Regular paper

Sensitivity of microwave radiometers with square – law and linear detectors

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Abstract — Stochastic analysis of modulation microwave radiometers with square – law and linear detectors is presented in the paper. Assuming ideal detector characteristics it is shown that in typical applications, i.e., in very low power measurements, a type of detector used is of no influence on total radiometer sensitivity. Other aspects of use of a particular detector are also presented.

Keywords — microwave thermograph, radiometer, thermal radiation, sensitivity.

1. Introduction

A modulation radiometer [3], presented in Fig. 1, was used for analysis. A high frequency (HF) amplifier can be described by the statistically equivalent pass band B_{HF} defined as [1, 4, 5]:

$$B_{HF} \overset{df.}{=} \frac{1}{2\pi} \frac{\left[\int\limits_{-\infty}^{\infty} G(\omega)\mathrm{d}\omega\right]^2}{2\int\limits_{-\infty}^{\infty} G^2(\omega)\mathrm{d}\omega}, \tag{1}$$

where $G(\omega)$ is the double-sided characteristics of RMS power gain of a HF amplifier.
B_{HF} is a pass band of a hypothetical rectangular filter that transfers signal with the same statistical error of mean-square value as a real filter when white noise is present at its input.
Low frequency (LF) amplifier is characterised by the noise pass band B_{LF} defined as [1, 4]:

$$B_{LF} \overset{df.}{=} \frac{1}{2\pi} \frac{\int\limits_{0}^{\infty} |H(\omega)|^2\mathrm{d}\omega}{|H(0)|^2}, \tag{2}$$

where $H(\omega)$ is the transmittance function of an LF amplifier and output filter system, defined for real frequencies.
B_{LF} is a pass band of a hypothetical rectangular filter that transfers signal with the same mean-square value as a real filter, when white noise is present at its input. Therefore, a band defined like this can be a useful measure of a bandwidth applied in narrowband measurements of mean-square values.
A nonlinear inertialess system was used as a detector. It was characterised by a double-half square function in the form of

$$U_{det} = \beta U_{in}^2 \tag{3}$$

or by single-half linear function in the form of

$$U_{det} = \begin{cases} \gamma U_{in} & \text{for} \quad U_{in} \geqslant 0 \\ 0 & \text{for} \quad U_{in} < 0 \end{cases}. \tag{4}$$

A thermal noise of the equivalent noise temperature T is an input signal of the detector. It is assumed that this noise formed by HF amplifier is narrowband, has normal distribution and, moreover, the noise entering detector has no DC component.
Double-sided characteristics of RMS power gain of a HF amplifier is approximated by the exponential function:

$$G(\omega) = G_{ps\,\max}\left[\mathrm{e}^{-\alpha(\omega+\omega_0)^2} + \mathrm{e}^{-\alpha(\omega-\omega_0)^2}\right]. \tag{5}$$

Consequently, a double-sided spectral power density of a detector input-signal, presented in the normalised form in Fig. 2, can be described as

$$S_{in}(\omega) = \frac{N}{2} G(\omega) = \\ = \frac{N_{\max}}{2}\left[\mathrm{e}^{-\alpha(\omega+\omega_0)^2} + \mathrm{e}^{-\alpha(\omega-\omega_0)^2}\right] \quad \left[\frac{\mathrm{W}}{\mathrm{Hz}}\right], \tag{6}$$

where

$$N_{\max} = G_{ps\,\max} kT. \tag{7}$$

Using definition (1) and relation (5) we get

$$B_{HF} = \frac{1}{\sqrt{2\pi\alpha}}. \tag{8}$$

According to Wiener-Kintchine theorem, the autocorrelation function of input noise is equal to

$$R_{in}(\tau) = \frac{1}{2\pi}\int\limits_{-\infty}^{\infty} S_{in}(\omega)\mathrm{e}^{\mathrm{j}\omega\tau}\mathrm{d}\omega = \frac{N_{\max}}{2\sqrt{\pi\alpha}}\mathrm{e}^{\frac{-\tau^2}{4\alpha}}\cos\omega_0\tau. \tag{9}$$

Assuming that input noise is an ergodic process and because of absence of DC component, we can estimate its variance as

$$\sigma_{in}^2 = R_{in}(0) = \frac{N_{\max}}{2\sqrt{\pi\alpha}}. \tag{10}$$

Taking dependence (8) into account we find relation between the band width B_{HF} and input noise variance

$$\sigma_{in}^2 = \frac{N_{\max}B_{HF}}{\sqrt{2}}. \tag{11}$$

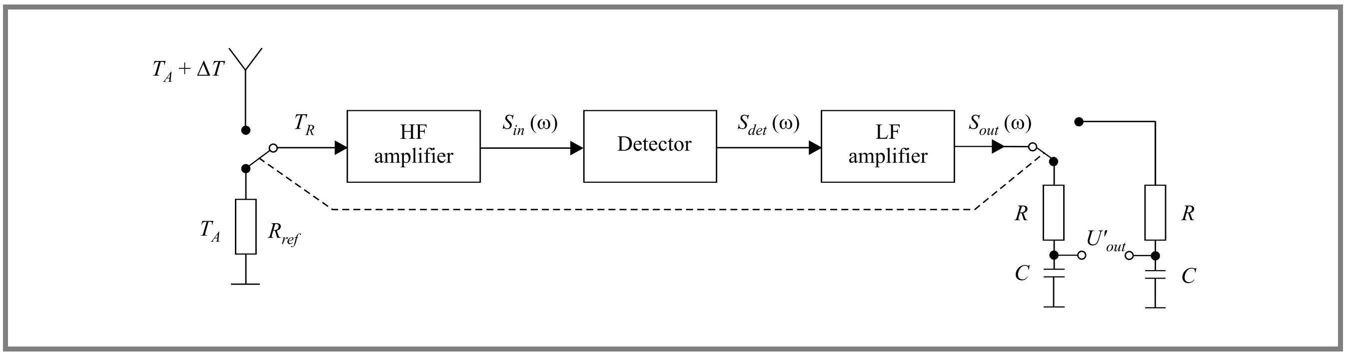

Fig. 1. Simplified block diagram of the modulation radiometer.

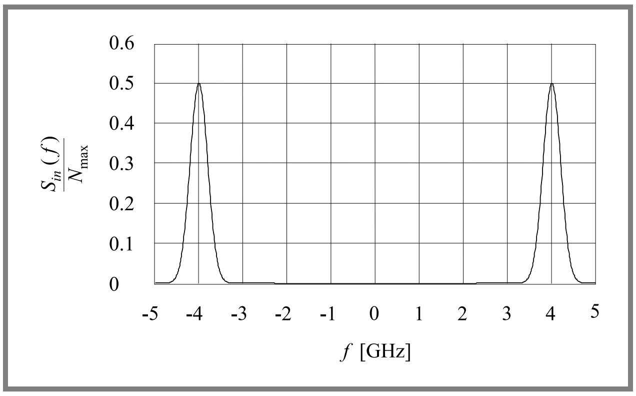

Fig. 2. Double-sided spectral power density of input noise.

Introducing the autocorrelation function envelope concept

$$\rho_{in}(\tau) = \mathrm{e}^{\frac{-\tau^2}{4\alpha}} \tag{12}$$

we finally get

$$R_{in}(\tau) = \sigma_{in}^2 \rho_{in}(\tau) \cos \omega_0 \tau \,. \tag{13}$$

2. Square – law detector

Considering transmission of stationary normal noise without component through a double-half inertialess system with square characteristics specified by relation (3) we find an expression which describes autocorrelation function of a process at the detector output [2]:

$$R_{det}(\tau) = \beta^2 \sigma_{in}^4 + 2\beta^2 R_{in}^2(\tau) \,. \tag{14}$$

Using relation (13) we have

$$R_{det}(\tau) = \beta^2 \sigma_{in}^4 \left[1 + \rho_{in}^2(\tau) + \rho_{in}^2(\tau) \cos 2\,\omega_0 \tau\right]. \tag{15}$$

If a fast-varying component that does not appear at an LF amplifier output (playing a role of a post-detector filter) is neglected, an autocorrelation function of an output process can be written as

$$R_{det}(\tau) = \beta^2 \sigma_{in}^4 \left[1 + \rho_{in}^2(\tau)\right]. \tag{16}$$

According to Wiener-Kintchine theorem a noise spectrum at the detector output is

$$S_{det}(\omega) = \int_{-\infty}^{\infty} R_{det}(\tau)\, \mathrm{e}^{-\mathrm{j}\omega\tau} \mathrm{d}\tau = $$
$$= \beta^2 \sigma_{in}^4 \left[2\pi\delta(\omega) + \sqrt{2\pi\alpha}\, \mathrm{e}^{\frac{-\alpha\omega^2}{2}}\right]. \tag{17}$$

The first component describes power of DC component. Its value at the LF amplifier output equals:

$$P_{DET} = \left|H(0)\right|^2 \frac{1}{2\pi} \int_{-\infty}^{\infty} \beta^2 \sigma_{in}^4 2\pi\delta(\omega)\, \mathrm{d}\omega = $$
$$= \frac{1}{2} \left|H(0)\right|^2 \beta^2 N_{\max}^2 B_{HF}^2 \,. \tag{18}$$

The second component represents spectrum of a low-varying component of output noise shown for real frequencies in Fig. 3. In a very narrow pass band of an LF amplifier as compared to the band B_{HF} (typically $B_{LF}/B_{HF} \cong 10^{-8}$) a varying exponential factor practically equals unity and a spectral density of output noise can be considered as constant

$$S_{det\,LF}(\omega) = \beta^2 \sigma_{in}^4 \sqrt{2\pi\alpha}\, \mathrm{e}^{\frac{-\alpha\omega^2}{2}} \approx $$
$$\approx \beta^2 \sigma_{in}^4 \sqrt{2\pi\alpha} = S_{det\,0} \,. \tag{19}$$

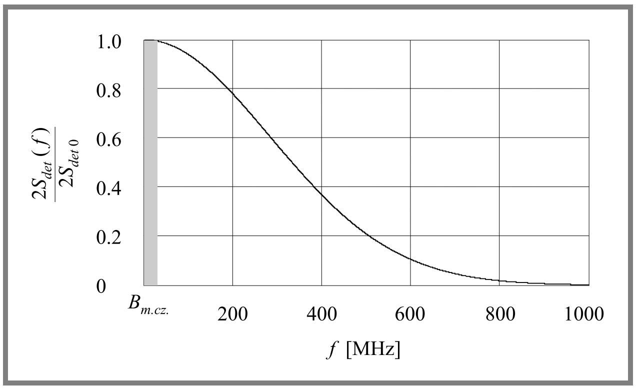

Fig. 3. Normalised one-sided noise spectral density at detector output.

Using relations (8) and (11), Eq. (19) can be expressed as

$$S_{det\,0} = \frac{1}{2}\beta^2 N_{\max}^2 B_{HF}\,. \tag{20}$$

Power of variable component of a noise at LF amplifier output is described by integral:

$$P_{det} = \frac{1}{2\pi}\int_0^\infty 2S_{det\,0}\left|H(\omega)\right|^2 \mathrm{d}\omega = $$
$$= 2S_{det\,0}\frac{1}{2\pi}\int_0^\infty \left|H(\omega)\right|^2 \mathrm{d}\omega\,. \tag{21}$$

Using definition (2) of a noise pass band for a low frequency channel, we get

$$P_{det} = 2S_{det\,0}B_{LF}\left|H(0)\right|^2 = $$
$$= \left|H(0)\right|^2\beta^2 N_{\max}^2 B_{HF}B_{LF}\,. \tag{22}$$

Including Eqs. (18) and (21), the ratio of constant and variable components can be expressed as

$$\rho_{square} = \frac{P_{DET}}{P_{det}} = \frac{B_{HF}}{2B_{LF}}. \tag{23}$$

DC voltage component at the LF amplifier output can be written down as

$$U_{OUT} = a_s\sqrt{P_{DET}} = \frac{b_s}{\sqrt{2}}N_{\max}B_{HF}\,,$$
$$b_s = a_s\beta\left|H(0)\right|, \tag{24}$$

where a_s is the proportionality factor depending on particular system design and b_s additionally includes also detector characteristics and LF channel amplification for DC and very low frequencies components.

An RMS value of a noise variable component at LF amplifier output – resulting from low-pass filtration – is equal to

$$U_{out} = a_s\sqrt{P_{det}} = b_s N_{\max}\sqrt{B_{HF}B_{LF}}\,. \tag{25}$$

Introducing a constant

$$c_s = b_s G_{ps\,\max}k \tag{26}$$

we get the following final expressions describing DC component and RMS value of AC component of the LF amplifier output voltage:

$$\begin{cases} U_{OUT} = \dfrac{c_s}{\sqrt{2}}T\,B_{HF} \\ U_{out} = c_s T\sqrt{B_{HF}B_{LF}} \end{cases}. \tag{27}$$

Equivalent noise temperature of input signal is a sum of RMS input temperature of radiometer noise, antenna noise, and measured temperature changes if antenna is connected to the system input

$$T = T_R + T_A + \Delta T = T_{sys} + \Delta T \tag{28}$$

or it equals

$$T = T_{sys} \tag{29}$$

if properly selected the reference resistance R_{ref} is connected to the input.

In a modulation receiver, constant voltage related to system noise is eliminated in an output differential circuit and as a consequence, DC output voltage depends only on antenna temperature rise and is described by relation:

$$U'_{OUT} = \frac{c_s}{\sqrt{2}}\left(T_{sys}+\Delta T\right)B_{HF} - \frac{c_s}{\sqrt{2}}T_{sys}B_{HF} = $$
$$= \frac{c_s}{\sqrt{2}}\Delta T\,B_{HF}\,. \tag{30}$$

On the other hand, we can neglect – as a very small – the component ΔT in expression describing fluctuation component and then the RMS value of AC component at the LF amplifier output is

$$U_{out} = c_s\left(T_{sys}+\Delta T\right)\sqrt{B_{HF}B_{LF}} \overset{\Delta T<<T_{sys}}{\approx} c_s T_{sys}\sqrt{B_{HF}B_{LF}}. \tag{31}$$

At the differential output, because of summation of non-correlated fluctuation components, the RMS value of AC component is twice as large; as result we have:

$$U'_{out} = 2\,c_s\,T_{sys}\sqrt{B_{HF}\,B_{LF}}\,. \tag{32}$$

Radiometer sensitivity is defined as the minimum input signal rise (in temperature units) $\Delta T_{\min}$ that ensures that DC component and AC component RMS value of output voltage are equal [4]:

$$U'_{OUT}\left(\Delta T_{\min}\right) = U'_{out}. \tag{33}$$

Thus, using (30) and (32) we get

$$\Delta T_{\min} = 2\,T_{sys}\sqrt{\frac{2\,B_{LF}}{B_{HF}}}\,. \tag{34}$$

3. Linear detector

Qualitatively different problem appears for single-half linear detector. Since such detector reproduces input noise envelope (envelope is described by Rayleigh distribution for narrowband Gaussian noise), and assuming that detector characteristics slope equals 45°($\gamma = 1$), as well as neglecting components close to central frequency of HF amplifier and its harmonics, we obtain the following expression for spectral density of the output noise power [2]:

$$S_{det}(\omega) = \frac{\pi\sigma_{in}^2}{2}\left[2\pi\delta(\omega) + \frac{1}{4}\sqrt{2\pi\alpha}\,\mathrm{e}^{\frac{-\alpha\omega^2}{2}}\right]. \tag{35}$$

As previously, we find expressions for powers of DC and AC components:

$$P_{DET} = \left|H(0)\right|^2\frac{1}{2\pi}\int_{-\infty}^{\infty}\frac{\pi\sigma_{in}^2}{2}2\pi\delta(\omega)\mathrm{d}\omega = $$
$$= \frac{\pi}{2\sqrt{2}}\left|H(0)\right|^2 N_{\max}B_{HF}\,, \tag{36}$$

$$P_{det} = \frac{1}{2\pi}\int_0^\infty 2\frac{\pi\sigma_{in}^2}{8}\sqrt{2\pi\alpha}\left|H(\omega)\right|^2 \mathrm{d}\omega = $$
$$= \frac{1}{2}\,\frac{\pi}{2\sqrt{2}}\left|H(0)\right|^2 N_{\max}B_{LF}\,. \qquad (37)$$

Using Eqs. (36) and (37) we express the ratio of DC and AC component powers as

$$\rho_{linear} = \frac{P_{DET}}{P_{det}} = \frac{2B_{HF}}{B_{LF}}\,. \qquad (38)$$

As in the previous section, we get:

$$\begin{cases} U_{OUT} = a_l\sqrt{P_{DET}} = c_l\sqrt{TB_{HF}} \\ U_{out} = a_l\sqrt{P_{det}} = \dfrac{c_l}{\sqrt{2}}\sqrt{TB_{LF}} \end{cases}, \qquad (39)$$

where:

$$c_l = a_l\left|H(0)\right|\sqrt{\frac{\pi}{2\sqrt{2}}G_{ps\,\max}k}\,. \qquad (40)$$

At the differential output:

$$\begin{cases} U'_{OUT} = c_l\sqrt{\left(T_{sys}+\Delta T\right)B_{HF}} - c_l\sqrt{T_{sys}B_{HF}} \\ U'_{out} = 2\dfrac{c_l}{\sqrt{2}}\sqrt{\left(T_{sys}+\Delta T\right)B_{LF}} \approx \sqrt{2}c_l\sqrt{T_{sys}B_{LF}} \end{cases}. \qquad (41)$$

Using definition (33), we obtain:

$$\sqrt{1+\frac{\Delta T_{\min}}{T_{sys}}} = 1+\sqrt{\frac{2B_{LF}}{B_{HF}}}\,. \qquad (42)$$

It is common in radiometry that the ratio $\Delta T_{\min}/T_{sys}$ is very small. Thus, we can perform power series expansion of the left side of above expression and, without any significant error, limit to the first two terms:

$$\sqrt{1+\frac{\Delta T_{\min}}{T_{sys}}} = 1+\frac{1}{2}\,\frac{\Delta T_{\min}}{T_{sys}} - \frac{1}{8}\left(\frac{\Delta T_{\min}}{T_{sys}}\right)^2 + \ldots \approx$$
$$\approx 1+\frac{1}{2}\,\frac{\Delta T_{\min}}{T_{sys}}\,. \qquad (43)$$

Consequently, we have

$$\frac{1}{2}\,\frac{\Delta T_{\min}}{T_{sys}} \approx \sqrt{\frac{2B_{LF}}{B_{HF}}} \qquad (44)$$

and, finally, the expression for sensitivity of a radiometer with a linear detector takes the following form:

$$\Delta T_{\min} \approx 2T_{sys}\sqrt{\frac{2B_{LF}}{B_{HF}}}\,. \qquad (45)$$

4. Summary

Comparison of relations (23) and (38):

$$\frac{\rho_{linear}}{\rho_{square}} = \frac{\dfrac{2B_{HF}}{B_{LF}}}{\dfrac{B_{HF}}{2B_{LF}}} = 4 \qquad (46)$$

shows that independently of bands ratio the output ratio of DC and AC components in linear detector is 6 dB better than for square detector. However, the sensitivities described by dependencies (34) and (45), most important in radiometry are almost the identical.

In real conditions, radiometer sensitivity does not depend on type of detector used, while it is a function of system temperature and a bandwidth ratio of pre- and post-detection channels, respectively.

Linear detector is characterised by linear dependence of output power on measured temperature while for square detector output voltage depends linearly on temperature. Since voltage measurement is related to lower measurement error than power measurement and because of good availability of square detectors such as semiconductor diodes for very low signals, single-half or double-half square detectors are commonly used in microwave amplifiers with direct amplification.

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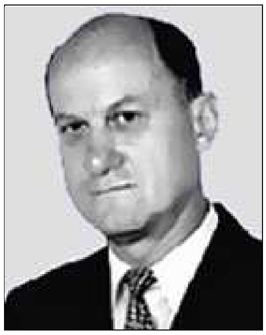

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