A Convex Optimization-based Approach for Sidelobe Level Suppression and Null Control in Antenna Arrays by Displacing a Minimum Number of Elements

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https://doi.org/10.26636/jtit.2025.3.2198

Abstract — This paper introduces two methods for peak sidelobe level (PSLL) reduction and null steering in the pattern of linear arrays using position control. While most research on this topic uses stochastic optimization techniques, here convex optimization and the off-grid compressive sensing framework were used to accomplish the required goals. For the first method, the problem of minimizing the PSLL and forming prescribed nulls in the pattern of linear arrays by controlling the elements' positions is cast as a convex optimization problem with the help of first-order Taylor approximation. For the second method, the goals are achieved by perturbing the locations of as few array elements as possible. Towards this end, the problem of forming prescribed nulls in the pattern of non-uniformly spaced linear arrays for a predefined PSLL by elements' position control is formulated as a sparse recovery problem within the off-grid compressive sensing framework. Simulations were performed to evaluate the efficacy of the proposed methods, and the results were compared to results obtained using stochastic optimization techniques.

Keywords — compressive sensing, convex optimization, mechanically adaptive arrays

1. Introduction

In phased array antenna, the radiation pattern can be altered so that the radiation pattern adds up to boost the radiation in the wanted direction while canceling out the radiation in the undesired directions. Numerous algorithms have been studied to create radiation pattern nulls by changing the excitation of elements' amplitude only [1]–[3], phase only [4], [5], amplitude and phase (complex) [6]–[8], or inter-element spacing [9]–[11].

Compressed sensing (CS) is a fairly recent signal processing method to sample and reconstruct signals efficiently by obtaining solutions of underdetermined linear systems [12]. The potential to defy established wisdom in data acquisition based on Shannon's theory [13] and permit the recovery of specific signals from much less observations than standard approaches has received much attention [14], [15].

The cornerstone of CS-based approaches is that a lot of physical variables, both intrinsically or extrinsically sparse, can be portrayed using just a few of nonzero expansion coefficients, given the appropriate expansion bases. The basic objective of CS methods is to determine an approximation of the solution $\mathbf{x}$ to the linear system $\mathbf{y} = \mathbf{A}\mathbf{x}$, where $\mathbf{x}$ must have as few nonzero elements as possible [12].

The evolution of CS has centered on signals having sparse representation in finite discrete dictionaries. The majority of signals we encounter in the actual world, particularly those used in remote sensing, sonar and radar, are characterized by continuous parameters. A discretization process is used to create a finite collection of grid points from the continuous parameter space in order to apply CS theory. When the true signal is not precisely supported on the grid points, performance loss occurs for the traditional CS methods, also known as on-grid CS. This problem is referred to as the basis mismatch problem [16].

Off-grid CS techniques aim to address the basis mismatch issues without trying to solve the problem by using denser discretization, since CS theory suggests that utilizing a finer grid could not improve performance and even might increase the coherence of the dictionary, which contradicts the restricted isometry property necessary for guaranteeing accurate estimation of sparse recovery problems [17], [18].

Off-grid CS framework has been used to synthesize uniformly weighted concentric ring arrays in [19]. A method based on off-grid CS for the synthesis of planar sparse arrays was proposed in [20]. In [21], an alternating algorithm to synthesis planar sparse antenna arrays with complex-excitation and reconfigurable pattern was proposed.

Most research on the topic of optimizing antenna arrays by position-only control employs stochastic optimization techniques [10], [22], [23]. Stochastic optimization techniques suffer from several limitations, including high computational cost, particularly for large array sizes. Additionally, there is no guarantee that the obtained solution is the optimal one, as it may be trapped in a local minimum. Another

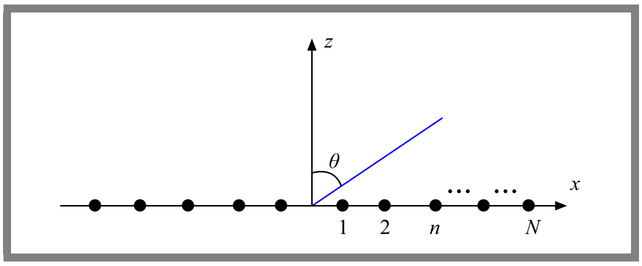

Fig. 1. Geometry of a $2N$ element linear array along the x-axis.

drawback is the inconsistent findings achieved during each run, which necessitates numerous independent runs. Some of the drawbacks of stochastic optimization methods can be overcome by using convex optimization.

In this paper, two methods for peak sidelobe level (PSLL) reduction and null steering in uniformly weighted linear arrays by controlling the position of the array elements are proposed. For the first method, the synthesis problem is cast as a convex optimization problem with the help of first-order Taylor approximation. Here, all the array elements' positions are perturbed to reduce the PSLL and impose prescribed nulls with a predetermined upper bound on the null depth in the radiation pattern. The problem is solved iteratively with a small position perturbations every iteration to minimize the approximation error.

For the second method, the problem is to impose prescribed nulls with a specified upper bound on the null depth while perturbing the positions of as few array elements as possible to achieve a predefined PSLL. The problem is formed as a sparse recovery problem using the off-grid compressive sensing framework. Here, instead of minimizing the ℓ_0 norm of the weight vector, the ℓ_0 norm of the position perturbation vector is minimized. An algorithm based on iterative reweighted ℓ_1 norm minimization of the position perturbation vector is proposed, where the position perturbations are kept small per iteration to control for the approximation error.

The remainder of the paper is structured as follows. Section 2 introduces the problem formulation. Method 1 is presented in Section 3. Method 2 is detailed in Section 4. Section 5 presents the simulation results. Finally, Section 6 draws the conclusions.

2. Problem Formulation

Figure 1 shows a $2N$ element linear array placed symmetrically along the x–axis. In the $x-z$ plane, the array factor is given by:

$$F(\theta) = 2\sum_{n=1}^{N} I_n\, e^{j\left(\frac{2\pi}{\lambda}x_n \sin\theta + \phi_n\right)}\,, \tag{1}$$

where λ is the wavelength, x_n is the position of the n-th element and I_n and ϕ_n are the excitation and the phase of the n-th element, respectively. For a uniformly excited array, i.e., $I_n = 1$ and $\phi_n = 0$, Eq. (1) can be written as:

$$F(\theta) = 2\sum_{n=1}^{N} \cos\left[\frac{2\pi}{\lambda}x_n \sin\theta\right]\,. \tag{2}$$

In this work, the array synthesis problem is modeled as an off-grid CS problem. Suppose that the n-th element position x'_n is not located on the grid points, but is situated at an unknown displacement from the closest grid point x_n. To find the element displacement from the nearest grid point, we present position perturbation to x_n.

Let $a(x_n) = 2\cos\left[\frac{2\pi}{\lambda}x_n \sin\theta\right]$. Using first order Taylor expansion:

$$a(x'_n) \approx a(x_n) + \delta_n \frac{\partial a(x)}{\partial x}\bigg|_{x=x_n}\,, \tag{3}$$

where δ_n is the position perturbation variable for the n-th element and $|\delta_n| \leqslant \Delta d_x/2$. Initially we start with the uniform equally spaced array with interelement spacing Δd_x, then the array elements can have a controlled displacement from their initial locations using the position perturbation variables. The use of $|\delta_n| \leqslant \Delta d_x/2$ ensures that successive elements do not overlap since they can only move by half the interelement spacing in either direction.

The array radiation pattern with the elements' position perturbations may be represented using the first-order Taylor approximation as:

$$F(\theta) \approx \sum_{n=1}^{N}\left[a(x_n) + \delta_n \frac{\partial a(x)}{\partial x}\bigg|_{x=x_n}\right]\,, \tag{4}$$

Equation (4) may be represented in matrix form by sampling the radiation pattern as:

$$\mathbf{F} = (\mathbf{A} + \mathbf{A}_x \Lambda_\delta)\mathbf{1}\,, \tag{5}$$

where $\mathbf{F} = [F(\theta_1), F(\theta_2), \ldots, F(\theta_J)]^T$ is a vector containing the samples of $F(\theta)$ at J directions θ_j, $j = 1, 2, \ldots, J$. $\mathbf{A} = [\boldsymbol{a}(1), \ldots, \boldsymbol{a}(J)]^T$ and $\boldsymbol{a}(j) = [a(x_1), \ldots, a(x_N)]^T$ with $\theta = \theta_j$. $\mathbf{A}_x$ is the partial derivative of $\mathbf{A}$ with respect to x. $\mathbf{\Lambda}_\delta = \text{diag}(\boldsymbol{\delta})$, where $\boldsymbol{\delta}$ is a vector of position perturbations $\boldsymbol{\delta} = [\delta_1, \ldots, \delta_N]^T$. $\mathbf{1} \in \mathbb{R}^N$ is the one vector.

3. Method 1

In this section, we are interested in reducing the PSLL and impose nulls with a predefined upper limit on the null depth in the array's radiation pattern by position control of all the array elements. Towards this end, the array synthesis problem may be expressed as:

$$\min_{\delta}\ \tau_s \tag{6a}$$

$$\text{subject to } |(\mathbf{A} + \mathbf{A}_x\Lambda_\delta)\mathbf{1}| \leqslant \tau_s, \quad \theta \in \Omega^{sl} \tag{6b}$$

$$|(\mathbf{A} + \mathbf{A}_x\Lambda_\delta)\mathbf{1}| \leqslant \tau_n, \quad \theta \in \Omega^{nl} \tag{6c}$$

$$|\delta_n| \leqslant \Delta d_x/2, \quad n = 1, 2, \ldots, N\,, \tag{6d}$$

where τ_s is a slack variable that represents an upper bound on the array response in the sidelobe region, τ_n is an upper bound on the null depth, Ω^{sl} is the sidelobe region, and Ω^{nl} is the null region (directions).

Since the suggested method relies on the first-order Taylor approximation in Eq. (3), the approximation error needs to be minimal to ensure the model's accuracy. It is obvious that the inaccuracy will increase as the values of the position perturbations, δ_n, rise. The modeling error can be reduced by reducing Δd_x in Eq. (6)-d to a smaller value $\Delta d'_x < \Delta d_x$, but this will limit the degrees of freedom provided to the algorithm and might reach a solution with a high PSLL.

Here, an iterative algorithm is proposed to mitigate this problem by restricting the value of δ_n in each iteration to $|\delta_n^k| \leqslant \Delta d'_x/2$, where δ_n^k is the value of δ_n in iteration k and $\Delta d'_x < \Delta d_x$. By doing so, we will be able to improve the model's accuracy without facing the aforementioned issues.

Initially, a uniformly spaced array is considered, i.e. the position perturbations δ_n^0, $n = 1, 2, \ldots, N$ are set to zero, $\mathbf{A}^0$ and $\mathbf{A}_x^0$ are calculated accordingly for the sidelobe region and the null directions. The optimization problem at the k-th iteration may be expressed as:

$$\min_{\delta^k} \tau_s \tag{7a}$$

$$\text{subject to } |(\mathbf{A}^{k-1} + \mathbf{A}_x^{k-1}\Lambda_\delta^k)\mathbf{1}| \leqslant \tau_s, \quad \theta \in \Omega^{sl} \tag{7b}$$

$$|(\mathbf{A}^{k-1} + \mathbf{A}_x^{k-1}\Lambda_\delta^k)\mathbf{1}| \leqslant \tau_n, \quad \theta \in \Omega^{nl} \tag{7c}$$

$$|\delta_n| \leqslant \Delta d'_x/2, \quad n = 1, 2, \ldots, N\ . \tag{7d}$$

The optimization problem in Eq. (7) is a convex optimization problem and can be solved using off-the-shelf packages, such as CVX [24]. After solving the optimization problem in Eq. (7), the array elements' positions are adjusted in accordance with their perturbation values:

$$x_n^k = x_n^{k-1} + \delta_n^k, \quad n = 1, \ldots, N\ , \tag{8}$$

where x_n^{k-1} is the position of the n-th element at the past iteration $k-1$. Finally, $\mathbf{A}^k$ and $\mathbf{A}_x^k$ are updated according to the new element positions, and the optimization problem in Eq. (7) is solved again for another iteration. The algorithm continues until the maximum number of iterations is reached. The maximum number of iterations is set experimentally to 10. This algorithm is referred to as method 1 for the remaining of the paper.

4. Method 2

To minimize the amount of elements that needs to be perturbed from their original positions under a predefined PSLL and null depth, the optimization problem may be formulated as:

$$\min_{\boldsymbol{\delta}} \|\boldsymbol{\delta}\|_0 \tag{9a}$$

$$\text{subject to } |(\mathbf{A} + \mathbf{A}_x\Lambda_\delta)\mathbf{1}| \leqslant \tau_s, \quad \theta \in \Omega^{sl} \tag{9b}$$

$$|(\mathbf{A} + \mathbf{A}_x\Lambda_\delta)\mathbf{1}| \leqslant \tau_n, \quad \theta \in \Omega^{nl} \tag{9c}$$

$$|\delta_n| \leqslant \Delta d_x/2, \quad n = 1, 2, \ldots, N\ , \tag{9d}$$

where $\|\cdot\|_0$ is the ℓ_0 norm, which is the number of the non-zero entries of its argument. The optimization problem in Eq. (9) is an NP-hard optimization problem due to the non-convex objective function. To achieve a convex optimization problem, the convex and sparsity-promoting ℓ_1 norm can be used in place of the ℓ_0 norm:

$$\min_{\boldsymbol{\delta}} \|\boldsymbol{\delta}\|_1 \tag{10a}$$

$$\text{subject to } |(\mathbf{A} + \mathbf{A}_x\Lambda_\delta)\mathbf{1}| \leqslant \tau_s, \quad \theta \in \Omega^{sl} \tag{10b}$$

$$|(\mathbf{A} + \mathbf{A}_x\Lambda_\delta)\mathbf{1}| \leqslant \tau_n, \quad \theta \in \Omega^{nl} \tag{10c}$$

$$|\delta_n| \leqslant \Delta d_x/2, \quad n = 1, 2, \ldots, N\ , \tag{10d}$$

where $\|\cdot\|_1$ is the ℓ_1 norm, which is the sum of the absolute values of its argument. That is $\|\boldsymbol{\delta}\|_1 = \sum_{n=1}^{N} |\delta_n|$.

To lower the approximation error and reduce the number of perturbed element, an algorithm based on the iterative reweighted ℓ_1 norm minimization is proposed [25]. Initially, a uniformly spaced array is considered with zero position perturbations $\delta_n^0 = 0$, $n = 1, 2, \ldots, N$. The matrices $\mathbf{A}^0$ and $\mathbf{A}_x^0$ are calculated accordingly for the sidelobe region and the null directions. The optimization problem at the k-th iteration can be expressed as:

$$\min_{\boldsymbol{\delta}^k} \sum_{n=1}^{N} |\psi_n^k \delta_n^k| \tag{11a}$$

$$\text{subject to } |(\mathbf{A}^{k-1} + \mathbf{A}_x^{k-1}\Lambda_\delta^k)\mathbf{1}| \leqslant \tau_s, \quad \theta \in \Omega^{sl} \tag{11b}$$

$$|(\mathbf{A}^{k-1} + \mathbf{A}_x^{k-1}\Lambda_\delta^k)\mathbf{1}| \leqslant \tau_n, \quad \theta \in \Omega^{nl}\ , \tag{11c}$$

with δ_n^k being the n-th element of $\boldsymbol{\delta}$ at iteration k. $\psi_n^k = 1/(|\delta_n^{k-1}| + \xi)$, where δ_n^{k-1} is the value of δ_n at iteration $k-1$. ξ is a small positive number utilized to retain numerical stability. In this work, ξ is set to 0.0001.

With this relation between ψ_n^k and δ_n^{k-1}, small elements in $\boldsymbol{\delta}$ will be penalized because they are multiplied by a large value ψ_n^k. This will result in even smaller values for the small entries in $\boldsymbol{\delta}$ in the following iteration and boosting the sparsity of the solution [25]. At the first iteration, $\psi_n^1, n = 1, 2, \ldots, N$ are set to one.

After solving the optimization problem in Eq. (11) using CVX, the values of the position perturbations are limited to $|\delta_n| \leqslant \Delta d'_x/2$, i.e., $\delta_n \in \left[-\frac{1}{2}\Delta d'_x,\ \frac{1}{2}\Delta d'_x\right]$. The final values of the position perturbations at iteration k are calculated using:

$$\tilde{\delta}_n^k = \begin{cases} \delta_n^k, & \text{if } \delta_n^k \in \left[-\frac{1}{2}\Delta d'_x,\ \frac{1}{2}\Delta d'_x\right] \\ -\frac{1}{2}\Delta d'_x, & \text{if } \delta_n^k < -\frac{1}{2}\Delta d'_x \\ \frac{1}{2}\Delta d'_x, & \text{otherwise}\ . \end{cases} \tag{12}$$

The array elements' positions are then updated in accordance with their position perturbation values:

$$x_n^k = x_n^{k-1} + \tilde{\delta}_n^k, \quad n = 1, \ldots, N\ , \tag{13}$$

where x_n^{k-1} is the position of the n-th element at the previous iteration $k-1$.

Finally, $\mathbf{A}^k$ and $\mathbf{A}_x^k$ are updated according to the new element positions for the sidelobe region and the null directions. Then, the optimization problem in Eq. (11) is solved again for another iteration. The algorithm continues until it reaches the maximum number of iterations or $\|\boldsymbol{\delta}^k\|_2 \leqslant \epsilon$, where ϵ is a tolerance parameter. Here, ϵ is set to be 0.0001 experimentally.$\|\cdot\|_2$ is the ℓ_2 norm. This indicates that there

Tab. 1. Geometry of the optimized 10-element array using method 1 (normalized with respect to $\lambda/2$).

n	Position	n	Position
1	±0.4880	4	±3.0000
2	±1.0625	5	±4.2082
3	±2.0642		

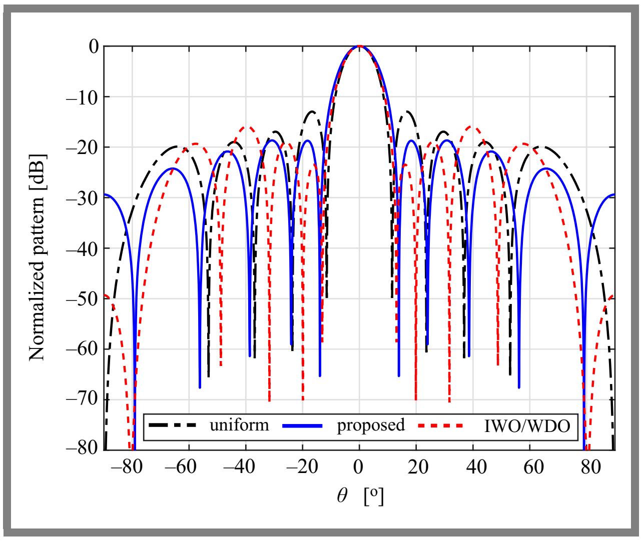

Fig. 2. Patterns of the uniform 10-element array, method 1 and the hybrid IWO/WDO from [10].

is no meaningful change in the positions of the array elements in the current iteration. The maximum number of iterations is set to 10. This algorithm is referred to as method 2 for the remaining of the paper.

5. Simulation Results

For the first example, consider the synthesis of a 10-element linear array (i.e. $N = 5$) with a minimum PSLL. The initial uniformly spaced array has an inter-element spacing of $\lambda/2$. This array was optimized using particle swarm optimization (PSO) in [22], comprehensive learning particle swarm optimizer (CLPSO) in [23], and hybrid invasive weed optimization and wind driven optimization (IWO/WDO) in [10]. The best obtained result was a PSLL of -15.9 dB for the normalized pattern using the hybrid IWO/WDO from [10]. Applying method 1 resulted in obtaining a PSLL of -18.67 dB for the normalized pattern compared to -15.9 dB for the hybrid IWO/WDO. The normalized patterns of the uniform array, method 1 and the hybrid IWO/WDO are shown in Fig. 2. The geometry of the optimized array is given in Tab. 1 with respect to $\lambda/2$.

The second example demonstrates the synthesis of a 28-element linear array ($N = 14$) with a minimum PSLL and three prescribed nulls at 30°, 32.5°, and 35° as in [10]. The initial uniformly spaced array has an inter-element spacing of $\lambda/2$. This array was optimized using particle PSO in [22], CLPSO in [23], and hybrid IWO/WDO in [10]. The hybrid IWO/WDO resulted in the best PSLL of -13.19 dB for the normalized pattern. Applying method 1 resulted in a PSLL of -18.45 dB. The normalized patterns of method 1, the uniform array and the hybrid IWO/WDO are depicted in Fig. 3. Table 2 lists the element positions for the optimized array using method 1 with respect to $\lambda/2$.

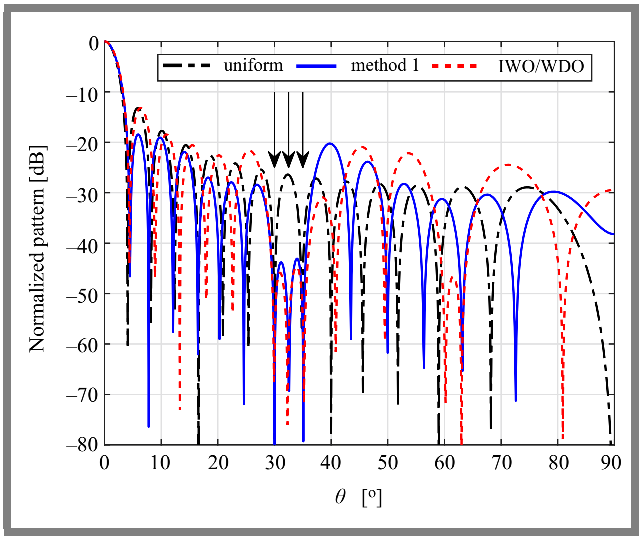

Fig. 3. Patterns of the uniform 28-element array, method 1 and the hybrid IWO/WDO from [10]. The directions of the nulls are indicated by the arrows.

Next, for the third example, we apply method 2 for the 28-element linear array ($N = 14$) with initial inter-element spacing of $\lambda/2$. We set the upper bound on the array response in the sidelobe region to that obtained using the hybrid IWO/WDO from [10]. The objective for method 2 is to achieve this PSLL and the three imposed nulls at 30°, 32.5°, and 35° by perturbing the positions of a minimum number of that array elements. Applying method 2 resulted in perturbing the positions of only 6 out of the total 28 array elements while satisfying all the constraints on the radiation pattern. The patterns of the uniform 28-element array, the hybrid IWO/WDO from [10] and the pattern of method 2 are shown in Fig. 4. It

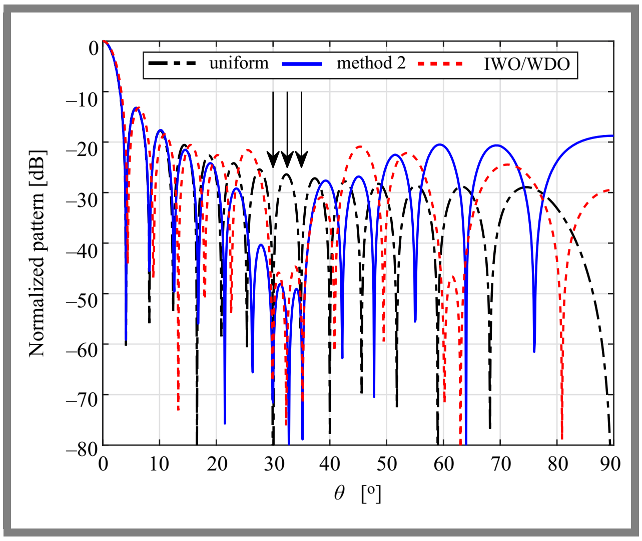

Fig. 4. Patterns of the uniform 28-element array, method 2 and the hybrid IWO/WDO from [10]. The directions of the nulls are indicated by the arrows.

Tab. 2. Geometry of the optimized 28-element array normalized with respect to $\lambda/2$. The perturbed elements using method 2 are marked in bold.

n	Method 1	Method 2
1	±0.6763	±0.5000
2	±1.0383	±1.5000
3	±2.2080	**±2.4339**
4	±2.7874	±3.5000
5	±3.9208	±4.5000
6	±4.6472	±5.5000
7	±5.5773	±6.5000
8	±6.7073	±7.5000
9	±7.8581	±8.5000
10	±8.7781	±9.5000
11	±10.1495	±10.5000
12	±11.2278	±11.5000
13	±12.3734	**±12.1901**
14	±13.8323	**±13.6986**

can be seen from the figure the all the constraints on the radiation pattern are met by perturbing the locations of only 6 array elements. The optimized array geometry using method 2 is given in Tab. 2.

6. Conclusions

In this paper, two methods for the synthesis of aperiodic linear arrays were presented. For the first method, the problem of PSLL reduction and forming prescribed nulls in the radiation pattern of the array by optimizing the position of array elements was formulated as a convex optimization problem and solved iteratively.

For the second method, only a small number of the array elements are perturbed from their original positions to achieve a predefined upper bound on the PSLL and form prescribed nulls in the radiation pattern with an upper bound on the null depth. The two methods were compared to results from the literature using stochastic optimization techniques such as PSO, CLPSO, and hybrid IWO/WDO. The results showed the effectiveness of the proposed methods.

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