Paper

Modeling of mixed traffic for mobile cellular network

Imdadul Islam, Jugal Krishna Das, and Siddique Hossain

Abstract— The most convenient way of presenting one- or two-dimensional offered traffic in a network by Markovian chain to evaluate quality of service (QoS). Pictorial presentation of chain becomes very complicated for three dimensional traffic case, required for voice data integrated service/mixed traffic of mobile cellular network; hence application of cut or node equations become a cumbersome job. This paper proposes a model of three-dimensional traffic in a network for both unlimited and limited user case. Here direct analytical method is introduced instead of Markovian chain to achieve traffic parameters.

Keywords— quality of service, Markovian chain, time and call congestion, probability states and voice data integrated network.

1. Introduction

In teletraffic engineering two different cases of traffic, i.e., limited and unlimited users are prevalent. Both types of traffic and their combination could be modeled using state transition diagram/Markovian chain, to reveal different probability states of offered traffic and their transition based on average arrival and termination rate like in [1–5]. After modeling the chain, cut or node equations are applied to achieve relation between different probability sates. Finally each probability state is normalized by dividing it by entire sample space.

A good example of multi-dimensional traffic is mobile cellular network where two different arrivals, i.e., new originating call and handoff arrival traffic are prevalent, summarized in [6–9]. Of course, bandwidth (BW) of either traffic is same since both are voice signals. If there is a provision of another offered traffic of different bandwidth, say video/text/image data is added to conventional mobile cellular system like voice data integrated network of [10, 11].

Three-dimensional traffic model of this paper would be consistent with voice data integrated service and could be an useful tool for a network planar to estimate performance of his network.

Here the concept of two-dimensional traffic is adopted to achieve generalized equation of probability state and blocking probability for three-dimensional traffic case. Section 2 of the paper reveals theoretical analysis of three-dimensional traffic, Section 3 proposes mathematical modeling of 3D traffic, Section 4 depicts the results of previous sections and finally Section 4 concludes the entire paper.

2. Mixed traffic model

Let us now consider a three-dimensional Markov process where any probability state $P_{u,v,w}$ reveals that a cell is occupied by u voice calls of new arrival, v packet/data calls and w handover voice calls. Assuming the bandwidth of data call is $h \geq 2$ times wider than that of voice call. State transition chain of three-dimensional mixed traffic is shown in Fig. 1 based on general packed radio services (GPRS) network of [13], birth-death process of IS-95 handoff of [14] and multimedia traffic of [15].

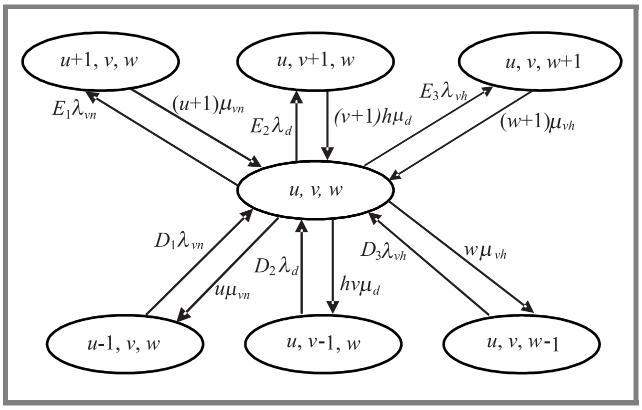

Fig. 1. State transition diagram for mixed traffic.

Entire set of probability states of the transition diagram is

$$S = \left\{ (u,v,w) \middle| 0 \leq v + vh + w \leq n, 0 \leq u \leq n, 0 \leq v \leq \left\lfloor \frac{n}{h} \right\rfloor \right. \\ \left. \text{and } 0 \leq w \leq n \right\}, \tag{1}$$

where $\lfloor x \rfloor$ is a floor function.

For all valid states:

$$\sum_{(u,v,w)\in S} P(u,v,w) = 1\,, \tag{2}$$

where S is the set of valid probability states.

Several states are unreachable, based on following six conditions:

$$D_1 = \begin{cases} 1;\ u-1+hv+w \leq n-1 \quad \text{and} \quad (u-1,v,w) \in S \\ 0;\ \text{otherwise} \end{cases}, \tag{3}$$

$$D_2 = \begin{cases} 1; \ u+h(v-1)+w \le n-h \quad \text{and} \quad (u,v-1,w) \in S \\ 0; \ \text{otherwise} \end{cases}, \tag{4}$$

$$D_3 = \begin{cases} 1; \ u+hv+w-1 \le n-1 \quad \text{and} \quad (u,v,w-1) \in S \\ 0; \ \text{otherwise} \end{cases}, \tag{5}$$

$$E_1 = \begin{cases} 1; \ u+hv+w \le n-1 \quad \text{and} \quad (u+1,v,w) \in S \\ 0; \ \text{otherwise} \end{cases}, \tag{6}$$

$$E_2 = \begin{cases} 1; \ u+hv+w \le n-h \quad \text{and} \quad (u,v+1,w) \in S \\ 0; \ \text{otherwise} \end{cases}, \tag{7}$$

$$E_3 = \begin{cases} 1; \ u+hv+w \le n-1 \quad \text{and} \quad (u,v,w+1) \in S \\ 0; \ \text{otherwise} \end{cases}. \tag{8}$$

Now the sum of the probabilities of complete occupation of channels in Eq. (12) gives call blocking probability based on [12]:

$$B = \sum_{C \in \left\{ (u,v,w) \middle| u+hv+w=n, \ 0 \le u \le n, \ 0 \le v \le \left\lfloor \frac{n}{h} \right\rfloor, \ 0 \le w \le n \right\}} P(u,v,w) \ . \tag{9}$$

Let us now apply node equation at (u,v,w) of Fig. 1:

$$\begin{aligned} & P_{u,v,w}\left\{E_1\lambda_{vn} + E_2\lambda_d + E_3\lambda_{vh} + u\mu_{vn} + hv\mu_d + w\mu_{vh}\right\} \\ = \ & P_{u+1,v,w}(u+1)\mu_{vn} + P_{u,v+1,w}(v+1)h\mu_d \\ & + P_{u,v,w+1}(w+1)\mu_{vh} + P_{u,v,w-1}D_3\lambda_{vh} \\ & + P_{u,v-1,w}D_2\lambda_d \, . \end{aligned} \tag{10}$$

Any probability state could be determined from a set of such linear equations at different nodes.

3. Proposed traffic model

It would be a laborious job to determine quality of service (QoS) or probability states of the network from a series of node equations like Eq. (10). In this section an alternate model is proposed to determine normalized probability states or QoS directly. One of the convenient ways of presenting two-dimensional traffic is in triangular matrix form, where corresponding terms of the following two series of A_1 and A_2 (A_1 and A_2 are the offered traffic of $M/M/n$ case) are multiplied:

$$1, \ \frac{A_2}{1!}, \ \frac{A_2^2}{2!}, \ \frac{A_2^3}{3!}, \ \ldots, \ \frac{A_2^{n-2}}{(n-2)!}, \ \frac{A_2^{n-1}}{(n-1)!}, \ \frac{A_2^n}{n!}$$

$$1, \ \frac{A_1}{1!}, \ \frac{A_1^2}{2!}, \ \frac{A_1^3}{3!}, \ \ldots, \ \frac{A_1^{n-2}}{(n-2)!}, \ \frac{A_1^{n-1}}{(n-1)!}, \ \frac{A_1^n}{n!}.$$

For limited trunk case, only the probability states, $P_{x,y}$ satisfying the condition $x+y \le n$ (n is the total number of channels) will remain in the table, form a triangular matrix. Diagonal elements the table/matrix will be the complete occupied states and sum of those states in normalized form gives the call blocking probability the network.

Now rows of the triangular matrix for n channels network would be like:

$$\left(1 \ \ \frac{A_2}{1!} \ \ \frac{A_2^2}{2!} \ \ \frac{A_2^3}{3!}, \ \ldots, \ \frac{A_2^{n-2}}{(n-2)!} \ \ \frac{A_2^{n-1}}{(n-1)!} \ \ \frac{A_2^n}{n!}\right)$$

$$A_1\left(1 \ \ \frac{A_2}{1!} \ \ \frac{A_2^2}{2!} \ \ \frac{A_2^3}{3!}, \ \ldots, \ \frac{A_2^{n-2}}{(n-2)!} \ \ \frac{A_2^{n-1}}{(n-1)!}\right)$$

$$\frac{A_1^2}{2!}\left(1 \ \ \frac{A_2}{1!} \ \ \frac{A_2^2}{2!} \ \ \frac{A_2^3}{3!}, \ \ldots, \ \frac{A_2^{n-2}}{(n-2)!}\right)$$

$$\vdots$$

$$\frac{A_1^{n-3}}{(n-3)!}\left(1 \ \ \frac{A_2}{1!} \ \ \frac{A_2^2}{2!} \ \ \frac{A_2^3}{3!}\right)$$

$$\vdots$$

$$\frac{A_1^n}{n!} \, .$$

Therefore complete set of occupied states (utilization of n channels), i.e., diagonal elements of the matrix are:

$$\frac{A_2^n}{n!}, \ \frac{A_2^{n-1}}{(n-1)!}A_1, \ \frac{A_2^{n-2}}{(n-2)!}\frac{A_1^2}{2!}, \ \ldots, \ \frac{A_2^3}{3!}\frac{A_1^{n-3}}{(n-3)!}, \ \ldots, \ \frac{A_1^n}{n!} \, .$$

The normalized form of sum of complete occupied states is given as

$$\therefore \ S = \frac{\sum_{r=0}^{n} \frac{A_1^r}{r!}\frac{A_2^{n-r}}{(n-r)!}}{\sum_{p=0}^{n} \frac{A_1^p}{p!}\sum_{q=0}^{n-p}\frac{A_2^q}{q!}} \, . \tag{11}$$

Each element of the matrix is divided by the entire sampling space to achieve normalized probability states and blocking probability would be the sum of normalized completely occupied states shown in Eq. (11). Probability states of three-dimensional unlimited user's traffic with offered traffics, A_x, A_y and A_z erlangs of equal bandwidth could be presented like triangular matrix of two-dimensional traffic taking any single traffic like A_z as a parameter shown in Fig. 2. Here the states corresponding to complete utilization of channel are depicted with shaded block on right side for number of channel $n = 4$; of course all the states are shown here before normalization.

Now sum of complete utilized states in normalized form, i.e., blocking probability is derived as:

$$B(A_x, A_y, A_z, n) = \frac{\sum_{p=0}^{n}\frac{A_z^p}{p!}\sum_{r=0}^{n-p}\frac{A_x^r}{r!}\frac{A_y^{n-r-p}}{(n-r-p)!}}{\sum_{p=0}^{n}\frac{A_z^p}{p!}\sum_{r=0}^{n-p}\frac{A_x^r}{r!}\sum_{s=0}^{n-r-p}\frac{A_y^s}{s!}} \, . \tag{12}$$

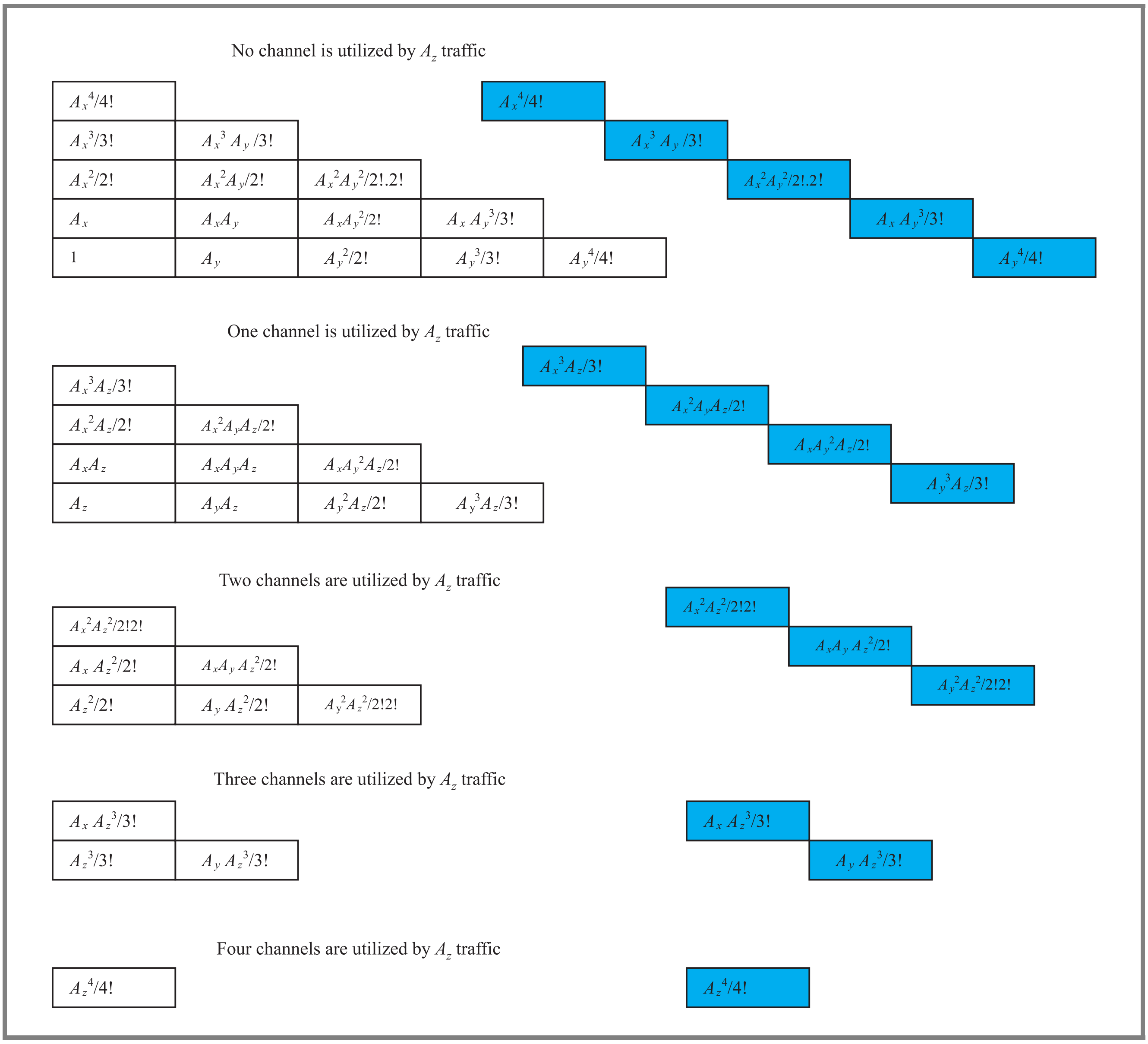

Fig. 2. Probability states of 3D traffic with complete utilization states before normalization ($n = 4$).

Probability states of Fig. 2 could be shown more explicitly in three-dimensional plain like Fig. 3, where each shaded state indicates complete utilization of channels by combination of traffic A_x, A_y and A_z. Arrows in z direction indicates blocking states of A_z traffic.

For non-uniform bandwidths above equations have to be modified according to the bandwidth of offered traffic A_x, A_y and A_z. Before going to mathematical derivation, let us assume that bandwidth of A_z traffic is twice wider than that of A_x or A_y. Now probability states of Fig. 3 would be modified like Fig. 4, where number of probability states are greatly reduced. Observing the trend of change of Figs. 3 and 4 would help us to derive generalized equation of probability state for non-uniform bandwidth traffic.

Let us consider a case when bandwidth of A_z traffic is h times greater than that of A_x or A_y of course bandwidth of A_x and A_y are equal. Blocking probability of Eq. (12) now would be modified like follows:

$$B(A_x,A_y,A_z,n,h) = \frac{\sum\limits_{p=0}^{\Omega} \frac{A_z^p}{p!} \sum\limits_{i=0}^{h-1} \sum\limits_{r=0}^{\phi} \frac{A_x^r}{r!} \frac{A_y^{\phi-r}}{(\phi-r)!}}{\sum\limits_{p=0}^{\Omega} \frac{A_z^p}{p!} \sum\limits_{i=0}^{n-hp} \frac{A_x^i}{i!} \sum\limits_{j=0}^{\phi} \frac{A_y^j}{j!}}, \qquad (13)$$

where $\Omega = \lfloor n/h \rfloor$ and $\phi = n - i - hp$ and $\phi \geq 0$.

Limited user's traffic is applicable to small networks like a micro/pico cell of a mobile cellular network with low offered traffic, cell of a wireless local loop (WLL)

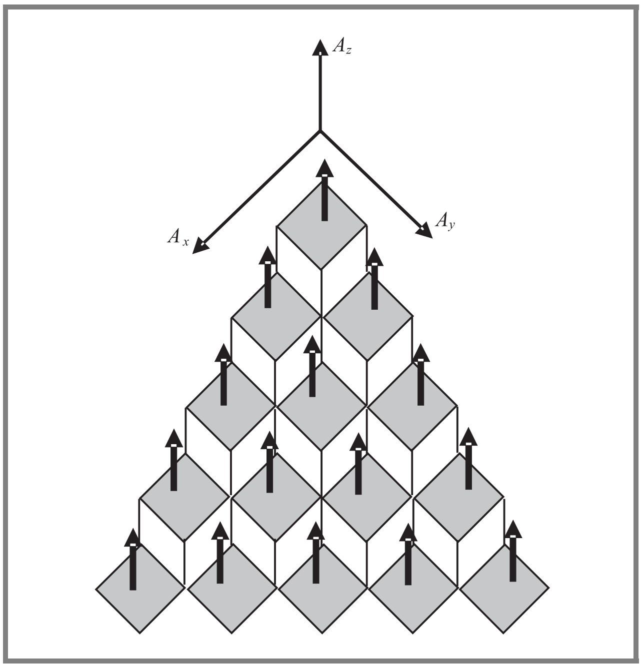

Fig. 3. Entire probability states in 3D plain for uniform BW ($n = 4$).

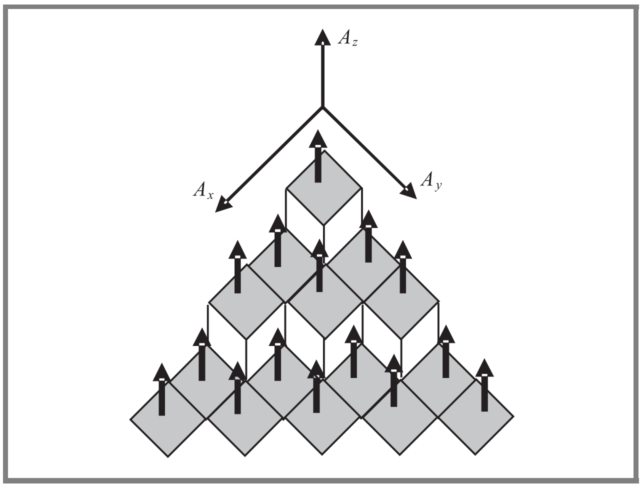

Fig. 4. Entire probability states in 3D plain for twice BW of A_z traffic ($n = 4$).

of rural area, pico net of Bluetooth, etc. For limited user case any probability state for uniform bandwidth would be like one shown below, based on Engset's traffic model of [18]:

$$P_{r,q,p} = \frac{\binom{N}{p} A_z^p \binom{M}{r} A_x^r \binom{O}{q} A_y^q}{\sum_{p=0}^{n} \binom{N}{p} A_z^p \sum_{r=0}^{n-p} \binom{M}{r} A_x^r \sum_{s=0}^{n-r-p} \binom{O}{s} A_y^s}, \qquad (14)$$

where M, O and N are the numbers of users for A_x, A_y and A_z traffic. Of course A_x, A_y and A_z are now considered as traffic intensity per user.

Probability of time congestion, i.e., percentage of total observation period, when the network remains blocked is derived based on papers [18, 19], like:

$$B_T = \frac{\sum_{p=0}^{n} \binom{N}{p} A_z^p \sum_{r=0}^{n-p} \binom{M}{r} A_x^r \binom{O}{n-r-p} A_y^{n-r-p}}{\sum_{p=0}^{n} \binom{N}{p} A_z^p \sum_{r=0}^{n-p} \binom{M}{r} A_x^r \sum_{s=0}^{n-r-p} \binom{O}{s} A_y^s}. \qquad (15)$$

Probability of call congestion, i.e., percentage of lost call is derived as

$$B_c = \frac{\sum_{p=0}^{n} (N-p) \binom{N}{p} A_z^p \sum_{r=0}^{n-p} (M-r) \binom{M}{r} A_x^r (O-n+r+p) \binom{O}{n-r-p} A_y^{n-r-p}}{\sum_{p=0}^{n} (N-p) \binom{N}{p} A_z^p \sum_{r=0}^{n-p} (M-r) \binom{M}{r} A_x^r \sum_{s=0}^{n-r-p} (O-s) \binom{O}{s} A_y^s}. \qquad (16)$$

For non-uniform traffic, Eqs. (15) and (16) can be modified according to the trend of Eqs. (12) and (13). Equations (12), (13), (15) and (16) reveal performance of a network in context of QoS.

4. Results and discussions

The number of complete occupied states by A_z traffic (shaded states) is 15 in both Figs. 3 and 4. In Fig. 3 the length of sample space is 35 but that of Fig. 4 is only 21; hence blocking probability of A_z traffic would be greater in the second case ($h = 2$) for the same number of channels. This phenomenon is depicted graphically in Figs. 5 to 7, where call blocking probability is varied against A_x, A_y for a fixed value of A_z as parameters. Values of A_x/A_y is varied from 0 to 5 erl with $h = 1$, 2, 3, $A_z = 2$ erl and $n = 14$. For limited user case call and time congestion are plotted in similar way varying A_x/A_y

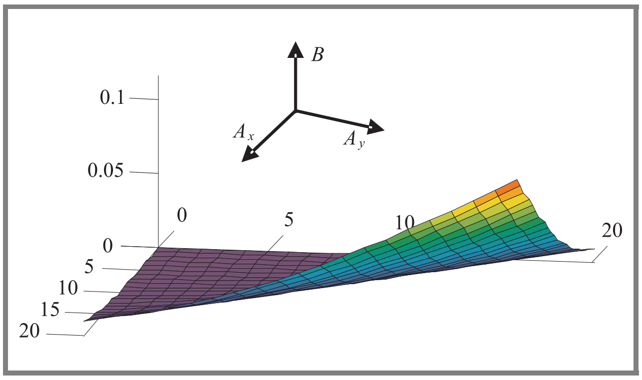

Fig. 5. Variation of blocking probability against A_x and A_y for a fixed value of A_z ($h = 1$, $A_z = 2$, $n = 14$).

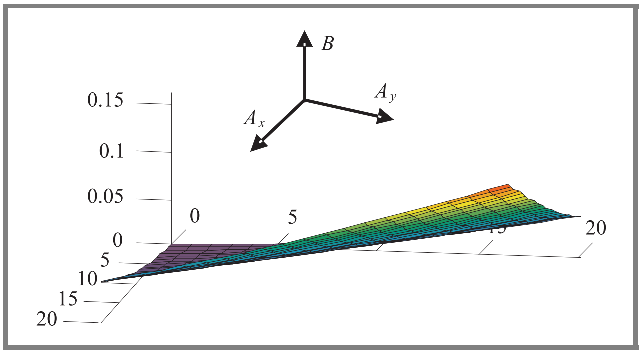

Fig. 6. Variation of blocking probability against A_x and A_y for a fixed value of A_z ($h = 2$, $A_z = 2$, $n = 14$).

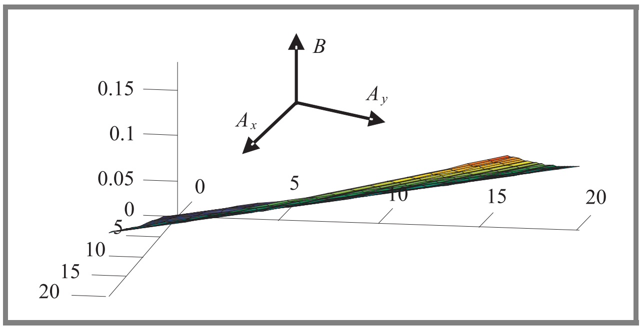

Fig. 7. Variation of blocking probability against A_x and A_y for a fixed value of A_z ($h = 3$, $A_z = 2$, $n = 14$).

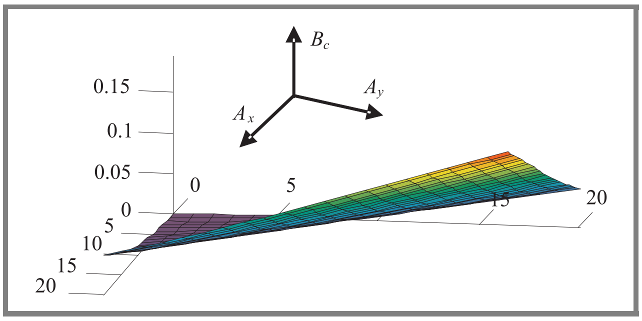

Fig. 8. Variation of call congestion against A_x and A_y for a fixed value of A_z ($h = 1$, $M = 100$, $N = 100$, $O = 100$, $n = 14$, $A_z = 0.05$ erl/user).

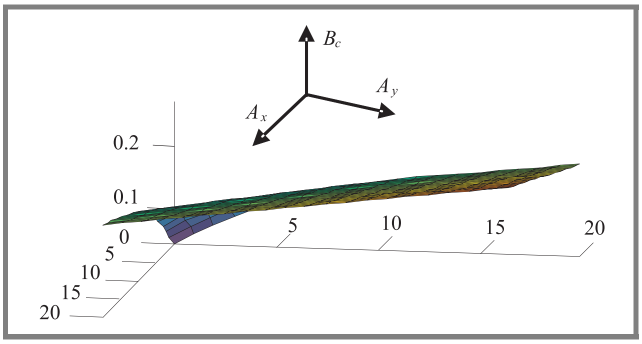

Fig. 9. Variation of call congestion against A_x and A_y for a fixed value of A_z ($h = 3$, $M = 100$, $N = 100$, $O = 100$, $n = 14$, $A_z = 0.05$ erl/user).

from 0 to 0.05 erl/user, with $h = 1$ and 3, $M = 100$, $N = 100$, $O = 100$, $n = 14$, $A_z = 0.05$ erl/user. Comparison of Figs. 8 and 9 or Figs. 10 and 11 reveals the same impact of h like unlimited users case.

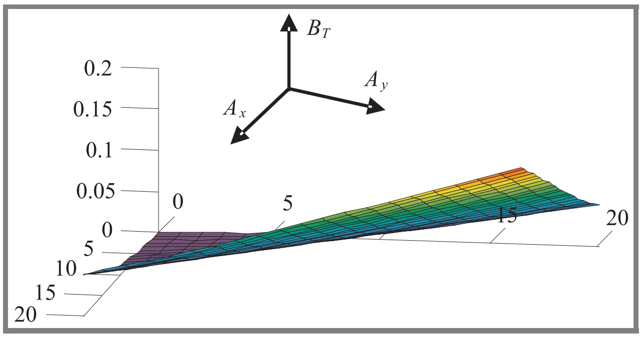

Fig. 10. Variation of time congestion against A_x and A_y for a fixed value of A_z ($h = 1$, $M = 100$, $N = 100$, $O = 100$, $n = 14$, $A_z = 0.05$ erl/user).

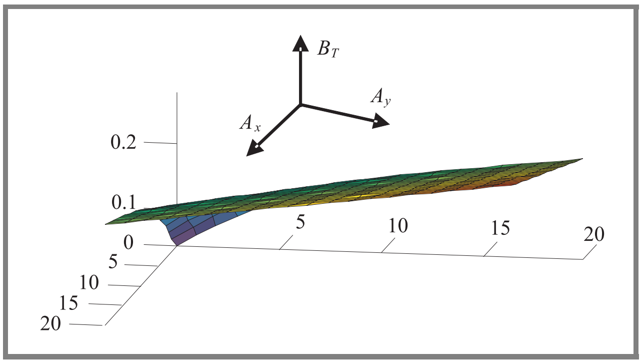

Fig. 11. Variation of time congestion against A_x and A_y for a fixed value of A_z ($h = 3$, $M = 100$, $N = 100$, $O = 100$, $n = 14$, $A_z = 0.05$ erl/user).

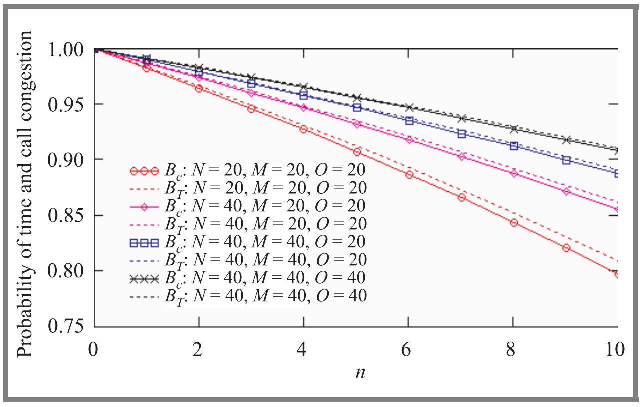

Fig. 12. Comparison of time and call congestion against number of channel (n).

Figure 12 makes a comparison of time and call congestion probability, where dotted lines are represent time and solid lines represent call congestion probability taking, $M = N = O = 20$ and $M = N = O = 40$. Here the number of users is lowered to visualize distinct separation between time and call congestion. All the graphs in this paper show very high blocking probabilities because of taking large offered traffic compared to number of channels. In real life environment, however, the number of channels would be selcted based on offered traffic and required QoS, using Eqs. (12), (13) and (15), (16). We have expected before that the entire analysis presented here can be done using three-dimensional Markovian chain, node equation and inverse matrix technique to solve the linear node equation like described in [20–23] but the analysis would be very complicated in comparison to our model.

5. Conclusion

This paper proposes a mathematical model of analyzing 3D limited and unlimited user's traffic of dissimilar bandwidth. All the graphs shown in this paper (Figs. 5–12) yield a logical result. Combination of limited and unlimited user's traffic even different call admission schemes like "duel threshold bandwidth reservation", "dynamic partition" could be done quite comfortably based on concept of the paper.

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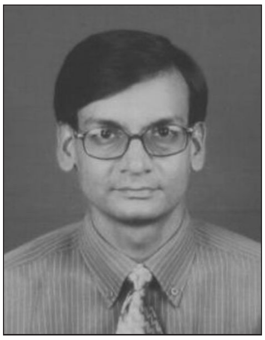

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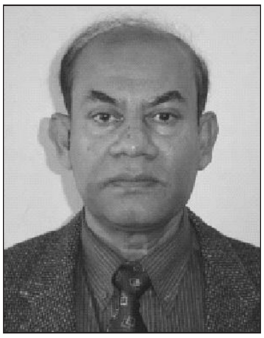

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