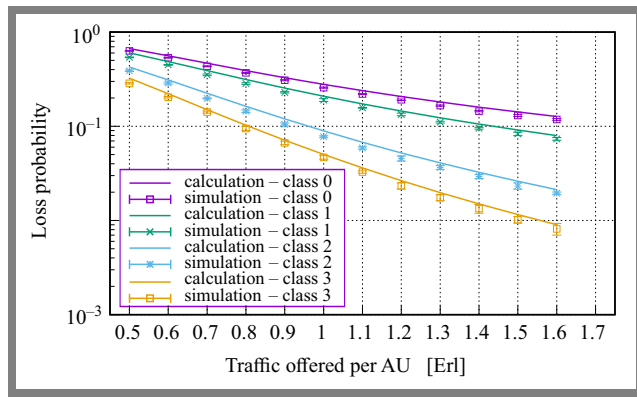
Fig. 4. Probability of availability $s = 2$ of specific component resources in System 3.

Fig. 5. Probability of availability $s = 2$ of specific component resources in System 4.

Fig. 6. Probability of availability $s = 2$ of specific component resources in System 5.

volume of traffic a offered to one allocation unit increases. The lowest probability of availability of the selected resources is obtained for class three (graphs *class* 3), which requests the largest number of AUs to handle the request $t_{Er}(4) = 10$. In contrast, request of class zero (graphs *class* 0) are most likely to complete calls in the selected LAR component resources. The results for System 2 are shown in Fig. 3. In this case, the number of selected component resources was changed in the system ($s = 3$). Observing the graphs shown, it is easy to see that for low traffic volumes ($a < 0.7$ Erl), the results obtained differ little from those of System 1 (Fig. 3). As the traffic volume a increases, the differences become more significant and for high traffic volumes ($a > 1.4$ Erl), a two- or even threefold reduction in the probability of availability of $s = 3$ selected component resources can be observed compared to System 1, in which $s = 2$ component resources were selected. In System 3 (Fig. 4), the capacity of the separated resource is reduced twice (relative to System 1) to $f = 20$. In System 4 (Fig. 5), the capacity of the separated resource is identical to System 1 ($f = 40$), while the number of AUs requested by the submissions of each class is larger. Comparing Systems 1, 3 and 4, it can be seen that increasing the requests of individual classes has a much greater impact on the probability of availability of the selected component resources than reducing the capacity of the component resources. Reducing the capacity of the component resources by half (System 3) did not result in major changes in availability probabilities, while increasing the number of requested AUs for individual class requests (System 4) resulted in a significant decrease in availability probabilities.

Figure 6 shows the results of the tests of System 5. The system is characterized by identical structural parameters and requests of the different traffic classes as System 1, while the traffic types for the *class* 1 class (from Erlang to Engset) and *class* 2 (from Erlang to Pascal) were changed. This change resulted in a slight decrease in the probability of availability of selected component resources for the Pascal-type traffic class *class* 2, compared to the Erlang-type traffic offered in System 1 (Fig. 2). In contrast, the availability probability for the Engset type traffic class *class* 1 is slightly higher compared to the Erlang type traffic offered in System 1.
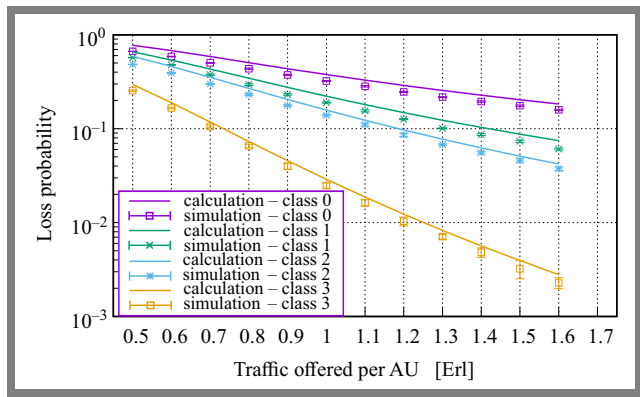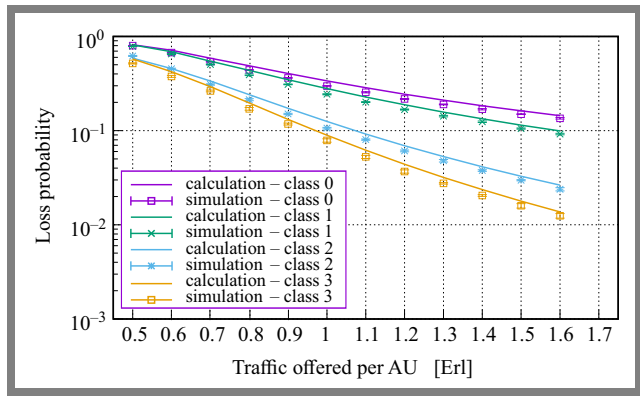
4. Conclusions

The article presents a new analytical model for assessing the probability of availability s of selected constituent resources in offered resources. The proposed analytical model has been subjected to a number of tests, which have confirmed its satisfactory accuracy for different configurations of LAR structural parameters and different mixtures offered traffic.

Given the many parameters and their interdependencies, the model described can be used in ICT network design and optimization work and by network operators when analyzing the amount of resources that need to be provided in order to ensure the uninterrupted provision of services of a certain quality.

This model can find it's applications in distributed databases that horizontally partition data across multiple servers or shards. For example when a complex query involves aggregating data from multiple shards, it requires the simultaneous availability of the relevant servers to process the query efficiently. This is common in analytical queries that span different partitions of the database. Another application of the model can be found in distributed systems employing consensus algorithms, the simultaneous availability of a majority of nodes is crucial for reaching a consensus on important decisions or ensuring the replication of data across the nodes for fault tolerance. Applications of this model can also be found

in content delivery networks (CDN). CDNs often distribute multimedia content globally to users. When a user requests a video stream, the CDN may fetch and deliver content from multiple edge servers simultaneously. Each server could be responsible for delivering a different segment of the video. The availability of N servers simultaneously ensures a seamless and uninterrupted streaming experience.

The examples mentioned above confirm the considerable application potential of the model as a tool for modelling, dimensioning and optimizing resources.

Acknowledgments

References

[1] P. Andruloniw, K. Kowalik, B. Partyka, and P. Zwierzykowski, "Multi-layer QoE Learning System Implemented by Fiberhost", *Applied Sciences*, vol. 13, no. 4, art. no. 2300, 2023 (https://doi.org/10.3390/app13042300).

[2] K. Kowalik, P. Andruloniw, B. Partyka, and P. Zwierzykowski, "Telecom Operator's Approach to QoE", *Journal of Telecommunications and Information Technology*, no. 2, pp. 26–34, 2022 (https://doi.org/10.26636/jtit.2022.159522).

[3] B. Nowak, M. Piechowiak, M. Stasiak, and P. Zwierzykowski, "An Analytical Model of a System with Priorities Servicing a Mixture of Different Elastic Traffic Streams", *Bulletin of the Polish Academy of Sciences. Technical Sciences*, vol. 68, no. 2, pp. 263–270, 2020 (https://doi.org/10.24425/bpasts.2020.133111).

[4] D. Aguilar, L. Erazo, P. Leon, and C. Aguilar, "Electronic Prototype for Measuring Network Parameters on the Service Offered Delivered by ISP", in: *2016 IEEE International Conference on Automatica (ICA–ACCA)*, Curico, Chile, 2016 (https://doi.org/10.1109/ICA-ACCA.2016.7778428).

[5] S. Hanczewski, M. Stasiak, and M. Weissenberg, "A Multiparameter Analytical Model of the Physical Infrastructure of a Cloud-Based System", *IEEE Access*, vol. 9, pp. 100981–100990, 2021 (https://doi.org/10.1109/ACCESS.2021.3097157).

[6] M. Stasiak, M. Sobieraj, and P. Zwierzykowski, "Modelling of Multi-service Switching Networks with Multicast Connections", *IEEE Access*, vol. 10, pp. 5359–5377, 2022 (https://doi.org/10.1109/ACCESS.2022.3141689).

[7] A. Al-Darrab, I. Al-Darrab, and A. Rushdi, "Software-defined Networking Load Distribution Technique for an Internet Service Provider", *Journal of Network and Computer Applications*, vol. 155, art. no. 102547, 2020 (https://doi.org/10.1016/j.jnca.2020.102547).

[8] W. Jiang, M. He, and W. Gu, "Internet Traffic Prediction with Distributed Multi-agent Learning", *Applied System Innovation*, vol. 5, no. 6, art. no. 121, 2022 (https://doi.org/10.3390/asi5060121).

[9] N. Ferdosian, M. Othman, K.Y. Lun, and B.M. Ali, "Overload-state Downlink Resource Scheduling and its Challenges Towards 5G Networks", in: *2016 IEEE 21st International Workshop on Computer Aided Modelling and Design of Communication Links and Networks (CAMAD)*, Toronto, Canada, 2016 (https://doi.org/10.1109/CAMAD.2016.7790349).

[10] T. Uhl, J. Klink, and C. Hoppe, "Suitability of Transport Techniques for Video Transmission in IP Networks", *International Journal of Electronics and Telecommunications*, vol. 66, no. 1, pp. 133–138, 2020 (https://doi.org/10.24425/ijet.2020.131854).

[11] D. Strzeciwilk, "Performance Analysis of VoIP Data over IP Networks", *International Journal of Electronics and Telecommunications*, vol. 67, no. 4, pp. 743–750, 2021 (https://doi.org/10.24425/ijet.2021.139801).

[12] M. Stasiak, M. Głąbowski, A. Wiśniewski, and P. Zwierzykowski, *Modeling and Dimensioning of Mobile Wireless Networks: From GSM to LTE*, John Wiley & Sons, 315 p., 2010 (https://doi.org/10.1002/9780470976036).

[13] P. Zwierzykowski, *Modeling Traffic Management Mechanisms in Multi-service Mobile Networks* (in Polish), Habilitation Thesis, Department of Electronics and Telecommunications, Poznan University of Technology, 2014.

[14] M. Stasiak, "Blocking Probability in a Limited-availability Group Carrying Mixture of Different Multichannel Traffic Streams", *Annales Des Télécommunications*, vol. 48, no. 1–2, pp. 71–76, 1993 (https://doi.org/10.1007/BF03005233).

[15] M. Głąbowski and M. Stasiak, "Multi-rate Model of the Group of Separated Transmission Links of Various Capacities", in *Telecommunications and Networking – ICT 2004*, pp. 1101–1106 (https://link.springer.com/chapter/10.1007/978-3-540-27824-5_143).

[16] M. Głąbowski, S. Hanczewski, M. Stasiak, and J. Weissenberg, "Modeling Erlang's ideal grading with multi-rate BPP traffic", *Mathematical Problems in Engineering*, vol. 2012, pp. 1–35, 2012 (https://doi.org/10.1155/2012/456910).

[17] J. Roberts, "A service system with heterogeneous user requirements – application to multi-service telecommunications systems", in: *Proceedings of Performance of Data Communications Systems and their Applications*, pp. 423–431, 1981.

[18] M. Logothetis and I. Moscholios, *Efficient Multirate Teletraffic Loss Models Beyond Erlang*, John Wiley & Sons, 498 p., 2019 (https://doi.org/10.1002/9781119426974).

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