Paper

CDMA wireless system with blind multiuser detector

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Abstract— In this paper we present an approach capable of countering the presence of multiple access interference (MAI) in code division multiple access (CDMA) channels. We develop and implement a blind multiuser detector, based on an independent component analysis (ICA) to mitigate both MAI and noise. This algorithm has been utilized in blind source separation (BSS) of unknown sources from their linear mixtures. It can also be used for estimation of the basis vectors of BSS. The aim is to include an ICA algorithm within a wireless receiver in order to reduce the level of interference in CDMA systems. This blind multiuser detector requires less precise knowledge of the channel than does the conventional single-user receiver. The proposed blind multiuser detector is made robust with respect to imprecise knowledge of the received signature waveforms of the user of interest. Several experiments are performed in order to verify the validity of the proposed learning algorithm.

Keywords— code division multiple access, independent component analysis, blind source separation.

1. Introduction

Code division multiple access (CDMA) multiuser detection has undergone rapid evolution through significant research and development activity in telecommunications [12, 13]. With the ever-growing sophistication of signal processing and computation, multiuser detection exploits the potential needs to increase capacity in multiuser radio channels. It deals with the demodulation of mutually interfering signals in applications such as cellular telephony, satellite communication and digital radio.

In general, multiuser detection is also known as cochannel interference suppression, multiuser demodulation, and interference cancellation to deal with the demodulation of digitally modulated signals in the presence of a multiaccess interference. Motivated by the channel environment encountered in many CDMA applications, the design of multiuser detectors for channels with fading, multipath, or noncoherent modulation has attracted considerable attention [6, 12]. An adaptive multiuser detector which converges to the minimum mean squared error (MMSE) detector without requiring training sequences is proposed in [6]. This proposed blind multiuser detector is designed with imprecise knowledge of the received signature waveform of the desired user. In [15] a blind adaptive multiuser detector based on Kalman filtering in both a stationary and a slowly time-varying environment is proposed. The author showed that the steady-state excess output energy of the Kalman filtering algorithm is identically zero for a stationary environment. Also, Verdu presented an overview of the adaptive tentative-decision based detectors in [13]. Verdu mentioned that the linear MMSE has the features of the decorrelating detector, except that it requires knowledge of the received amplitudes. On the other hand, the tentative decision based multiuser detector is the simplest idea for successive cancellation, but the disadvantage is that it requires extremely accurate estimation of the received amplitudes [12, 13]. Meanwhile, Verdu's work has provided exceptional important reference and guidance for the implementation of the following work.

The goal of this paper is to introduce a blind multiuser detector that adaptively recovers the signals from multiple users. In this context, the blind (or non-data aided) multiuser detector means *it requires no training data sequence, but only the knowledge of the desired user signature sequence and its timing* [9]. The proposed blind multiuser detector employs iterative an independent component analysis (ICA) algorithm at the outputs of a bank of matched filters. The main motivation of employing blind multiuser detectors in CDMA is to recover the original users' sequences from the received signals that are corrupted by multiple access interference (MAI), without the help of training sequences and a priori knowledge of the channel.

The rest of this paper is organized as follows. Section 2 gives a description of the blind multiuser detector model. Section 3 discusses the proposed ICA algorithm. A performance analysis and system capacity discussion is given in Section 4 and concluding remarks are given in Section 5.

2. Blind multiuser detector

2.1. Channel model

In DS-CDMA, each user spreads its information signal in frequency by direct sequence modulation before transmission via the common channel (Fig. 1).

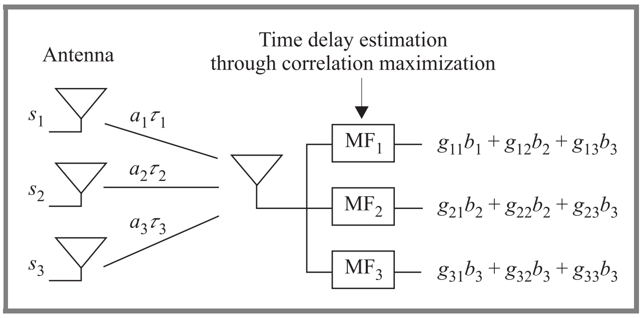

Fig. 1. K = 3-users detector for multiple access Gaussian channel.

We consider the K-user binary phase shift keying (BPSK) asynchronous DS-CDMA white Gaussian model as given in Eq. (1):

$$r(t) = \sum_{i=-J}^{J} \sum_{k=1}^{K} A_k b_k(i) s_k(t - iT - \tau_k) + \sigma n(t)\,, \qquad (1)$$

where:

- $2J{+}1$ symbols are sent by each of the K users;
- the kth signature waveform s_k is assumed to have unit energy ($\|s_k\| = 1$); $\tau_k \in [0, T)$ is the kth user's offset, where T is the symbol period; these signature sequences are independent of the data symbol, and have a chip rate much higher than that of the desired user information;
- A_k is the received amplitude of the kth user;
- b_k is the independent input data symbol of the kth user, $b_k \in \{-1, +1\}$;
- the kth signature waveform s_k is determined by the random pseudo-noise (PN) spreading sequence c_k and pulse shape waveform $p(t)$:

  $$s_k(t) = \sum_{i=0}^{N_{PG}-1} c_k(i) p(t - iT_c)\,, \qquad (2)$$

  where $s_k(t)$ is assumed to have unit energy over the symbol interval:

  $$\begin{aligned} T = N_{PG}T_c & \quad \text{symbol interval}, \\ T_c & \quad \text{chip interval}, \\ N_{PG} & \quad \text{processing gain}; \end{aligned}$$

  in this paper, we consider gold code spreading sequences; these signature sequences are independent of the data symbols and have a chip rate much higher than the symbol rate;
- the additive white Gaussian noise $n(t)$ is stationary and memoryless with unit power spectral density;
- σ^2 is the variance of noise.

We assume the users transmit completely asynchronously. In this context, when there are timing errors, each user's code experiences a random delay during the transmission and the received signal is no longer aligned with the locally generated codes [4].

For simplicity, we consider only one symbol interval. The representation for the signal during one symbol interval is written in vector form as

$$r(t) = \sum_{k=1}^{K} A_k b_k(i) s_k(t - \tau_k) + \sigma n(t)\,. \qquad (3)$$

At the receiver, the signal in Eq. (1) is chip-matched filtered and sampled at the bit rate $(1/T_b)$. The chip-matched filtered signal can be represented as

$$x_m(t) = \frac{1}{T}\int_0^T r(t) s_m(t - \tau_m + \Delta\tau_m) dt\,, \qquad (4)$$
$$m = 1, \ldots, K,$$

where we assume a correlation maximization (or similar) operation is performed to approximately time-align the mth matched filter to the time delay τ_m of the mth user signal.

Following the sampling operation, we have:

$$x_m(i) \;=\; \text{sampled}[x_m(t)] = \sum_{k=1}^{K} g_{mk} b_k(i) + \sigma_m n(i)\,. \qquad (5)$$

The set of match-filtered signals can be represented as

$$\mathbf{x}(i) = \mathbf{G}\mathbf{b}(i) + \sigma\mathbf{n}(i)\,, \qquad (6)$$

where $\mathbf{G}$ is the matrix $\{g_{mk}\}$, m = 1, …, K, k = 1, …, K, $\mathbf{b}(i)$ = $[b_1(i), b_2(i), \ldots, b_k(i)]^T$ and $\mathbf{n}(i)$ is a (K x 1) vector of noise samples.

2.2. Source independence

In the CDMA receiver, both code timing and channel estimation are often prerequisite tasks. Detection of the desired user's symbols in the CDMA system is far more complicated than in the simpler time division multiple access (TDMA) and frequency division multiple access (FDMA) systems used previously in mobile communications. Our main goal is to estimate and recover the original transmitted symbols. Several techniques are available

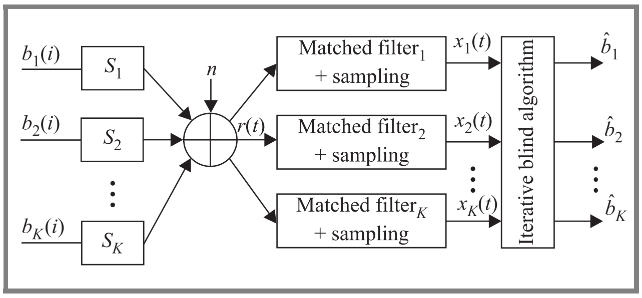

Fig. 2. K-user detection model.

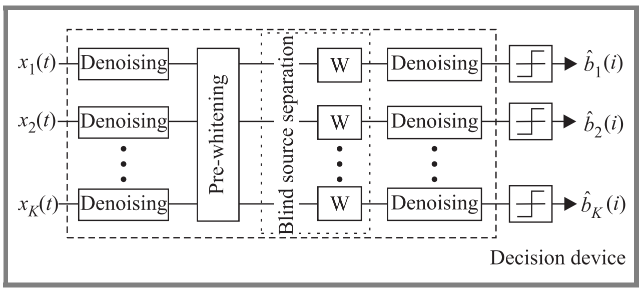

Fig. 3. The proposed blind receiver consists of PCA pre-whitening, ICA-BSS and wavelet denoising stages.

to estimate the desired user's symbols. In general, the matched filter (correlator) is the simplest estimator, but it performs well only if different users' chip sequences are orthogonal or the users have equal powers [2].

Recently, there have been attempts to apply blind and semi-blind signal processing models and algorithms in a wide variety of digital communications applications, for example multi access communications systems, multi sensor sonar and radar systems. Several good algorithms are also available for solving the basic linear and nonlinear ICA problem [1, 3, 5, 7, 10].

We propose to apply independent component analysis to design a new blind CDMA receiver. The main reason for using ICA in the CDMA receiver is because each path and user symbol sequence is typically independent of each other. The proposed multiuser detection algorithm is applied at the receiver after the wavelet denoising [8] (Figs. 2 and 3).

3. Proposed multiuser detection algorithm

The proposed algorithm is generalized from Amari's natural gradient algorithm [11]. This algorithm involves minimization of a multivariate cost function according to the stochastic gradient descent algorithm, as discussed later. The proposed algorithm, includes a pre-processing stage involving principal component analysis (PCA) (see [2, chap. 6, pp. 125–144] and [3]) of the measured sensor signals. Pre-processing is employed to pre-whiten the received signal vector, as discussed below.

3.1. Principle component analysis

Whitening of the received data $\mathbf{x}(i)$ is a common pre-processing task in ICA. In particular, a pre-whitening procedure is used mainly to decorrelate the sensor signals before separation. This makes the subsequent separation task easier, the separating matrix is then constrained to be orthogonal. There is no explicit assumption on the probability density of the vectors made in PCA [2, chap. 6, pp. 125–144], as long as the first and second order statistics are known or can be estimated from the mixture. The pre-whitened signal vector is given by

$$\mathbf{u}(i) = \boldsymbol{D}^{-1/2}\mathbf{E}^T\mathbf{x}(i)\,, \tag{7}$$

where $\mathbf{u} = [u_1, \ldots, u_K]^T$, $\mathbf{E} = (e_1, \ldots, e_K)$ is the matrix whose columns are the unit-norm eigenvectors of the covariance matrix $C_x = E\{\ \mathbf{x}(i)\ \mathbf{x}(i)^T\}$ and $\boldsymbol{D} = \mathrm{diag}(d_1, \ldots, d_K)$ is the diagonal matrix of the eigenvalues of C_x.

3.2. Proposed ICA algorithm

We now discuss the proposed ICA algorithm to unmix the source signals in the presence of noise. In the multiuser channel, $\mathbf{y}(i) = \mathbf{W}(i)\mathbf{u}(i)$, where $\mathbf{u} = [u_1, \ldots, u_K]^T$. The output components become $\mathbf{y} = g(\mathbf{u}(i))$, where the $g(\mathbf{u}(i))$ is an invertible nonlinearity. Bell and Sejnowski have shown [3] that by maximizing the join entropy of $H(\mathbf{y})$ for the neural process output can approximately minimize the mutual information among the output components $\mathbf{y}$. In this case, maximizing the joint entropy $H(y_1, y_2)$ of $K = 2$ output symbols, y_1 and y_2, consists of maximizing the individual entropy of each output while minimizing the mutual information $\Im(y_1, y_2)$ shared between these two output symbols [3]. The mutual information $\Im(\mathbf{y})$ between K output symbols can be deduced via Kullback-Leibler divergence:

$$\begin{aligned}\Im(\mathbf{y}) &= -H(\mathbf{y}) + \sum_{k=1}^{K} H_k(y_k)\\ &= \int_{-\infty}^{\infty} p(\mathbf{y})\log p(\mathbf{y})d\mathbf{y} - \sum_{k=1}^{K}\int_{-\infty}^{\infty} p(\mathbf{y})\log p_k(y_k)d\mathbf{y}\\ &= \int_{-\infty}^{\infty} p(\mathbf{y})\log\frac{p(\mathbf{y})}{\sum\limits_{k=1}^{K} p_k(y_k)}d\mathbf{y}\,,\end{aligned} \tag{8}$$

when the mutual information $\Im(\mathbf{y})$ is equal to zero, these K variables are statistically independent.

Then, the above mentioned differential entropy H of a random vector y_i with density $p(y_i)$ can be rewritten as

$$H(\mathbf{y}) = H(y_1)+,\ldots,+H(y_k) - \Im(\mathbf{y})\,, \tag{9}$$

where $H(y_1) = -E\Big\{\log\ \frac{p(u_1)}{|\frac{\partial y_1}{\partial u_1}|}\Big\}$,

$$\begin{aligned}H(\mathbf{y}) &= -\sum_{k=1}^{K} E\Big\{\log\frac{p(u_k)}{|\frac{\partial y_k}{\partial u_k}|}\Big\} - \Im(\mathbf{y})\\ &= -E\Big\{\log\frac{p(u_1)}{|\frac{\partial y_1}{\partial u_1}|}\Big\} + \ldots - E\Big\{\log\frac{p(u_k)}{|\frac{\partial y_k}{\partial u_k}|}\Big\} - \Im(\mathbf{y}).\end{aligned} \tag{10}$$

The goal is to learn the elements of the linear unmixing matrix $\mathbf{W}$ and the set of parameters for the nonlinearities $g(u_k(i))$. This algorithm is used to update the unmixing matrix $\mathbf{W}$. In detail, $\mathbf{W}$ is an estimate of the unknown mixing matrix of $\mathbf{u}(i)$. Using a gradient ascent algorithm, we consider the derivative of the entropy function with respect to $\mathbf{W}$ and the parameters of the nonlinearity is:

$$\begin{aligned}\frac{\partial}{\partial\mathbf{W}}(\Im(\mathbf{y})) &= -\frac{\partial H(\mathbf{y})}{\partial\mathbf{W}} - \frac{\partial}{\partial\mathbf{W}}\sum_{k=1}^{K} E\Big\{\log\frac{p(u_k)}{|\frac{\partial y_k}{\partial u_k}|}\Big\}\\ &= -(\mathbf{W}^T)^{-1} - \left(\frac{\frac{\partial p(\mathbf{u})}{\partial\mathbf{u}}}{p(\mathbf{u})}\right)\mathbf{u}^T.\end{aligned} \tag{11}$$

Following the work of [3, 10], we employ the following learning rule for $\mathbf{W}$

$$\Delta\mathbf{W}(\mathrm{p}) = -\alpha\frac{\partial\Im(\mathbf{y})}{\partial\mathbf{W}}\mathbf{W}^T\mathbf{W}\,, \tag{12}$$

where p is the iteration index and α the learning rate (refer to Appendix).

After initializing the weight matrix $\mathbf{W}$ and choosing α (sufficiently small value, e.g., 0.0001), the weights are iteratively updated according to the learning rule. In our observation, the learning process usually depends on the activities of the weights $\mathbf{W}$, the learning rate α, the input and output values of the mixture:

$$\mathbf{W}(\mathsf{p}+1) = \mathbf{W}(\mathsf{p}) + \alpha(\mathbf{I} - g(\mathbf{y})\mathbf{y}^T + \mathbf{y}(g(\mathbf{y}))^T)\mathbf{W}(\mathsf{p}), \quad (13)$$

where p is the iteration index.

The proposed algorithm for complex signals performs as follows:

1. Chip-matched filtered signals, wavelet denoising.
2. PCA pre-whitening the signals.
3. Select an initial separating matrix $\mathbf{W}_0$ and learning rate α.
4. Determine and estimate the initial, $\mathbf{y} = \mathbf{W}_0\mathbf{u}$.
5. Update the separating matrix by $\mathbf{W}_{\mathsf{p}+1} \longleftarrow \mathbf{W}_{\mathsf{p}} + \alpha(\mathbf{I} - g(\mathbf{y})\mathbf{y}^T + \mathbf{y}(g(\mathbf{y}))^T)\mathbf{W}_{\mathsf{p}}$, where $\mathbf{I}$ is the identity matrix.
6. Decorrelate and normalize $\mathbf{W}_{\mathsf{p}+1}$.
7. If $|(\mathbf{W}_{\mathsf{p}+1})^T\mathbf{W}_{\mathsf{p}}|$ is not close enough to 1, then $\mathsf{p} = \mathsf{p}+1$, and go back to Step 5. Else, output the vector $\mathbf{W}_{\mathsf{p}}$.
8. Wavelet denoising.
9. Output detector, sgn $(\mathbf{y})$.

3.3. Error measure

The performance during the learning process was monitored by an error measure based on:

$$\mathbf{PI} = \frac{1}{K^2}\Bigg(\sum_{i=1}^{K}\Big(\sum_{j=1}^{K}\frac{|PD_{ij}|}{\max_k|PD_{ik}|} - 1\Big) + \sum_{j=1}^{K}\Big(\sum_{i=1}^{K}\frac{|PD_{ij}|}{\max_k|PD_{kj}|} - 1\Big)\Bigg), \quad (14)$$

where PD_{ij} is the (i,j)th element of $\mathbf{PD} = \mathbf{WG}$, $\mathbf{G}$ is the unknown mixing matrix and K is the number of users. $\mathbf{PD}$ is close to the permutation of the scaled identity matrix when the sources are separated. This corresponds to $\mathbf{PI}$ = 0.

4. Numerical experiments

The proposed blind multiuser detector has been examined in various experimental situations. Several results are presented to compare the proposed blind multiuser detector with correlating detector, matched filter bank and blind MMSE detectors [6]. For each run, these 4 detectors are applied at the same time. The following experiments are mainly to demonstrate the performance of the multiuser detectors with varying signal-to-noise ratio (SNR) levels and power levels. These experiments are also to demonstrate the performance of the proposed method in multiuser interference (MAI).

We consider using a simulated DS-CDMA data with additive white Gaussian noise (AWGN) channel and two antenna elements in the reception with a half a carrier wavelength spacing, unless mentioned otherwise. All CDMA signals are generated with BPSK data modulation and gold codes of length 61 are used as the spreading codes. The length of the block was 40 non-coherent BPSK symbols, during which the channel was fixed. The number of signals distribution, and the path delays were randomly chosen. Matched-filter bank, decorrelating detector and blind MMSE detector receivers were used as reference methods.

We first present the performance of the proposed algorithm by presenting the numerical values of the bit error rate (BER) as a function of SNR in Fig. 4. The system consists of K = 2 users and both users are assigned with equal power. The proposed ICA detector based method shows the lowest BER compared to blind MMSE method, matched filter bank and decorrelating detector especially at lower SNR. The convergence of the gradient approach took place in 10–15 iterations in this case. The ICA detector displays better performance compared to the matched filter bank, and decorrelating detectors. However, the performances of the proposed ICA and adaptive blind MMSE detector are very close to each other. The adaptive blind MMSE detector slightly outperforms the ICA detector from 9 dB to 11 dB. Then, ICA detector shows better performance after 12 dB onwards. The margin of improvement becomes larger with increased SNR.

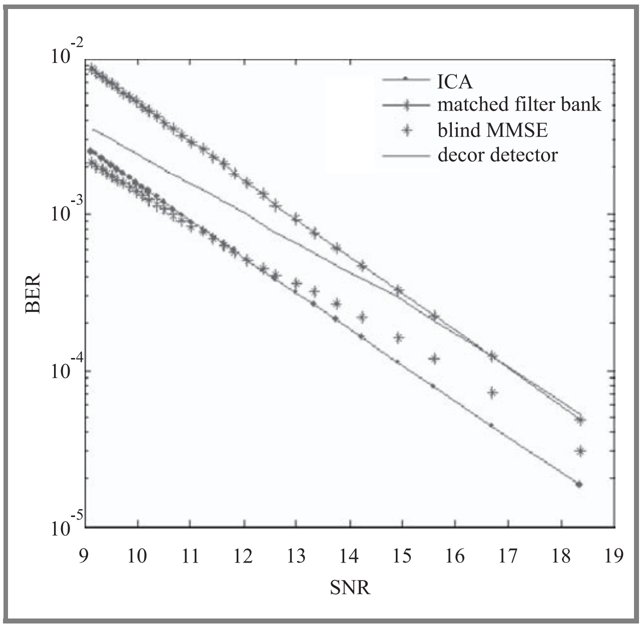

Fig. 4. Bit error rate as a function of SNR for decorrelating detector, matched filter bank, blind MMSE and ICA detectors.

We then implement the multiple access interference channels to demonstrate the ability of the mentioned detectors dealing with large number of users within the same channel as shown in Fig. 5. ICA analysis proved to be better performing technique; it is then followed by the blind MMSE detector, matched filter bank and the decorrelating detectors. We observed that the matched filter and decorrelating detector are not able to work in multiple access interference environment (which industries require BER $\leq 10^{-3}$), which the figure shown high BER with increasing MAI.

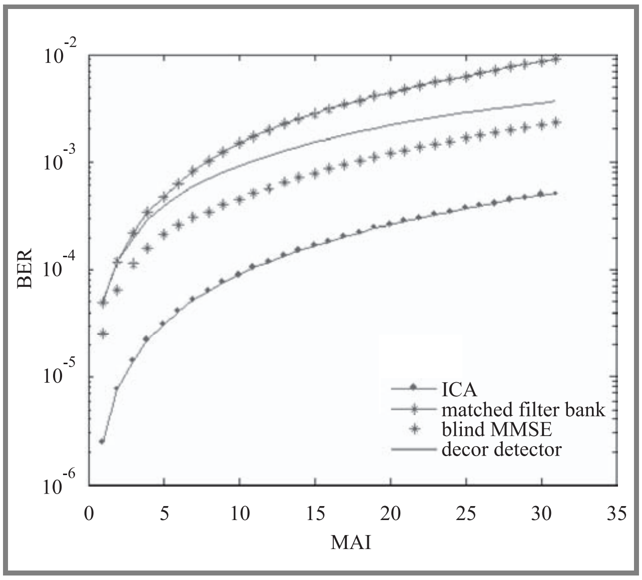

Fig. 5. Bit error rate plots versus K users in MAI channel using ICA, blind MMSE, matched filter bank and decorrelating detectors.

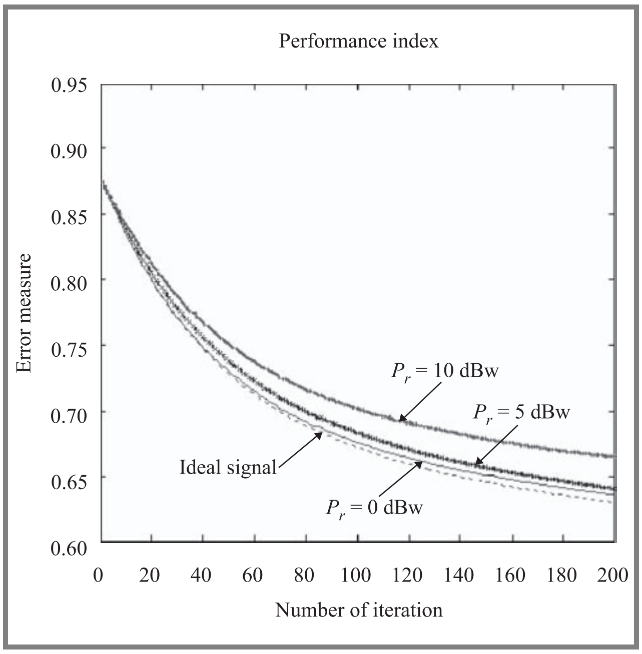

Fig. 6. Error measure for various power levels of multi access interference using the ICA multiuser detector.

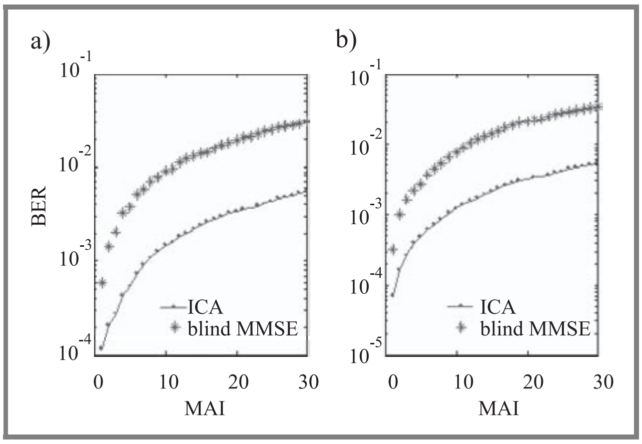

Fig. 7. Bit error rate versus MAI for SNR = 8 dB for various levels of signal power: (a) 15 dBw; (b) 5 dBw.

The error measure due to MAI is illustrated graphically in Figs. 6 and 7 for CDMA system. In this experiment, power level of the interfering signals (P_r) are 0 dBw, 5 dBw and 10 dBw respectively with SNR of 10 dB and the desired signal at 0 dBw. For comparison purposes, we include an "ideal" case, coresponding to no MAI and an SNR of 16 dB. Clearly, the ICA detector shows better performance, in which the error measure for the intefereing signal cases of 0 dBw, 5 dBw and 10 dBw is 0.635, 0.64 and 0.665, respectively. This is due to the proposed detector's denoising nature dealing with noisy channels.

5. Discussion

We have proposed a new methodology for the design of asynchronous multiuser CDMA system. The design is based on blind source separation in the DS-CDMA communication system by means of independent component analysis. The blind CDMA detectors are interference cancellers with ICA analysis to decrease the cross correlation between the users by employing multiple matched filters at the receiver. Since the signature sequences are known a priori, the accuracy obtained when estimating these parameters becomes high. The experimental results show that the proposed blind ICA multiuser detectors perform better in multiaccess interference than the blind MMSE, matched filter bank and decorrelating detectors. In particular, the main reasons for considering ICA as an additional tuning element in the next generation CDMA system are the following:

- ICA is worth considered as an additional element, attached to some existing receiver structure to perform the task of user identification.
- Since the original CDMA detection and estimation methods do not exploit the powerful but realistic independence assumption [2], ICA (with the independence of the source signals is utilized) would offer an

additional interference suppression capability to the CDMA detection [14].

- Since the receiver has some prior information on the communication system; typically at least the spreading code of the desired user is known, ICA can easily function in CDMA receiver.
- ICA is particularly to unmix the mixed signals to recover the original source signals. Therefore, it is able to mitigate additional multiple access interference to enhance the performance of detectors.

6. Conclusion

In this paper, we have designed a blind ICA multiuser detector based on the ICA algorithm. Several simulation results show that the blind multiuser detector provides a significant performance improvement compared to other multiuser detectors. We conclude that blind ICA detector is suitable for the next generation wireless CDMA communication system.

Appendix

The update for the mixing matrix $\mathbf{W}$ is determined via the gradient of the mutual information with respect to the elements of $\mathbf{W}$. Essentially, $\mathbf{W}$ is an estimate of $\mathbf{G}^{-1}$, where $\mathbf{G}$ is the unknown mixing matrix of $\mathbf{u}(i)$.
The updated elements of $\mathbf{W}$ in the natural gradient based optimization algorithm are given by

$$\mathbf{W}_{\mathsf{update}} = \mathbf{W} + \triangle\mathbf{W} = \mathbf{W} - \frac{\partial \Im(y_1,\ldots,y_K)}{\partial \mathbf{W}}\mathbf{W}^T\mathbf{W}, \quad (15)$$

where $\Im(y_1,\ldots,y_K)$ is the mutual information between the output signals where:

$$\begin{aligned}\Im(y_1,...,y_K) &= E\{\log\ p(\mathbf{u})\} - \log(\det\mathbf{W}) \\ &\quad - \sum_{k=1}^{K} E\{\log p_k(y_k)\}. \end{aligned} \quad (16)$$

When the mutual information $\Im(\mathbf{y})$ is equal to zero, these variables $y_1,\ldots,y_K$ are statistically independent. The gradient of $\Im(y_1,\ldots,y_K)$ with respect to $\mathbf{W}$ can be expressed as

$$\begin{aligned}&\frac{\partial \Im(y_1,\ldots,y_K)}{\partial \mathbf{W}} \\ &= \frac{\partial E\{\log(p(\mathbf{u}))\}}{\partial \mathbf{W}} - \frac{\partial\{\log(\det\mathbf{W})\}}{\partial \mathbf{W}} - \frac{\partial \sum_{k=1}^{K} E\{\log\ p(y_k)\}}{\partial \mathbf{W}} \\ &= -\frac{\partial\{\log(\det\mathbf{W})\}}{\partial \mathbf{W}} - \sum_{k=1}^{K}\frac{\partial E\{\log\ p(y_k)\}}{\partial \mathbf{W}} \end{aligned} \quad (17)$$

since the first term, $E\{\log\ p(\mathbf{u})\}$ does not involve $\mathbf{W}$. We will analyze the two remaining terms separately. In the case of the first term, we have:

$$\begin{aligned}\frac{\partial\{\log(\det\mathbf{W}\}}{\partial \mathbf{W}} &= \frac{1}{\det\mathbf{W}}\frac{\partial \det\mathbf{W}}{\partial \mathbf{W}} \\ &= \frac{1}{\det\mathbf{W}}(adj(\mathbf{W}))^T \\ &= (\mathbf{W}^{-1})^T. \end{aligned} \quad (18)$$

From the second term in Eq. (19), we have incorporated the density function $p_k(y_k)$:

$$\begin{aligned}&\sum_{k=1}^{K}\frac{\partial E\{\log(p(y_k))\}}{\partial \mathbf{W}} \\ &= \sum_{k=1}^{K} E\left\{\frac{1}{p_k(y_k)}\frac{\partial p_k(y_k)}{\partial(y_k)}\frac{\partial y_k}{\partial \mathbf{W}}\right\} \\ &= E\begin{pmatrix} \frac{1}{p_1(y_1)}\frac{\partial p_1(y_1)}{\partial(y_1)}u_1 & \ldots & \frac{1}{p_1(y_1)}\frac{\partial p_1(y_1)}{\partial(y_1)}u_K \\ \vdots & & \vdots \\ \frac{1}{p_K(y_K)}\frac{\partial p_K(y_K)}{\partial(y_K)}u_1 & \ldots & \frac{1}{p_K(y_K)}\frac{\partial p_K(y_K)}{\partial(y_K)}u_K \end{pmatrix} \\ &= E\left\{\frac{1}{p(\mathbf{y})}\frac{\partial p(\mathbf{y})}{\partial(\mathbf{y})}\mathbf{u}^T\right\}, \end{aligned} \quad (19)$$

where by $p(\mathbf{y})$ we mean $(p_1(y_1),\ldots,p_K(y_K))$.

The natural gradient of $\Im(y_1,\ldots,y_K)$ is given in Eq. (20). The minimum mutual information algorithm for ICA will repeatedly perform an update of the matrix $\mathbf{W}$:

$$\begin{aligned}\Delta\mathbf{W}_{\mathsf{p}} &= \mathbf{W}_{\mathsf{p+1}} - \mathbf{W}_{\mathsf{p}} \\ &= -\frac{\partial \Im(\mathbf{y})}{\partial \mathbf{W}}\mathbf{W}^T\mathbf{W} \\ &= [\mathbf{I} - g(\mathbf{y})\mathbf{y}^T]\mathbf{W}, \end{aligned} \quad (20)$$

where $\mathbf{I}$ is the identity matrix and

$$g(\mathbf{y}) = \frac{1}{p(\mathbf{y})}\frac{\partial p(\mathbf{y})}{\partial \mathbf{y}} = \frac{\partial}{\partial \mathbf{y}}\log(p(\mathbf{y})). \quad (21)$$

The multiplication with the natural gradient not only preserves the direction of the gradient but also speeds up the convergence process.
The formulation of Eq. (20) requires that each $\{g_k(y_k)\}_{k=1}^{K}$ is a nonlinear function corresponding to a symmetric density. Ideally the nonlinear function $g_k(y_k)$ approximates

the probability density function of y_k. The nonlinear function applied in this work is as follows:

$$g_k(y_k) = \mathrm{abs}(y_k^{0.9}(i)) \cdot \mathrm{sgn}(y_k(i))\,. \qquad (22)$$

After initializing the weight matrix $\mathbf{W}_0$ with identity matrix, and choosing α as sufficiently small constant, e.g., 0.0001, the weights are iteratively updated according to the learning rule in Eq. (23). Indeed, the learning process usually depends on the activities of the weights $\mathbf{W}$, learning rate α, nonlinearity g($\mathbf{y}$), input and output values of the mixture. The Eq. (20) is extended as

$$\mathbf{W}_{\mathsf{p}+1} = \mathbf{W}_{\mathsf{p}} + \alpha\big(\mathbf{I} - g(\mathbf{y})\mathbf{y}^T + \mathbf{y}(g(\mathbf{y}))^T\big)\mathbf{W}_{\mathsf{p}}\,, \qquad (23)$$

where p is the iteration index, $\mathbf{I}$ is the identity matrix, and the estimated output $\mathbf{y}_{\mathsf{p}}(i) = \mathbf{W}_{\mathsf{p}}\mathbf{u}(i)$.

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