Regular paper

# Near fields of elliptic dielectric lenses

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**Abstract — The focusing properties of an elliptic dielectric cylinder taken as a 2D model of dielectric lens are studied for the plane wave illumination. An algorithm based on the concept of analytical regularization is applied for the numerical solution of the corresponding wave scattering problem. Numerical results for the near-field patterns are presented.**

***Keywords — elliptic dielectric lens, method of analytical regularization, antennas.***

## 1. Introduction

Planar slot or strip elements combined with dielectric lenses have the potential to be used in mm and sub-mm wave receivers [1, 2]. The wide attention they have been attracting recently is due to their capability of integration with electronic components such as detecting diodes, local oscillators and mixers. Furthermore, they provide good efficiency with respect to other antennas printed on homogeneous substrates. Besides, lenses are frequently employed in the laser technologies for the compression of the light beams. The elliptical shape of the lens provides focusing properties if its eccentricity is properly related to the dielectric constant. On the other hand the lens interface gives rise to reflection inside the lens that may significantly affect the input impedance and the radiation sensitivity. This aspect has not been properly investigated in the literature up to now, however, it is a critical point in the overall design of lens antennas. Although various analytical techniques have been applied for the dielectric lens analysis, they were commonly based on high-frequency approximations, neglected the lens curvature and finite beam size, and failed to characterize resonances. This appears a rough model description, as the actual size of the lens is usually of the scale of few wavelengths only [1, 2]. On the other hand, direct numerical simulations like FDTD may suffer from unclear and uncontrollable accuracy. As a consequence, there is a need to develop a reliable simulation technique capable to model wavelength-scale effects in addition to geometrical optics ones.

Our main efforts will be concentrated around the analysis of elliptic lenses in receiving mode. It is known [1, 2] that if the eccentricity of the ellipse is related to the lens dielectric constant as $e = 1/\sqrt{\varepsilon}$, all the rays outgoing from the focal point that impinge on the lens interface above the middle section form a parallel beam, i.e. plane wave. However, in practice it is very important to take into account finiteness of source size, and to select a proper source position and the lens geometry (Fig. 1).

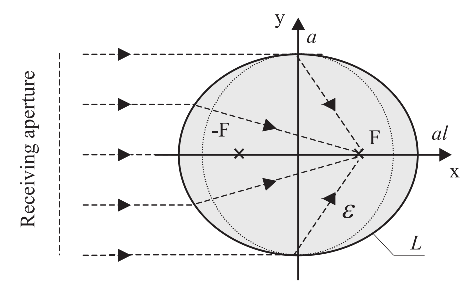


***Fig. 1.*** Geometry and notations of the problem.

In the receiving mode, it is important to know the actual size and location of the focal domain for different electric sizes of lenses and different angles of arrival of the incident wave. Although those effects escape conventional asymptotic analyses, they will be accurately quantified with our full-wave mathematically correct integral-equation method applied here. We emphasize that there is no limitation on the dielectric constant contrast between the lens material and background medium.

## 2. Outline of the solution

We use an efficient algorithm for the solution of 2D problem of wave scattering by a smooth dielectric cylinder that can be built on the concept of analytical regularization [3, 4]. The basic idea of the approach is as follows. The total field has to satisfy the Helmholtz equation with the coefficient $k$ and $k_e = k\sqrt{\varepsilon}$ outside and inside the lens, respectively. Field functions are presented as single-layer potentials with the density functions to be determined:

$$U_{outside}(\mathbf{r}) = \int_L p(\mathbf{r}_s) G_0(\mathbf{r}, \mathbf{r}_s)\, d\, l_s + U_0\,, \qquad (1)$$

$$U_{inside}(\mathbf{r}) = \int_L q(\mathbf{r}_s) G_e(\mathbf{r}, \mathbf{r}_s)\, d\, l_s\,. \qquad (2)$$

Here $U_0$ is the incident field, and the kernels are the Green's functions of the free space and uniform media of relative permittivity $\varepsilon$, respectively:

$$G_0(\mathbf{r},\mathbf{r}_s) = \frac{i}{4}H_0(k|\mathbf{r}-\mathbf{r}_s|)\,, \tag{3}$$

$$G_e(\mathbf{r},\mathbf{r}_s) = \frac{i}{4}H_0(k_e|\mathbf{r}-\mathbf{r}_s|)\,, \tag{4}$$

and $H_0$ is the Hankel function of the 1st kind.

By applying the boundary conditions, a set of singular integral equations (IEs) of the first kind is obtained. Introducing a global parameterization $x(t)$, $y(t)$ of the contour $L$, they can be cast into the following form:

$$\begin{cases} \int_0^{2\pi} q(t_s)G_\varepsilon(t,t_s)L(t_s)dt_s - \int_0^{2\pi} p(t_s)G_0(t,t_s)L(t_s)dt_s = \\ = U_0(t_s) \\ \frac{q(t)+\alpha p(t)}{2\alpha} + \frac{1}{\alpha}\int_0^{2\pi} q(t_s)\frac{\partial}{\partial n}G_\varepsilon(t,t_s)L(t_s)dt_s + \\ -\int_0^{2\pi} p(t_s)\frac{\partial}{\partial n}G_0(t,t_s)L(t_s)dt_s = \frac{\partial}{\partial n}U_0(t)\,, \end{cases} \tag{5}$$

where $t,t_s \in [0,2\pi]$, and $L(t) = \sqrt{(dx/dt)^2+(dy/dt)^2}$.

Equations (5) are uniquely solvable provided that the wavenumber $k$ does not belong to the set $D$ of discrete eigenvalues of the interior Dirichlet problem for $L$ [5]. Direct discretization of IEs (5) is not efficient due to the singular character of the kernel functions. In order to transform them to the Fredholm second kind matrix equation with favorable features, the analytical regularization has to be done. Adding and subtracting the canonical-shape kernels

$$\hat{G}_0(t-t_s) = \frac{i}{4}H_0\left(2ka\left|\sin\frac{t-t_s}{2}\right|\right)\,, \tag{6}$$

$$\hat{G}_e(t-t_s) = \frac{i}{4}H_0\left(2k_ea\left|\sin\frac{t-t_s}{2}\right|\right)\,, \tag{7}$$

and their normal derivatives perform extraction of the singular parts of the IEs. Analytical inversion of the latter parts is carried out by using the IE discretization based on Galerkin's scheme with angular exponents as global basis functions. Thus, unknown density functions are sought as

$$\{p(t),q(t)\}L(t) = \frac{2}{i\pi}\sum_{m=-\infty}^{\infty}\{p_m,q_m\}e^{imt}. \tag{8}$$

Here, the angular exponents are the orthogonal eigenfunctions of the integral operators, for example,

$$\int_0^{2\pi} e^{imt_s}H_0\big(2kaSin|(t-t_s)/2|\big)dt_s = 2\pi e^{imt}J_m(ka)H_m(ka), \quad m = 0,\,\pm1,\,\pm2,\ldots \tag{9}$$

Resulting matrix equation is

$$Z + AZ = B, \tag{10}$$

where $Z = \left\{z_m^1, z_m^2\right\}\big|_{m=-\infty}^{\infty}$,

$$z_m^1 = p_mH_m(ka)J_m(ka) - \alpha q_mH_m(k_ea)J_m(k_ea) \tag{11}$$

$$z_m^2 = kap_mH_m'(ka)J_m(ka) - k_eaq_mH_m(k_ea)J_m'(k_ea) \tag{12}$$

and the elements $A_{nm}^{ij}$ and $B_m^j(i,\,j=1,2)$ depend on the Fourier-expansion coefficients of the smooth functions. The latter are the differences between kernels (3) and (4) on $L$ and on the canonical-shape contour, i.e. a circle of radius $a$. These matrix elements can be economically computed by using the DFFT algorithm. The coefficient $\alpha$ in Eqs. (11) and (12) is 1 or $\varepsilon$ for $E$- or $H$- polarization, respectively.

Such a regularization plays the role of analytic preconditioning and guarantees point-wise convergence of the numerical solution (provided that $k \notin D$), i.e., a possibility to minimize the error to machine precision by solving progressively greater matrices.

Here, one has to note that the resulting computational error is determined by several factors: accuracy of cylindrical functions calculation, accuracy of FFT in the coefficients calculation, accuracy of numerical integration, and finally, the truncation error. In our algorithm, Bessel functions are calculated with digital precision that is achieved by using the recursion technique (backward for Bessel and forward for Neumann function). The bottleneck of the algorithm is accuracy of the matrix element calculation that is controlled by the order of FFT.

Under these conditions the rate of convergence of the algorithm can be estimated by plotting the normalized computational error, $e(N)$, in the sense of the $l_2^2$ norm, versus the matrix truncation number $N$:

$$e(N) = \left\|Z^N - Z^{N+1}\right\| \cdot \left(\left\|Z^N\right\|\right)^{-1}, \tag{13}$$

where $Z^N = \left\{z_n^{1N}, z_n^{2N}\right\}$ are the expansion coefficients computed from the matrix equation with each block truncated after $N$ equations. Details of the algorithm properties can be found in [4].

As it has been noted, there is a discrete set of wavenumbers, $D$, which are defective for the developed algorithm. When intermediate operations are done with finite precision, the condition number of the matrix (10) blows up in the vicinity of $k \in D$ that is comparable in size with the precision. This entails a spurious resonance in the field characteristics of the scatterer that is also called "numerical resonance" in the contrast to natural physical resonances. This feature is a demerit of the algorithm. It is caused by the implementation of the single-layer potentials (3) and (4) and can be overcome when using, instead, linear combination of single and double-layer potentials [5]. Nevertheless our approach is free from the inaccuracies near to natural resonances that are intrinsic for conventional numerical approximations [6].

As the far-field characteristics are of interest, the large-$r$ approximation is used. This enables one to replace the Hankel functions with its asymptotic and to reduce the integral in formula (1) to

$$U^{rad}(t) = \left[\left(1/i\pi kr\right)^{1/2} \cdot e^{ikr}\right] \cdot \Phi(t)\,, \qquad (14)$$

where $\Phi(t)$ is the far-field radiation pattern determined as:

$$\Phi(t) = \frac{1}{2\pi} \sum_{n=-\infty}^{\infty} p_n \int_0^{2\pi} e^{-ik\left(\cos(t)x(t_s)+\sin(t)y(t_s)\right)+\text{int}_s} dt_s. \quad (15)$$

The frequency dependence of the total scattered power can be characterized by the cumulative scattering characteristic such as a total-scattering cross-section $\sigma_{tot}$ determined as:

$$\sigma_{tot} = \frac{2}{\pi k} \int_0^{2\pi} \left|\Phi(t)\right|^2 dt\,. \qquad (16)$$

## 3. Numerical results

The dielectric materials selected for the computations are high-density polyethylene (HDP), quartz, alumina, and silicon with $\varepsilon$ of 2.31, 4.0, 9.8, 11.7, respectively. The dimensions of the considered lenses are about several wavelengths. The axial ratio of the considered lenses is $l = \sqrt{\varepsilon/(\varepsilon-1)}$ to provide the eccentricity condition $e = 1/\sqrt{\varepsilon}$.

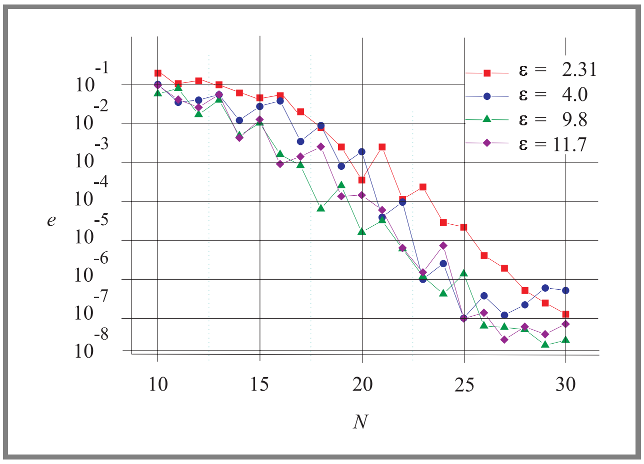


***Fig. 2.*** Relative computational error versus truncation number for different dielectric materials and $ka = 10$.

Figure 2 shows the dependence of the relative computational error versus the truncation number $N$. Rapid fall of the error with the growth of $N$ is due to the point-wise convergence of the solution that comes from the Fredholm nature of the considered matrix equation. All the following numerical results have been obtained with the error below $10^{-3}$.

In Fig. 3 the total scattering cross-section versus the normalized wavelength parameter $ka$ for various dielectric materials is shown. Sequences of extrema, which are well seen in the graph, are explained by the natural resonances. Extraordinarily high-Q maxima correspond to spurious numerical resonances that are involved in the true solution due to chosen representation of the fields.

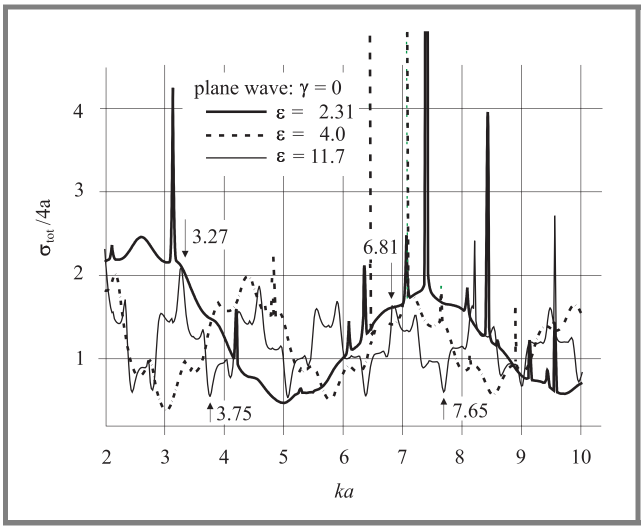


***Fig. 3.*** Normalized total scattering cross-section of the elliptic cylinder versus normalized frequency parameter for different dielectric materials ($E$-polarized plane wave illumination in the frontal mode).

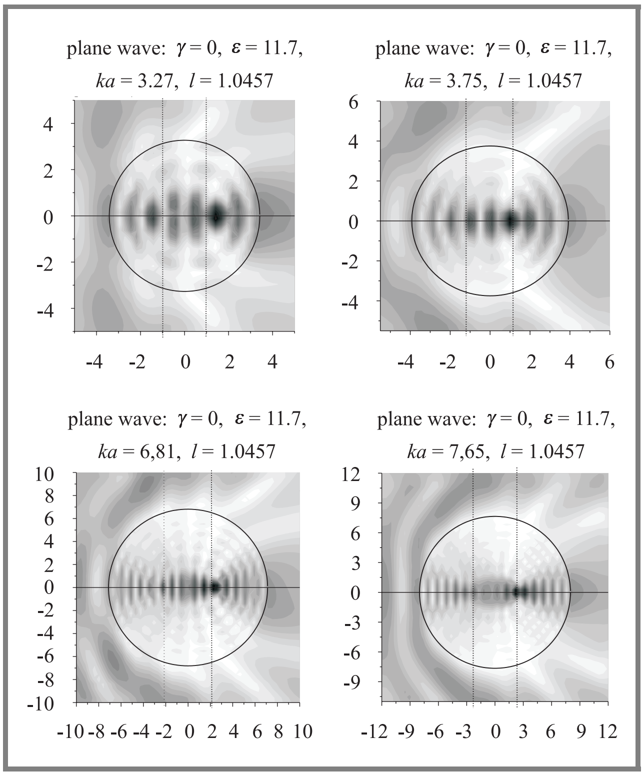


***Fig. 4.*** Near-field portraits for a silicon lens symmetrically illuminated by the $E$-polarized plane wave. Intersections of the plane of symmetry with the vertical dashed lines indicate the focal points of the lens. Corresponding $ka$ values are marked with arrows in Fig. 3.

There are two pairs of near-field portraits (Fig. 4) calculated for silicon lens with $ka = 3.27$, 3.75 and 6.81, 7.65, respectively. These values of the normalized wavelength parameter $ka$ are marked with arrows in Fig. 3 and correspond to the local maxima and minima in the total scattering cross-section. It can be seen that the focal domain has finite size: the greater $ka$ the smaller the area of the greatest field concentration. The shift of the focal domain from the geometrical focus of the ellipse is interrelated to the total scattering cross-section: maximum shift is for the maxima of $\sigma_{\text{tot}}$.

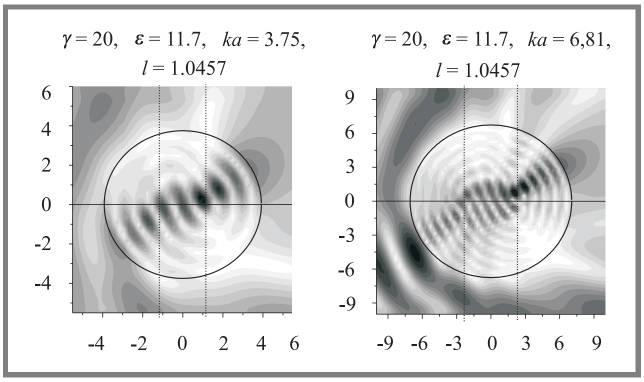


***Fig. 5.*** The same as in Fig. 4 for the incident wave arrival angle $\gamma = 20°$.

Figure 5 demonstrates the transformation of the focal domain in the case of the plane wave illuminating the lens at the angle of 20°. It is shown that the focal domain shifts from the ellipse geometrical focus and becomes wider.

## 4. Conclusions

An efficient and accurate numerical method, based on the concept of analytical regularization, has been applied for the solution of the 2D problem of the plane wave scattering by an elliptic dielectric cylinder taken as a model of typical lens used in the mm and sub-mm wave antenna applications. Obtained numerical results demonstrate the effects such as focal domain shift and transformation for a dielectric lens of the size comparable to the wavelength that cannot be analyzed in geometrical or physical optics approximations. As it is shown, the resonant nature of the effects is still a challenge for more careful analysis. The question of nonuniqueness of the solution of IEs used in the mathematical model also requires a further study. Nevertheless the possibilities of the approach as to the accurate analysis of the smooth arbitrary shaped dielectric lenses have been demonstrated.

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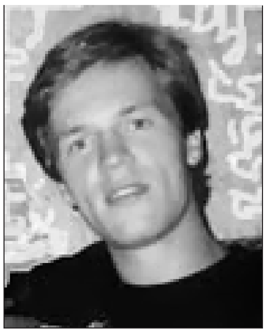

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