

Determining Speed and Reliability of Transmitting Indicator Information in Residual Class Systems

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<https://doi.org/10.26636/jtit.2026.1.2403>

Abstract — The article is devoted to the analysis and assessment of the efficiency of fiber optic systems whose primary objective is to transmit data. The efficiency of information transmission systems depends on numerous indicators, such as interference immunity, speed, energy efficiency, cost, development time, and design. However, from the user's point of view, quality of service is determined primarily by transmission speed and reliability. This article compares two systems. The first corresponds to the modern paradigm: one user – one transmission channel. In the other, the number of users is provided with a complex channel for transmitting symbols of the alphabet of a certain system of residual classes. At the same time, the transmission speed in the residual class system – compared to the classical multiplexing method – decreases slightly to 28/32, while the reliability (determined based on the probability of failures) increases by several orders of magnitude. The work proves a lemma that determining the optimal alphabets of residual class systems allows to optimally approximate the transmission speed of modules to binary coding systems. An analysis of the non-linear loss function, which considers the speed parameters and probabilistic reliability indicators, is performed as well.

Keywords — *efficiency criteria, fiber optic transmission systems, reliability, residual class systems, transmission speed*

1. Introduction

Joint consideration of energy efficiency (with such factors as reliability and speed taken into consideration) is of paramount importance in various technical systems. Speed and reliability may be contradictory requirements, while losses depend on other factors as well. For example, if a vehicle is operated at a supercritical speed over a significant amount of time, then it is logical that its reliability will decrease. On the contrary, if the vehicle is equipped with a heavy duty, damage-resistant suspension system, this will lead to a potential decrease in its maximum speed.

This work considers both transmission speed and reliability in analyzing the efficiency of fiber optic transmission systems (FOTS). The principles presented in this article can be applied to any transmission system. However, FOTS was chosen as an example solution, as this technology is used for transmitting the largest amounts of traffic and such systems may rely on a very large number of different multiplexing methods.

It should be noted that modern global data transmission systems are based on a hierarchical architecture: international traffic exchange nodes constitute the top level of the system. Below there are national node subsystems, operator nodes, and so on, all the way to the “last mile”, i.e. to the end user taking advantage of a given service. Channels connecting these nodes at all levels of the network must be reliable, meaning they need to minimize the probability of failures and ensure sufficient data transfer speeds for a given network node.

For a FOTS channel, a failure is the worst-case scenario, as it is tantamount to a complete loss of efficiency.

The classic approach to information distribution consists in providing each user, temporarily, with a separate data transmission channel. This principle is referred to as the classical principle of multiplexing (CPM). In this case, a failure of one transmission channel does not exert any impact on most other users.

However, the specific user affected by the failure may be extremely dissatisfied with the service. The relevance of information may be lost due the insufficient quality of service, which can lead to discontinuation of subscription. If the channel affected by the failure is used for transmitting operational information of national importance, the problem of minimizing the probability of failure becomes particularly significant.

The aim of this work is to develop an approach that combines FOTS-specific principles, according to which efficiency is analyzed based on such criteria as transmission speed and reliability. To achieve the goal, we study the transmission speed and failure probability of a complex channel used for transmitting symbols of the alphabet of a certain system of residual classes.

2. Literature Review

Efficiency, determined based on a set of probabilistic reliability and transmission speed (bandwidth) indicators, was not considered in depth in recent research dealing with FOTS. Reliability [1], [2] and bandwidth [3]–[5] were considered in different contexts, as mutually independent factors. Only work [6] took a comprehensive approach to assessing the ef-

efficiency of FOTSS, as it considered the criteria of reliability and bandwidth, and thus may be considered relevant.

Paper [6] presented a framework in which FOTS channels were multiplexed the logic of residual class systems (RCS). In this concept, the residues from different modules are transmitted over different channels (physical or logical). This method increases the reliability of FOTS, but achieves worse results for CPMs-based systems on binary arithmetic (SBA) logic in total bandwidth.

3. Transmission Methods in RCS

The research presented in this work is based on the theory of timer signal structures (TSS). TSS was initially developed for the purpose of transmitting signals using RCS-based sequences [7]–[8].

The use of RCS allows to increase cryptographic protection of transmission systems. This is caused by the fact that no codes are used in RCS, unlike in systems based on the transmission of SBA codes. However, at the channel level and in the subsystems tasked with routing and processing information flows, modern transmission systems are based on SBA in some way or another. Therefore, the problem of reducing the redundancy of RCS, compared to SBA, is becoming increasingly important.

When analyzing RCS, the focus is placed on the Chinese remainder theorem (CRT). CRT considers systems of mutually prime integers m_1, m_2, \dots, m_M , which are called RCS modules. The capacity of the RCS alphabet is defined as the following product: $E = m_1 \cdot m_2 \cdot \dots \cdot m_M$. The property of the CRT is that any number N within range $0 \leq N < E$ can be uniquely represented by the remainder of dividing this number N by the module.

However, due to the difference between RCS and SBA systems (SBA is a positional system, while RCS is not) during the transmission of RCS symbols in SBA, a certain number of additional bits is required, which can be determined for a separate module:

$$b(m) = \begin{cases} \log_2(m), & \text{fract}[\log_2(m)] \equiv 0 \\ \log_2(m) + 1, & \text{fract}[\log_2(m)] > 0 \end{cases}, \quad (1)$$

where $\text{fract}[\cdot]$ determines the fractional part of a number.

The proof of Eq. (1) is simple and follows from the fact that only for modules that are powers of two do the numbers of bits in the RCS and the SBA coincide. Therefore, there can be only one module that is an even number in the RCS. Information redundancy of the RCS with M modules, in the general case, will be approximately m bits per transmitted symbol.

For an SBA system with base n , the alphabet contains $D = 2n$ possible messages. Compared to SBA, a number of “residual” bits $\Delta B = M$ is added to the RCS if there are no even numbers between the RCS modules, if one of the modules is an even number, $\Delta B = M - 1$. In this case, the capacity of the RCS alphabet E , in the general case, is less than the capacity of the SBA alphabet D . Otherwise, if $E > D$, then an optimization task, based on the minimum redundancy in terms

of bits lost, is necessary. Simultaneously, the optimal RCS should transmit the minimum bit sequence and the number of symbols that are as close as possible to the D value.

Based on these considerations, the RCS optimization problem can be formulated as a system based on the following criteria:

$$\Delta B \rightarrow \min; \Delta E = D - E \rightarrow \min; M > 1, \quad (2)$$

where ΔB is bit redundancy and ΔE stands for information deficiency.

The number of RCS alphabets that satisfy condition (2) is very limited. This conclusion is proved in the following manner.

Let us consider an RCS system with base n .

If n is an even number ($n = 2k$), then the optimal RCS in the sense of criterion (2) will be:

$$m_1 = 2^k - 1, \quad m_2 = 2^k + 1. \quad (3)$$

If n is an odd number ($n = 2k + 1$), then the optimal RCS is the following.

$$m_1 = 2, \quad m_2 = 2^k - 1, \quad m_3 = 2^k + 1. \quad (4)$$

The proof is in:

$$E = (2^k - 1) \cdot (2^k - 1) = 2^n - 1 = D - 1.$$

Compared to the SBA alphabet, the RCS alphabet has only one symbol of information deficiency. Therefore, if $E = D$, then the RCS alphabet will include modules that are not mutually prime. Modules m_1 and m_2 are mutually prime numbers, since the remainder of the division by will never assume the value of 0, since:

$$\frac{m_2}{m_1} = \frac{m_1 + 2}{m_1} = \frac{1 + 2}{m_1} \Rightarrow \frac{2}{m_1 \neq 0}.$$

In this case, we have once again 1 bit of redundancy, and the information deficiency increases to 2 symbols.

Examples of RCS characteristics according to Eqs. (3) and (4), compared with the characteristics of the SBA, are given in Tab. 1.

One may notice that the relative difference of characteristics ΔB and ΔE compared to the alphabet capacity decreases with increasing base n of the RCS. For comparative analysis of the SBA and RCS in the general case, the relative indicators of bit redundancy and information deficiency can be used in the following form:

$$br = \frac{n - 1}{n}, \quad ir = \frac{E}{D}.$$

According to the definitions of modules from Eqs. (3) and (4), the graph of these characteristics is presented in Fig. 1.

4. Comparison of FOTS Models for SBA and RCS

An example of a CRT based on RCS is presented in Fig. 2. The figure shows two physically and logically separated channels for two different data transmission users. Each user is the owner of a separate optical fiber and transmits, in parallel code, 3 bits per clock interval only. The signals are transmitted

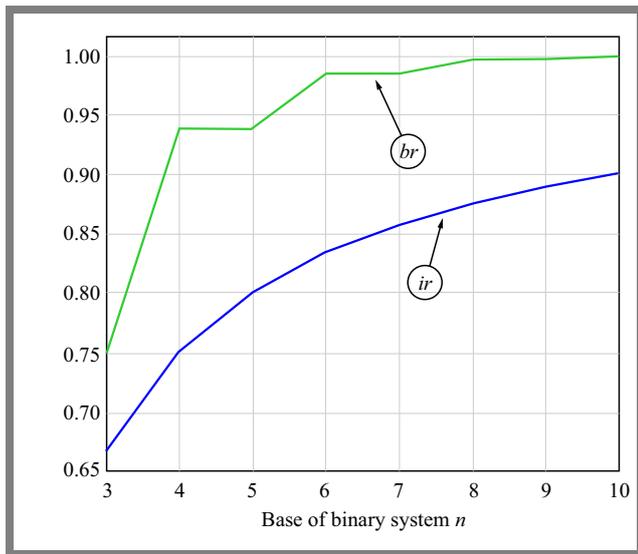


Fig. 1. Dependence of the relative indicators br and ir on SBA n .

by an elementary binary code (0, 1), and the subsequent bits of code packets B_0, B_1, B_2 are transmitted, in parallel code, over 3 frequency channels $F_{ch0}, F_{ch1}, F_{ch2}$, respectively.

In fact, each of the users has a separate physical channel (OF 1 or OF 2), in which 3 logical subchannels are organized by multiplexing in frequency domain. This scheme does not fully reflect a modern multichannel FOTS, where signals in 30 ... 90 frequency channels are transmitted simultaneously over a single optical fiber. However, only the basic principles are considered in this paper.

In the time domain, the transmission is also organized discretely, i.e. by using the clock signal. According to Fig. 2, in a CRT-based RCS system, each of the users can transmit, in one clock interval, one of the symbols of the alphabet with $D = 8$ possible values, and both users can transmit a total of 16 symbols in one clock interval.

The same multiplexing channels can be used to organize data transmission in the RCS. The schematic diagram of the RCS, which is close to the CRT-RCS from Fig. 2, is shown in Fig. 3. Here, the frequency channels and transmission channels over optical fibers are aggregated into a single common channel which is provided to both users.

Paths along two different fibers can be treated as one bit of information. Therefore, this subsystem can transmit the values of residue module 2. The frequency channels in an aggregated system can be in eight states. The RCS modulus closest to eight, which is mutually prime to 2, has the value

Tab. 1. Comparative characteristics of SBA and RCS.

n	D	m_1	m_2	m_3	E	ΔB	ΔE
4	16	3	5	–	15	1	1
5	32	2	3	5	30	1	2
6	64	7	9	–	63	1	1
7	128	2	7	9	126	1	2
8	256	15	17	–	255	1	1

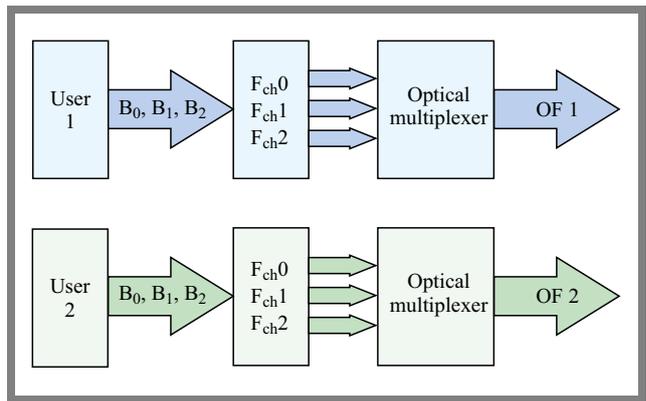


Fig. 2. Conventional system with two complex FOTS channels for a CRT-based RCS.

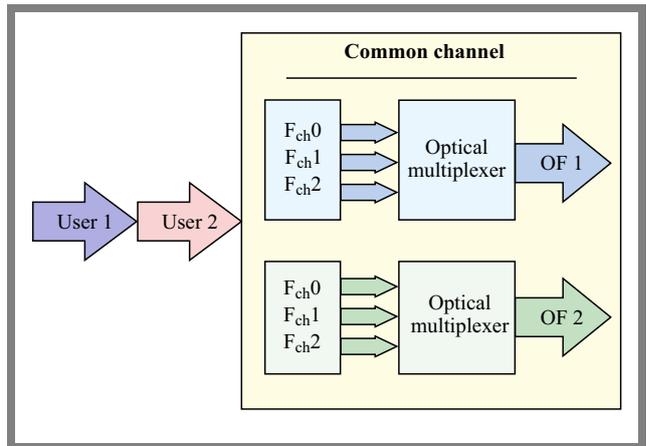


Fig. 3. System with an aggregated channel in RCS codes.

of 7. Therefore, in one clock cycle, it is possible to transmit symbols of the alphabet with module (2, 7), and thus the capacity of the alphabet is 14 variants in this case.

As a result, one RCS user transmits 14 variants in each clock cycle, while the SDC user only transmits 8. This difference does not mean that RCS is more efficient in terms of transmission speed. To compare RCS and SBA, it is necessary to consider the fact that users on the receiving end must act differently.

For the SBA case (Fig. 2), users transmit data by logically separated channels. Here, the distribution of data streams follows the TSC theory [8], where the transmission of user messages occurs sequentially over time. This method of distributing the total data stream is illustrated in Fig. 3, where users sequentially transmit their data, alternating according to FOTS clocks.

Within two clock periods, each SBA user will transmit symbols using an alphabet with a capacity of 16 symbols, while an RCS user will only transmit from an alphabet which contains 14 symbols. Accordingly, both users of the SBA can transmit one symbol out of 32 possible options in 2 clock cycles, while users of the RCS can transmit only 28. Therefore, transmitting the RCS alphabets using the SBA codes leads to losses in terms of the maximum speed achieved.

5. Comparison of FOTS Models Based on Reliability Criteria

FOTS system failures may be caused by various reasons:

- failure of frequency channels due to aging of active elements of optoelectronic subsystems,
- aging of optical fiber,
- damage to optical cable, individual fibers or patchcords,
- above-specification operation of attenuators,
- violation of optical fiber switching schemes,
- lightning strikes affecting the cable,
- intentional damage to FOTS components.

Due to the high complexity of these systems and optical components, such reasons must be analyzed during design, production, and operation of FOTS. Most likely, it is impossible to derive any generalized formulas for analyzing reliability at the system level. Simulation modeling methods and the determination of probabilistic reliability indicators can be helpful and effective. However, in this work, we consider a simple probabilistic model, where the probabilities of the operational state of all FOTS components are known according to the scheme shown in Fig. 2.

The reliability parameter of a separate frequency channel is defined as probability p of its operating state at a specific time during operation, for example, over a month, year, or decade. Consequently, the probability of failure in this component is: $q = 1 - p$. A malfunction of any frequency channel affecting each user means a critical service failure. In this case, the SBA signals cannot be transmitted with the base of the 3-bit binary system. Similarly, the failure of the entire channel for each user may be caused by damage to the optical fiber. In this case, the reliability of the system is defined as a linear function of the reliability of frequency channels $p^* = a \cdot p$, where a is the proportionality coefficient.

We assume that frequency channel failures and optical fiber damage are statistically independent. Then, according to Fig. 2, the logic of potential channel failures affecting one user is determined by the fact that specific system components, namely 3 frequency channels and 1 optical channel, are connected in series. Then, the probability of failure-free operation of the transmission channel for each user P_U and the probability of failure Q_U , can be formulated as follows:

$$P_U = a \cdot p^4, \quad Q_U = 1 - P_U. \quad (5)$$

In the case of RCS, the system failure logic is different and more complex. The failure model can be defined as the parallel connection of channels for the first and second users, according to Fig. 2. Then, the probability of a serviceable state P_R and the probability of failure Q_R for RCS is determined as follows:

$$\begin{aligned} P_R &= 1 - Q_U^2 = 1 - (1 - P_U)^2 = 2a p^4 - a^2 p^8 \\ Q_R &= 1 + a^2 p^8 - 2a p^4 \end{aligned} \quad (6)$$

The proportionality coefficient determining the comparative reliability of frequency channels and the optical channel may

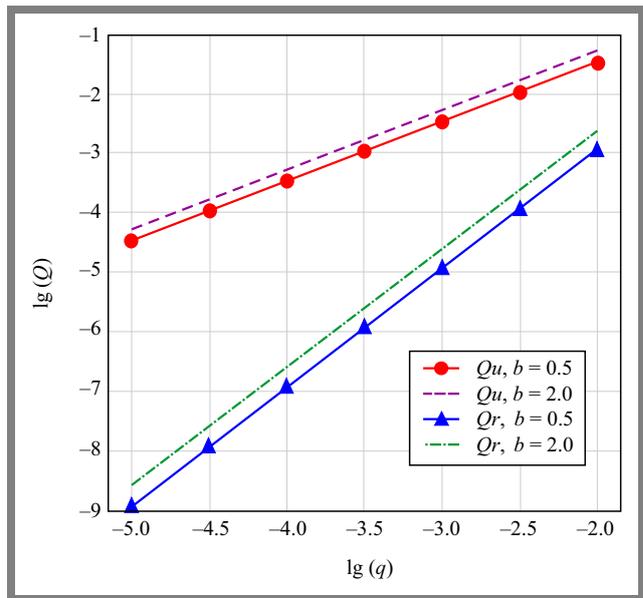


Fig. 4. Comparison of the probability of failure for SBA and RCS channels.

not be very intuitively acceptable, since the probabilities of operating states are large. In other words, if the probability of failure of frequency channel q and optical fiber q^* were defined as a proportional formula $q^* = b \cdot q$, then the clear description of the states of the system is described in example as: the probability of damage to the optical fiber is 10 times greater than the probability of failure of the frequency channel ($b = 10$). Moreover, coefficients a and b are related by a simple relationship, such as:

$$a = 1 + \frac{q(1-b)}{1-q}, \quad (7)$$

As a result of the fact that parameter b can vary significantly depending on the class and operating conditions of FOTS, the conventional model values of this parameter are subjected to further analysis using Eqs. (5)–(7), modeling of the probability of failures for the SBA and RCS was performed. Figure 4 shows two examples of the probability failure of frequency channels and failures along the optical fiber, when $b = 0.5$ and $b = 2$. Analysis of these graphs shows that the failure probabilities of the SBA are significantly higher when compared to the RCS and differ by orders of magnitude. In this case, for the SBA, $Q_U > q$, while for the RCS, $Q_R < q$ is always true.

6. Effectiveness of FOTS in Terms of Speed and Reliability

The effectiveness of FOTS in terms of service quality is perceived differently by operators and users. For operators, a decrease in quality may result in lost subscribers or corporate clients. For users, a decrease in quality of service depends on the reduction in the transmission speed V (percentage-wise) compared to the maximum achievable value V_{max} . Therefore, the speed reduction coefficient ($K = V_{max}/V$)

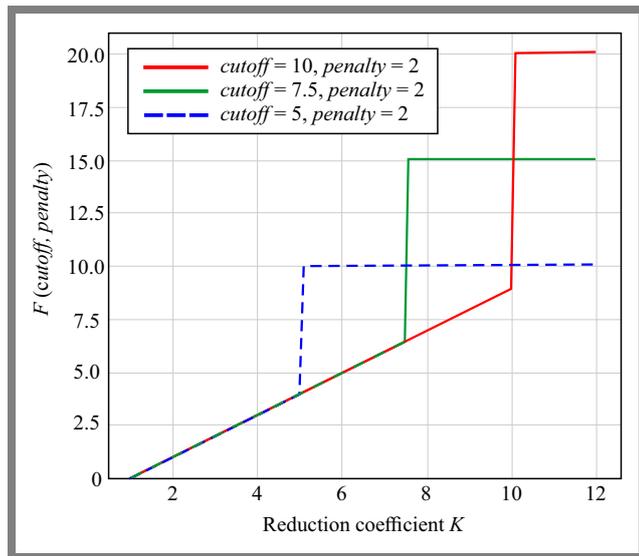


Fig. 5. Examples of penalty functions $F(V)$.

can be considered one of the important quality measures. The decrease in quality can be characterized by various factors, e.g., penalty function $F(V)$, with $F(V_{max}) = 0$.

Some operators base their sales policies on the type of quality of service (QoS). In such a business model, the user enjoys a guaranteed transmission speed V that is expected to remain within specific limits ($V_{min} \leq V \leq V_{max}$). Speed is reduced to the guaranteed minimum level V_{min} during periods in which peak network loads are experienced, for instance. In a situation in which the contract with the operator provides for a minimum transfer speed value $cutoff = V_{min}$, when the transmission rate falls below that threshold, the user may perceive this to constitute a critical failure. If the cutoff level is exceeded, an increase factor sanction > 1 is added to the penalty. For example, the penalty function corresponds to the following conditions:

$$F(V) = \begin{cases} \frac{V_{max}}{V}, & V \leq cutoff \\ \frac{penalty \cdot V_{max}}{V_{min}}, & V > cutoff \end{cases} \quad (8)$$

An example of $F(V)$ is shown in Fig. 5, where the value of the speed reduction coefficient K is plotted on the x -axis.

Using Eq. (8), an integral indicator of efficiency losses is formulated, which takes into account the total penalties over a given period of operation:

$$L(F, t) = \frac{1}{T} \int_0^T F(V(t)) dt \quad (9)$$

Equations (8) and (9) are suitable for assessing the efficiency of a FOTS system during operation and may serve as a foundation for analysis using simulation modeling methods. Then, considering the statistics concerning potential failures and the lead time required to restore the operation of the system after incidents, Eqs. (8) and (9) are extended by applying appropriate weighting factors that characterize the probabilities of the serviceable state and the state of parametric or critical failures.

7. Results and Discussion

Solving optimization problems using multiple criteria that may contradict each other requires using non-trivial approaches, especially in problems that require the development of non-standard indicators based on various informal requirements for the system.

This article proposes the basics of a methodology used for a comprehensive analysis of the efficiency of FOTS from the point of view of speed and reliability. The method is based on the theory of timer signal structures, the theory of reliability, and the arithmetic principles of residual class systems.

For the transmission of residues of different modules in the RCS, it is proposed to use different multiplication methods. Here, the simplest option is considered, where residues modulo 2 are transmitted over two different optical fibers (spatial multiplexing), and residues modulo 3 are transmitted using frequency multiplexing.

The analysis conducted shows that the transmission speed is in the RCS. Therefore, in the development of the theory of timer signal structures, optimal RCS have been determined, which are optimally close to the SBA based on the speed criterion. For the example considered, the ratio of the RCS and SBA transmission speed in bits was found to equal 28/32 bits per clock interval. Under equal conditions, the RCS according to the reliability criterion exceeds the RCS by orders of magnitude. To solve the complex problem of evaluating the effectiveness of FOTS in terms of quality of service, a non-linear penalty function has been proposed.

This study shows that the use of RCS relying on various multiplexing methods can promise a significant improvement in probabilistic reliability indicators of FOTS, with a very slight loss of transmission speed.

References

- [1] O.V. Bondarenko *et al.*, "Thermomechanical Influence on Optical Fibers during the Production and Operation of Optical Cable", *Scientific Works of ONAS*, pp. 23–32, 2019 (<https://doi.org/10.33243/2518-7139-2019-1-1-23-32>).
- [2] D.M. Stepanov, "Development of the Method of Improving the Selection of Dielectrical Strength Elements of Optical Cable Design", *Herald of Khmelnytskyi National University. Technical Sciences*, vol. 343, pp. 315–322, 2024 (<https://heraldts.khmnu.edu.ua/index.php/heraldts/en/article/view/570>).
- [3] N.A. Odegov and I.S. Balejev, "Number-analytical Method for the Synthesis of Optimal Irregular DWDM Frequency Plans", *Scientific Works of ONAS*, pp. 68–80, 2020 (<https://doi.org/10.33243/2518-7139-2020-1-2-70-81>).
- [4] N.A. Odegov, "Evaluation of the Limit Transmission Speed in the Magistral Fiber Optical System", *Scientific Works of ONAS*, pp. 61–76, 2019 (<https://doi.org/10.33243/2518-7139-2019-1-2-61-76>).
- [5] N.A. Odegov *et al.*, "Algorithm for The Synthesis of Optimal FOTS Frequency Plans for Nonlinear Transmission Environment (Boat Method)", *Herald of Khmelnytskyi National University. Technical Sciences*, vol. 327, pp. 88–97, 2023 (<https://heraldts.khmnu.edu.ua/index.php/heraldts/en/article/view/509>).
- [6] D. Bahachuk *et al.*, "Multiplex Technique of Data Transmission in Residual Class Systems", *Eastern-European Journal of Enterprise*

Technologies, vol. 6, pp. 23–31, 2023 (<https://doi.org/10.15587/1729-4061.2023.292504>).

- [7] M.V. Zakharchenko *et al.*, “Data Transmission Systems: Effectiveness of Block Coding”, *Scientific Works of ONAS*, 2014 (<https://metod.onat.edu.ua/download/555>).
- [8] M. Zakharchenko *et al.*, “Comparison of Positional and Timer Coding in the System of Residual Classes”, *2022 IEEE 16th International Conference on Advanced Trends in Radioelectronics, Telecommunications and Computer Engineering*, Lviv-Slavske, Ukraine, 2022 (<https://doi.org/10.1109/TCSET55632.2022.9766866>).

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