Paper

Whispering gallery resonator method for permittivity measurements

Krzysztof Derzakowski, Adam Abramowicz, and Jerzy Krupka

Abstract — The new method of measuring permittivity is described. The measurements are performed using whispering gallery mode open dielectric resonators. The accuracy is assured by applications of the mode matching method. Three resonant modes (HE_{511}, HE_{611} and HE_{711}) are used in measurement procedure. Accuracy of the method is much better than 0.3% for the relative permittivity having values from 20 to 50.

Keywords — dielectric resonator, permittivity, loss tangent, measurement method, whispering gallery modes.

1. Introduction

In the whispering gallery (WG) modes the electromagnetic field distribution is such that the most of the modal energy is confined in the dielectric cylinder (or ring) close to its edge. Thus due to small radiation losses the WG resonators can be used without shielding. At millimeter wave frequency range the low permittivity dielectrics are usually used due to dimensions of elements and possibility of fabrication. High permittivity resonators are interesting at lower frequencies. At microwave frequencies typical dielectric resonators are shielded and suffer from conductor losses. The WG resonators offer a most effective way to decrease conductor losses and as a result to increase the unloaded quality factor of the resonators. That is why the high permittivity whispering gallery mode resonators have been used in measurements of ultra-low loss dielectric materials [1]. Precise computations of the complex permittivities require the knowledge of the mode type and well defined structure what means dimensions of the resonator and a shield over resonator. Typically the resonant mode used in measurements of the material properties is TE_{011}.

In the case of open dielectric resonators operating in the whispering gallery modes one can resign from shielding. The other problem in using WG modes is their recognition which in general is not easy. Fortunately the distribution of the WG modes in open resonator structures is very convenient and at least three first modes can be recognised accurately. In our method an open dielectric resonator is measured. From the full spectrum of modes three resonant frequencies having consecutive mode subscripts are used to calculate the permittivity of the resonator. WG modes are easy to recognise because they have high quality factors, much higher than lower order modes. The resonant frequencies of the consecutive modes have almost linear dependence on the mode subscripts. Having resonant frequencies and resonator dimensions a permittivity can be calculated in the process of adjusting its value to the resonant frequencies of WG modes.

Moreover, an investigation of the WG modes behaviour brought us to the conclusion that the HE_{511}, HE_{611}, HE_{711} and HE_{811} WG modes are always first WG modes that can be observed. The difference between resonant frequencies of the HE_{511} and HE_{711} modes depends nearly linearly on the permittivity and can provide a simple and accurate method of the measurements.

2. Mode matching method computations

To calculate the resonant frequencies of the WG resonators a variety of methods can be used [2]. Our idea is to apply the radial mode matching method [3, 4] to the open structures. In the case of high permittivity dielectrics this concept works very well. In the computations we have considered shielded structure but the metal shield was located at the distance sufficiently large to have no influence on the resonant frequencies and field distribution of the WG modes.

The radial mode matching method is used in its classical form as described in [4]. As usual the multilayered cylindrical dielectric resonator structure is divided into separate regions in which the permittivities are independent of coordinate r, and are piecewise constant functions of z. In each region the electromagnetic field is represented as a superposition of the TE and TM multilayered radial waveguide modes. The E_z and H_z field components in each layer are given below:

$$E_z^{(\alpha)}(r,\varphi,z) = \sum_{m=0}^{\infty}\sum_{i=0}^{\infty}\left[a_i^{(\alpha)} J_m\left(\sqrt{t_i^{(\alpha)}}r\right) + b_i^{(\alpha)} N_m\left(\sqrt{t_i^{(\alpha)}}r\right)\right]\Psi_i^{(\alpha)}(z)\cos\left(m\varphi+\psi_i^{(\alpha)}\right), \quad (1)$$

$$H_z^{(\alpha)}(r,\varphi,z) = \sum_{m=0}^{\infty}\sum_{i=1}^{\infty}\left[c_i^{(\alpha)} J_m\left(\sqrt{\lambda_i^{(\alpha)}}r\right) + d_i^{(\alpha)} N_m\left(\sqrt{\lambda_i^{(\alpha)}}r\right)\right]\Phi_i^{(\alpha)}(z)\sin\left(m\varphi+\varphi_i^{(\alpha)}\right), \quad (2)$$

where a_i, b_i, c_i and d_i are the unknown expansion coefficients, λ_i and t_i are the eigenvalues of the eigenfunctions Φ_i and Ψ_i which form the complete and biorthogonal series,

α is the number of a region, i and m are the multilayered radial waveguide mode subscripts.

In the region I containing symmetry axis the Neumann function must be omitted from the field expressions. The eigenfunctions Φ_i and Ψ_i describe the dependence of the fields along z axis. Taking advantage of the axial symmetry the fields of resonant modes can be expressed as a linear combination of modes having the same mode subscript m. By applying the continuity relations at the discontinuity surfaces (for $\mathbf{r} = R^{\alpha}$), and by numerical matching the fields at the structure walls, for each m a set of linear equations is found. This system of infinite equations can be numerically solved only for a finite number of terms, but then equations are met with an error depending on the number of used terms. By rewriting the equations into a form expressing mean-square errors, and using variational methods, a set of homogeneous linear equations with a finite number of unknown coefficients is obtained, which has a following general form:

$$\overline{W}\left[a_i^I,..,c_i^I,..,a_i^{II},..,b_i^{II},..,c_i^{II},..,d_i^{II},..,a_i^{III},..\right]^T = [0], \qquad (3)$$

where $\overline{W}$ is a square matrix with elements depending on structure parameters and a frequency, T means a transposed matrix.

Table 1
Measured and computed resonant frequencies
(resonator dimensions $D = 63.43$ mm, $L = 6.36$ mm)

Number	Measured frequency f_m [MHz]	Computed frequency f_c [MHz]	Error [%] $100(f_c - f_m)/f_m$	Mode type
1	1882.4	1896.9	+0.767	$m = 2$
2	2172.9	2178.1	+0.239	$m = 3$
3	2442.6	2446.1	+0.143	$m = 4$
4	2704.2	2704.7	+0.0185	HE_{511}
5	2909.1	2915.3	+0.213	$m = 3$
6	2956.4	2956.4	0	HE_{611}
7	3196.2	3198.2	+0.0626	$m = 4$
8	3203.5	3203.1	−0.0125	HE_{711}
9	3446.8	3446.1	−0.0203	HE_{811}
10	3470.5	3471.1	+0.0115	HE_{521}
11	3687.4	3686.5	−0.0244	HE_{911}
12	3737.5	3737.6	+0.0027	HE_{621}

The nontrivial solution of the system exists if and only if the determinant of the matrix $\overline{W}$ is equal zero. All the resonant frequencies of the structure can found from the zeros of the determinant. After finding the resonant frequencies for each of them Eqs. (3) can be solved and all unknown expansion coefficients can be found. From the known field expressions, expansion coefficients and resonant frequencies we calculate the electromagnetic field distributions. The computer program written in Microsoft's FORTRAN PowerStation has the following limitations: up to 10 layers of dielectrics in up to 7 cylindrical regions and up to 30 radial waveguide modes taken into account in each region. The user can choose the electromagnetic field mapping accuracy by selecting a desired number of grid points.

Another limitation in the program is that the structure must be fully surrounded by the PEC or PMC walls (the types of walls can be mixed). This is a certain limitation because the open structures basically are not considered. Fortunately due to the WG modes characteristics the open WG resonators can be analysed with high level of accuracy.

The resonant modes having first index 5 or higher reveal the typical WG mode behaviour i.e. the resonant frequencies of shielded structures and without shield are nearly the same what can be observed experimentally. A comparison of the computed resonant frequencies with the measured ones shows an excellent agreement – for the WG modes the average error was on the level of 0.015% as can be seen in Table 1.

3. Numerical and experimental results

The transmission characteristic has been measured in the structure shown in Fig. 1. The dielectric resonator has been excited by the microstrip lines in such a way that coupling coefficient was small enough to observe resonances without disturbing them. The characteristic has been measured

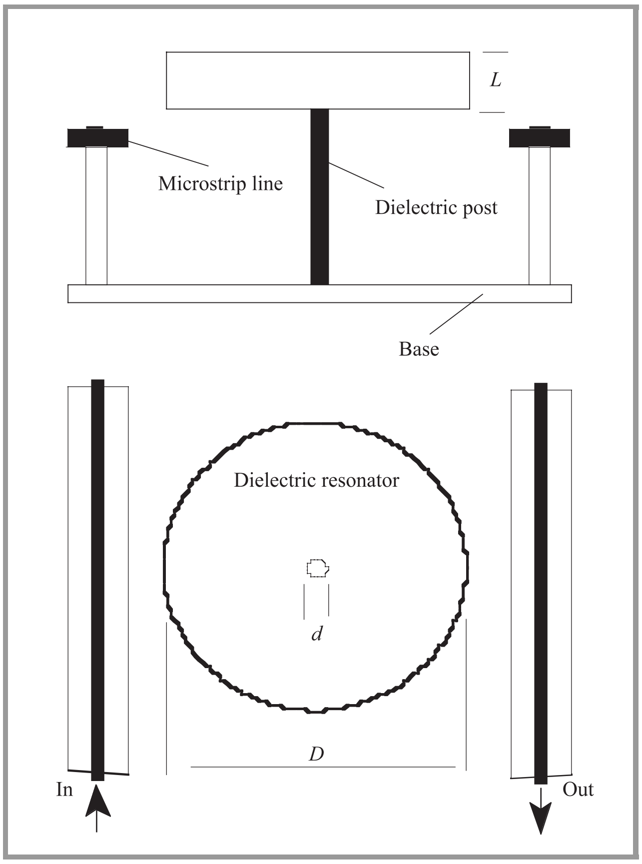

Fig. 1. Measured structure ($D = 63.43$ mm, $d = 6.15$ mm, $L = 6.35$ mm, $\varepsilon_r = 45$).

in the frequency range 1.7 – 3.8 GHz which allowed us to observe several lowest WG resonances. The measured results are shown in Fig. 2 and in Table 1. For the quality factor measurements the excitation through the coupling loops has been performed. The WG modes have unloaded quality factor on the level of 14000. Lower order modes have unloaded quality factors in the range 100 – 4000 thus the WG modes can be easily distinguished.

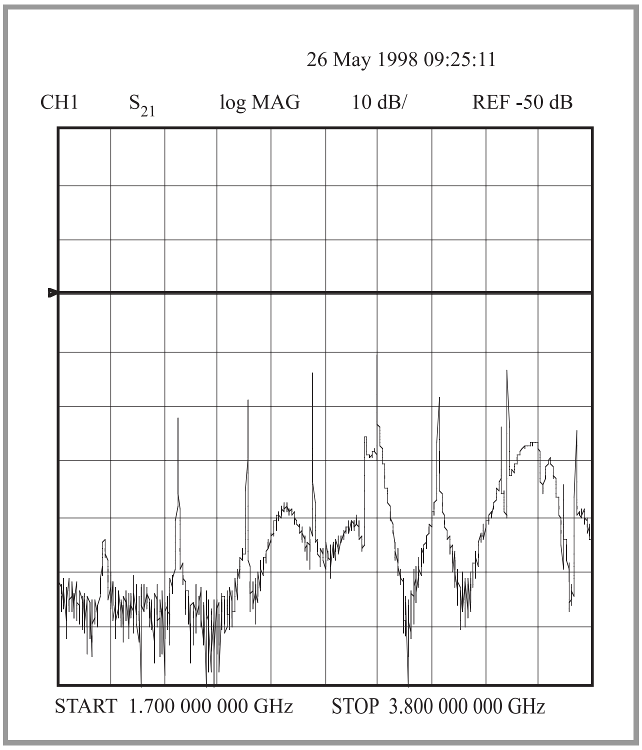

Fig. 2. Measured frequency response.

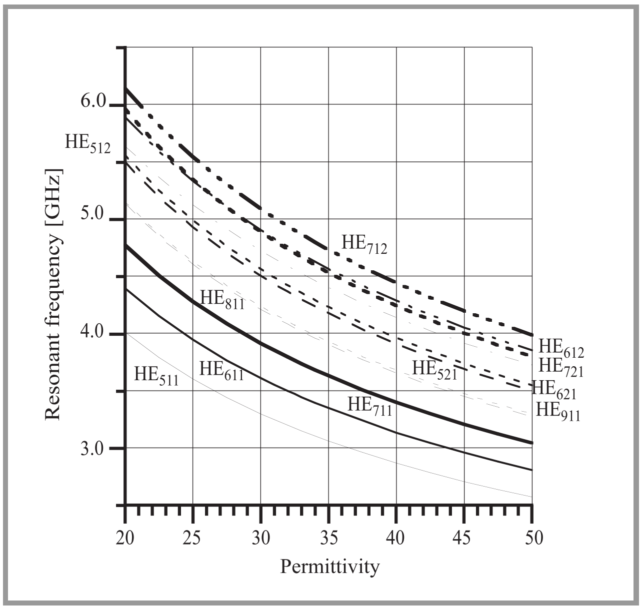

Fig. 3. Mode chart of the lowest WG modes.

The resonant frequencies of the resonator have been computed by means of the radial mode matching method assuming the permittivity of 45 as given by the producer (Trans-Tech). For WG modes the field distributions have been computed and then the mode indices found. The computed resonant frequencies and the mode designation is presented in Table 1.

The agreement between measured and computed resonant frequencies of the WG modes is better than 0.025%. The lower order modes have measured resonant frequencies shifted down in comparison with computed ones. This is an effect of metal walls surrounding the structure present in a case of simulations. The lower mode number the bigger discrepancy can be noticed. Having such a good agreement of measured and computed resonant frequencies one can be sure that the radial mode matching method can provide reliable results when used in measurements.

The computer simulations have been used to investigate the WG modes behaviour. The resonant frequencies of the structure have been computed for different values of the relative permittivity as shown in Fig. 3. It is easy to notice that four first WG modes are always the same: HE_{511}, HE_{611}, HE_{711} and HE_{811}. From the practical point of view the mode HE_{811} is not very useful because it has resonant frequency close to the next WG mode HE_{521} and their characteristics can interfere. Also the HE_{611} mode is close to the lower order mode but this mode ($m = 3$) has very low quality factor ($Q_0 \approx 400$) and should has no significant influence on the HE_{611} mode what we confirmed experimentally. The HE_{511} mode is separated from any other modes. What is also interesting the difference between resonant frequencies of the HE_{611} and HE_{511} modes is nearly equal

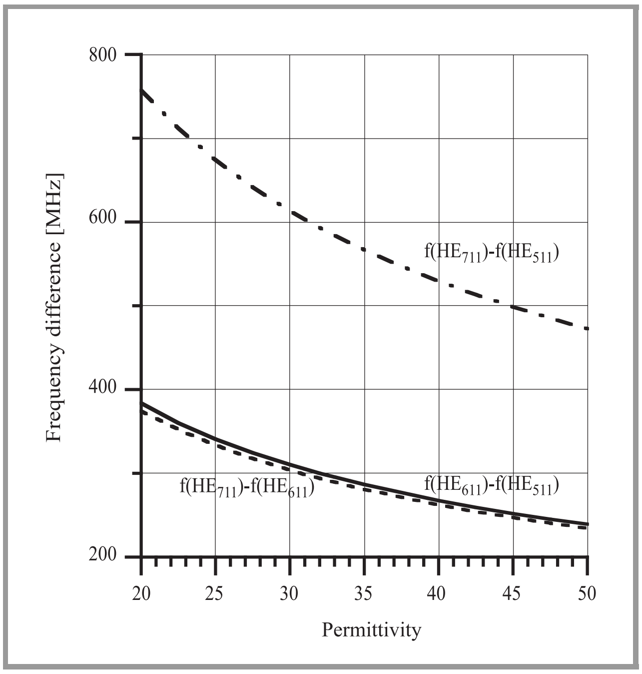

Fig. 4. Differences between resonant frequencies of the WG modes.

to the difference between resonant frequencies of the HE_{711} and HE_{611} modes as shown in Fig. 4. (In the Fig. 4 the difference between resonant frequencies HE_{711} and HE_{511} is also shown). In general three first WG modes can be always easily identified thus with the proper algorithm the resonant frequencies of this modes can be used to calculate the permittivity.

4. The measurement procedure

The measurements are done in a following way:

- first, the resonant frequencies and quality factors of the modes of open dielectric resonator are measured using network analyser;
- second, the WG modes are identified taking into account quality factors and distribution of modes;
- third, the real part of permittivity is found using distance between resonant frequencies HE_{711} and HE_{511}.

In our test measurements (resonator $D = 63.43$ mm, $L = 6.36$, WG modes HE_{511}, HE_{611}, HE_{711}) the determined permittivity was $\varepsilon_r = 44.95$. The agreement with classical methods (the producer values) is considerably high.

Table 2
Computed coefficients used in Eq. (4)
(diameter of the resonator $D = 63.43$ mm)

Resonator thickness L/mm	A	B
2	15.614413	1.837472
3	15.815942	1.900219
4	15.889140	1.928469
5	15.928919	1.944443
6.36	15.964069	1.957261
7.72	15.994913	1.965745

If one can assume the accuracy of the resonant frequency measurement better then 0.2 MHz the permittivity of the resonator can be established with the accuracy better than 0.15%, which is sufficiently enough for industrial practice. Depending on the resonator dimensions similar to that presented in Fig. 4 characteristics can be computed and applied in measurements of dielectrics. However this characteristics are good for the real part of the permittivity measurements only. Depending on the diameter to height (D/L) ratio the computed characteristics of the difference between resonant frequencies HE_{711} and HE_{511} differ in values but reveal the same behaviour that can be described with a following equation:

$$\varepsilon_r = e^{[A - B\ln(\Delta f)]}, \quad (4)$$

where Δf is frequency difference, A and B coefficients.
The coefficients A and B for several resonator dimensions are given in Table 2. To obtain the permittivity of the resonator it is enough to measure the difference between resonant frequencies of the HE_{711} and HE_{511} modes and to use Eq. (4).

5. Conclusions

The new, simple and precise measurement method is presented. Open WG dielectric resonator structures used to determine the permittivity of the resonators offer high accuracy of the measurements. The real part of the permittivity is directly related to the difference between resonant frequencies of the HE_{711} and HE_{511} modes. The method is so simple that there is no need for numerical computations and results are obtained from the simple equation.

References

[1] J. Krupka *et al.*, "Complex permittivity measurements of extremely low loss dielectric materials using whispering gallery modes", in *IEEE MTT-S Int. Microw. Symp. Dig.*, Denver, USA, 1997, pp. 1347–1350.

[2] Xiao Hu Jiao *et al.*, "Whispering-gallery modes of dielectric structures: applications to millimeter-wave bandstop filters", *IEEE Trans. Microw. Theory Techn.*, vol. 35, pp. 1169–1175, 1987.

[3] S. Maj and M. Pospieszalski, "A composite multilayered cylindrical dielectric resonator", in *IEEE MTT-S Int. Microw. Symp. Dig.*, San Francisco, USA, 1984, pp. 190–192.

[4] K. Derzakowski and A. Abramowicz, "Dielectric resonator figure of merit", *Bull. Polish Acad. Sci., Techn. Sci.*, vol. 44, no. 2, pp. 129–139, 1996.

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