Paper

Analysis of optical-microwave mixing process in electro-optical modulators

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Abstract — The principle of operation and the general parameters of the electro-optical Mach-Zehnder modulators are reminded in the paper. With the use of the mathematical relationships describing the transmission of the optical signal through the modulator, the theoretical model of the optical signal transmission with two modulating signals at two frequencies will be presented. The theoretical relationships describing the efficiency of the optical-microwave mixing and frequency multiplying in the two-tone operation will be derived. The analysis and the simulations will be performed for different operating points, where the nonlinearity of the transmission characteristic is specially strong.

Keywords* — *electro-optical modulator, nonlinear effects, microwave-optical mixing.

1. Introduction

During the last years the nonlinear effects in the processes of optical signals modulation and detection are investigated very intensively. It is related to the development in a new type of optical fibre links with capability of transmitting lots of electrical signals with the use of many carrier frequencies (subcarrier multiplexing) and achieving an operation without the intermodulation distortions.

The analysis of the nonlinear effects allows explaining the electrical-optical mixing processes and further it allows using them for different purposes.

In the photodetection processes the PIN photodiodes and the phototransistors are commonly used. In both cases the nonlinear effects are observed, who could be used in the frequency conversion process. The modulation of the optical signals can be obtained in different ways. When the laser generation conditions are changed in this process, thereby causing the changes in the power or the frequency of the generated signal – it is so-called internal modulation. Whereas when there is no disturbance of the generation process and the optical signal parameters are being changed during the transmission through the separate device placed at the laser output – it is so-called external modulation. There are two types of modulators, which are now commonly used: electro-absorption modulators and electro-optical modulators called Mach-Zehnder modulators. The characteristics of the second ones are well known and described.

The electro-optical modulators are readily used in the analogue optical links, for the sake of exceptional good transmission conditions while operating with the high levels of the optical power. The important problem that appears is the linearization of the transmission characteristics and the minimization of the nonlinear effects. However, the nonlinear effects could be used in the frequency conversion processes for obtaining required conversion products, for example, such as the harmonics of the modulation signal. The work presented here treats of analysis of the conditions to rise up such products.

2. Mach-Zehnder electro-optical modulator

The electro-optical effect appears when the refraction coefficient *n(E)* of an optical material is a function of electric field E

$$n(E) = n + a_1 E + \frac{a_2}{2} E^2 + \ldots \cong n - \frac{1}{2} n^3 r E \,. \qquad (1)$$

When the two of the first parts of the expression (1) are dominant in the electro-optical material, this phenomenon is called Pokels effect. In the expression (1) the Pockels coefficient is denoted as r and it is equal to $10^{-10} \ldots 10^{-12}$ m/V at the electric field $E = 10^6$ V/m, then $\Delta n = 10^{-6} \ldots 10^{-4}$. The materials having this effect are called the electro-optical ones and they are, for example, $LiNbO_3$, $LiTaO_3$, CdTe, GaAs [1].

With the use of the Pockels effect it is possible to make an optical signal phase modulator. In the material demonstrating the electro-optical effect, for example, $LiNbO_3$, a planar optical waveguide is made. This optical waveguide is placed in an electric field created by the electrodes. Then the electric field as a result of a signal connected to the electrodes causes a change of the phase of an optical signal transmitted through the optical waveguide.

The use of an interferometer allows constructing an optical signal amplitude modulator. A planar version of such modulator is shown in Fig. 1. An optical beam with the power P_0 is divided in two equal parts and directed into two branches. Within theses branches two electro-optical phase modulators are placed and they work in counter-phase. The next coupler sums the two beams. When the effects of a high frequency and the attenuation of the planar optical waveguides are neglected, the power transmission of the modulator can be described by a simple formula. The operation of both phase shifters in the interferometer branches is shown in Fig. 2a [2].

The output signal and further the transmission of the modulator can be obtained by summing the optical signals from two branches. It is shown in Fig. 2a.

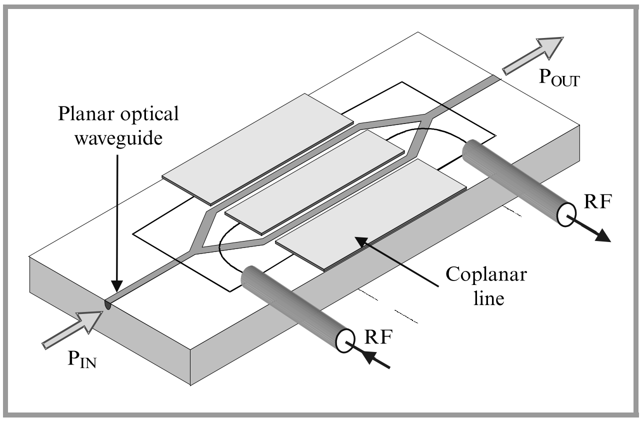

Fig. 1. Planar structure of electro-optical Mach-Zehnder modulator.

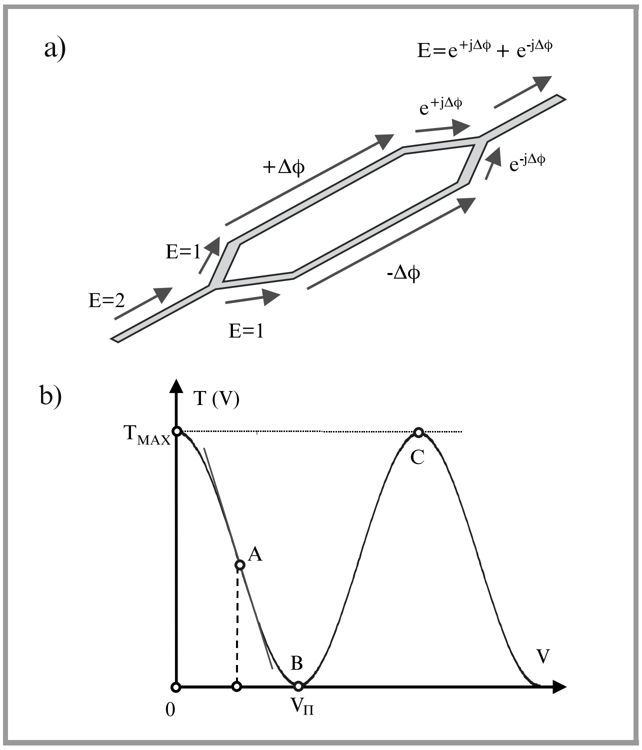

Fig. 2. (a) Illustration of Mach-Zehnder interferometer/modulator operation. (b) Characteristic of power transmission through M-Z modulator.

The transmission can be described as:

$$T(V_{RF}) = \frac{P_{o\,out}}{P_{o\,in}} = \left| \frac{E\left(e^{j\Delta\phi} + e^{-j\Delta\phi}\right)}{2E} \right|^2 =$$
$$= \cos^2\left[\Delta\phi\left(V_0 + V_{RF}\right)\right] \approx \frac{1}{2} - \Delta\phi\,\left(V_{RF}\right). \qquad (2)$$

The maximal transmission T_{MAX} of optical power is less than unity because the modulator has its own losses resulting from the planar optical waveguides losses. The formula describing the optical power transmission should include them. When the characteristic value of the modulator voltage (V_π) is considered the transmission can be expressed as (3):

$$T(V) = \frac{P_{o\,out}}{P_{o\,in}} = \frac{T_{MAX}}{2}\left\{1 + \cos\left[\frac{\pi V}{V_\pi}\right]\right\}. \qquad (3)$$

In this expression the voltage $V = V_0 + V_{RF}$ is the superposition of a bias voltage V_0 and a signal voltage V_{RF}. The full shape of the transmission characteristic is shown in Fig. 2b. The power transmission in the point of equality decreases almost to zero. When the symmetry of the power dividing and summing is achieved the ratio $P_{\max}/P_{\min}$ can be obtained up to 1000:1. In the point of inflexion of the $T(V)$ characteristic there is a long straight line section at $V_0 = V_\pi/2$ and with a slope S_{MZ}:

$$S_{MZ} = \left.\frac{\partial T(V)}{\partial V}\right|_{V=V_\pi} = -\frac{\pi T_{MAX}}{2V_\pi}. \qquad (4)$$

The modulator transmission $T(V_{RF})$ at $V_0 = V_\pi/2$:

$$T(V_{RF}) \cong \frac{T_{MAX}}{2} + S_{MZ}V_{RF} = \frac{T_{MAX}}{2}\left(1 - \frac{\pi V_{RF}}{V_\pi}\right). \qquad (5)$$

The pre-bias at point A allows the interferometer to operate as a linear amplitude modulator. This mode of operation is used in the analogue optical links [3]. The voltage V_{RF} can be a sine-form signal or to be a superposition of many carrier frequencies with the modulation side bands.

When the modulator is supplied by a rectangular shape wave with the signal levels corresponding to the points B and C, the device operates as two-state switch. The operation at the bias corresponding to the points B or C has the strongest nonlinearities. This mode of the operation is the most proper in the case of a mixing process.

When the RF signal frequency is being increased, the optical signal transit time τ, through the area of an alternating electric field, becomes comparable with the period T equal to $T = 1/f_{mod}$. When the signal transit time becomes equal to the period T, the modulation effects obtained during one half-period will be disturbed in the next half-period.

Let's assume that:

$$V_{RF} = V_M \sin(\omega t + \Theta). \qquad (6)$$

The presence of the alternating field causes the modulation of the angle $\Delta\phi$:

$$\Delta\phi = m(f)\frac{\pi V_M}{V_\pi}\sin\left(\omega t + \Theta\right), \qquad (7)$$

where $m(f)$ – the modulation depth coefficient depends on the modulation signal frequency:

$$m(f) = \left|\frac{\Delta\phi\,(f)}{\Delta\phi\,|0|}\right| = m_0\frac{\sin\left(\omega_{mod}\tau/2\right)}{\omega_{mod}\tau/2}. \qquad (8)$$

The finite transit time has the result in the decreasing of the optical signal modulation depth.

Another cause limiting the modulator working bandwidth is the RF signal path mismatching. The RF signal path is ended with the capacitance which reactance decreases with the increase of the frequency thereby causing the RF path almost short-circuited. A solution for this problem is a travelling wave modulator. In this construction the following situation is obtained. The optical signal speed in the planar waveguide is equal to the modulating microwave signal speed. There are many very interesting solutions for travelling wave modulator structures. However, they will not be described in this paper.

3. Optical-microwave mixing with single M-Z modulator – bias in inflexion point

It was mentioned above that the bias in the inflexion point of the transmission characteristic allows to linearly modulate the optical power transmitted through the modulator. It will be found out that in this point it is possible to work with the optical-microwave frequency mixing process with the use of the nonlinear modulator characteristics [4, 5]. The starting point of the theoretical analysis will be the formula (3), and it will be expressed in a new form (9):

$$T_N\left(V_0, V_{RF}\right) = \frac{P_{o\,out}}{P_{o\,in}\,T_{MAX}} = \\ = \frac{1}{2}\left\{1+\cos\left[\phi_0(V_0)+\frac{\pi V_{RF}}{V_\pi}\right]\right\} . \quad (9)$$

In the formula above, $T_N(V_0,V_{RF})$ is the power transmittance of the electro-optical modulator normalized to the maximal power transmittance. By setting appropriate value of the bias voltage to achieve $\phi_0(V_0) = 90°$ and to work in the inflexion point of the transmission characteristic the formula (9) can be written as:

$$T_N\left(V_{RF}\right) = \frac{1}{2}\left\{1-\sin\left[\frac{\pi V_{RF}}{V_\pi}\right]\right\} . \quad (10)$$

The optical-microwave mixing process analysed in this point can be performed in the system setup shown in Fig. 3.

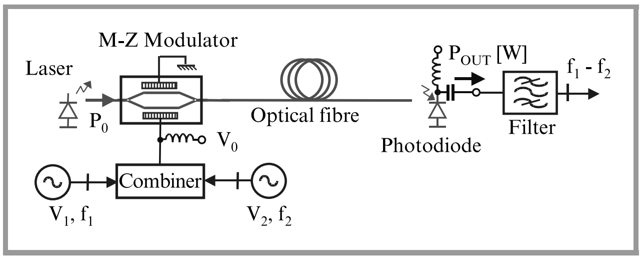

Fig. 3. System to perform optical-microwave mixing process with the use of M-Z modulator.

A combiner and an appropriate bias circuit allow inputting the bias voltage and two alternating sine-form voltages into the modulator. One can write:

$$V_{RF} = V_1 \sin \omega_1 t + V_2 \sin \omega_2 t . \quad (11)$$

The normalized transmission T_N can be written as:

$$T_N = \frac{1}{2}\left\{1-\sin\left[X_1 \sin \omega_1 t + X_2 \sin \omega_2 t\right]\right\} , \quad (12)$$

where X_1 and X_2 are respectively:

$$X_1 = \frac{\pi V_1}{V_\pi} , \quad X_2 = \frac{\pi V_2}{V_\pi} . \quad (13)$$

With the use of three well-known identity relations, presented in the Appendix, there is possible to expand the transmittance (12) into the Fourier series. This Fourier series expression contains Bessel functions. Considering only the first terms of the expansion one can obtain:

$$T_N = \frac{1}{2} - J_1(X_1)J_0(X_2)\sin \omega_1 t + \\ -J_0(X_1)J_1(X_2)\sin \omega_2 t - J_1(X_1)J_2(X_2)\times \\ \times\left\{\sin\left[\left(\omega_1-2\omega_2\right)t\right]+\sin\left[\left(\omega_1+2\omega_2\right)t\right]\right\}+ \\ -J_2(X_1)J_1(X_2)\left\{\sin\left[\left(2\omega_1-\omega_2\right)t\right]+ \\ +\sin\left[\left(2\omega_1+\omega_2\right)t\right]\right\}+\ldots . \quad (14)$$

The expression above allows analysing and explaining two phenomena related to the nonlinearities of the modulation process.

3.1. Intermodulation distortions during two-tone operation

During the two-tone operation the amplitudes of both sine-form signals are equal to each other $V_1 = V_2$. When the amplitude values are small, then $X_1 = X_2 << 1$. Considering only the first terms of the expansion into the series of the Bessel functions in (14): $J_0(X), J_1(X)$ and $J_2(X)$, according to the formula given in Appendix, one can obtain the following simplified form of the expression (14):

$$T_N = \frac{1}{2} - \frac{X_1}{2}\sin \omega_1 t - \frac{X_2}{2}\sin \omega_2 t + \\ -\frac{X_1 X_2^2}{16}\left\{\sin\left[\left(\omega_1-2\omega_2\right)t\right]+ \\ +\sin\left[\left(\omega_1+2\omega_2\right)t\right]\right\} - \frac{X_1^2 X_2}{16}\left\{\sin\left[\left(2\omega_1-\omega_2\right)t\right]+ \\ +\sin\left[\left(2\omega_1+\omega_2\right)t\right]\right\}+\ldots . \quad (15)$$

The power spectrum of the electrical signal at the photodetector output contains, beside the expected signals at the frequencies ω_1, ω_2 and the amplitudes which are proportional to V_1^2 and V_2^2, also the undesired intermodulation products $(2\omega_1 - \omega_2)$ and $(2\omega_2 - \omega_1)$ with the amplitudes which are proportional to the third power of the both signal powers at the input of the modulator.

3.2. Products of optical-microwave mixing at bias in the inflexion point

In the optical-microwave mixing process there are also two signals at the frequencies ω_1, ω_2 connected to the modulator. The amplitude of the first of them, called also the signal, is small so the condition $X_1 << 1$ is fulfilled. The second signal at the amplitude V_2 plays role of a heterodyne and usually $V_2 >> V_1$. The process products described above and undesired become now useful products of the frequency mixing. The values of the transmission T_N terms at the frequencies $\omega_1 (2\omega_2 - \omega_1)$ are equal to:

$$\begin{aligned} T_{N(\omega_1)} &= -\frac{X_1}{2} J_0(X_2) \sin \omega_1 t \,, \\ T_{N(2\omega_2-\omega_1)} &= -\frac{X_1}{2} J_2(X_2) \sin \left[(\omega_1 - 2\omega_2) t \right] . \end{aligned} \quad (16)$$

The results of the simulation calculations are presented in Fig. 4. It was assumed that the power of the signal at the M-Z modulator input is simply related to the amplitude $V_{1,2}$ and the characteristic impedance Z_0 of the modulator planar transmission line $P_{LO} = V_{1,2}^2 / 2Z_0$. In calculations it was assumed: $Z_0 = 50\ \Omega$, $V_\pi = 7$ V.

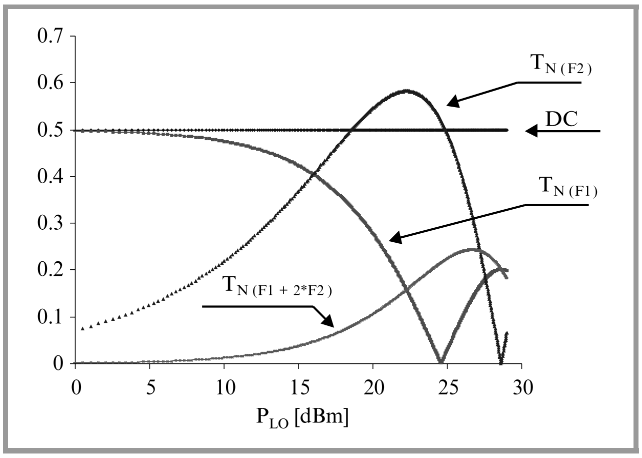

Fig. 4. Products of optical-microwave mixing for circuit shown in Fig. 3 and at bias in the inflexion point. Calculations have been made for $Z_0 = 50\ \Omega$ and $V_\pi = 7$ V.

The analysis of the Bessel functions curves presented in the Appendix allows explaining the role of the heterodyne. The increase of the amplitude V_2 causes decreasing of the amplitude of $T_{N(\omega_1)}$ until it disappears at $X_2 = 2.42$, when $J_0(2.42) = 0$. At the same time the amplitude of $T_{N(2\omega_2 - \omega_1)}$ increases obtaining the maximum at $X_{2OPT} = 3.05$, when $J_2(3.05) = 0.486$. The frequency conversion process is the most effective at this value of X_2 and the conversion losses have the smallest value equal about 6 dB.

The power of the signal led to the M-Z modulator is related in a simple way to the amplitude $V_{1,2}$ and the characteristic impedance Z_0 of the planar transmission line of the modulator. The optimal value of the heterodyne power P_{LOOPT}, at which $X_2 = X_{2OPT}$ and the conversion losses are the smallest, can be calculated with the use of the following formula:

$$P_{LOOPT} = \frac{V_2^2}{2Z_0} = \frac{V_\pi^2 X_{2OPT}^2}{2\pi^2 Z_0} . \quad (17)$$

One can notice that the lower sideband $(2\omega_2 - \omega_1)$ used in the mixing process corresponds to the harmonic mixing. The upper sideband $(2\omega_2 + \omega_1)$ can be used as well.

4. Optical-microwave mixing with single M-Z modulator – bias in point of maximal transmission

The bias at the maximum of the transmission characteristic, obtained when $\phi_0(V_0) = 0°$, causes also the frequency conversion process to take place. This characteristic is then expressed by:

$$T_N(V_{RF}) = \frac{1}{2} \left\{ 1 + \cos \left[\frac{\pi V_{RF}}{V_\pi} \right] \right\} . \quad (18)$$

When the RF voltage V_{RF} is a superposition of two parts according to the formula (11), the normalized transmission T_N can be written as:

$$T_N = \frac{1}{2} \left\{ 1 + \cos \left[X_1 \sin \omega_1 t + X_2 \sin \omega_2 t \right] \right\} . \quad (19)$$

With the use of the relations presented in the Appendix and by limiting analysis to few terms of the expansion, the transmission T_N can be expressed as:

$$\begin{aligned} T_N = {} & \frac{1}{2} \left[1 + J_0(X_1) J_0(X_2) \right] + \\ & + J_2(X_1) J_0(X_2) \cos(2\omega_1 t) + \\ & + J_0(X_1) J_2(X_2) \cos(2\omega_2 t) + \\ & - J_1(X_1) J_1(X_2) \left\{ \cos \left[(\omega_1 - \omega_2) t \right] + \right. \\ & \left. - \cos \left[(\omega_1 + \omega_2) t \right] \right\} + \ldots . \end{aligned} \quad (20)$$

The relationship obtained above allows using interesting applications of this characteristic mode of the operation of the M-Z modulator.

4.1. Operation in frequency doubler configuration

When the modulator has only one signal given at the input

$$V_{RF} = V_1 \sin \omega_1 t \quad (21)$$

then in the power spectrum at the photodetector output the strong product at the frequency $2\omega_1$ is observed, beside the constant product. The amplitude of the term $T_{N(2\omega_1)}$ increases at the beginning proportionally to V_1^2, and reach its maximum at $X_2 = 1.84$, when $J_2(1.84) = 0.581$. At this value of X_2 the frequency multiplying process has the maximal efficiency.

The frequency multiplying process is applied in the fibre-radio systems to overcome the limit resulting from the maximal modulation frequency of the electro-optical modulator.

4.2. Optical-microwave mixing products

The situation is similar like in the Sec. 3.2, in the optical-microwave mixing process two signals at the frequencies ω_1 and ω_2 are connected to the modulator. The amplitude of the first of them, called signal, is small, to fulfil the condition $X_1 << 1$. The second signal at the amplitude V_2, plays role of the heterodyne, and $V_2 >> V_1$. The transmittance term T_N at the frequency according to the lower sideband $(\omega_1 - \omega_2)$ is then expressed as:

$$T_{N(\omega_1-\omega_2)} = -\frac{X_1}{2} J_1(X_2) \sin\left[(\omega_1 - \omega_2)\, t\right] . \qquad (22)$$

The results of the calculations are shown in Fig. 5.

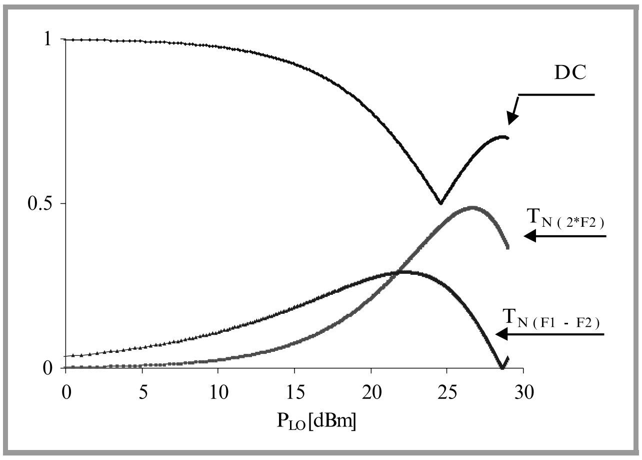

Fig. 5. Products of optical-microwave mixing for circuit shown in Fig. 3 and at bias in point of maximal transmission. Calculations have been made for $Z_0 = 50\ \Omega$ and $V_\pi = 7$ V.

The value of the lower sideband $(\omega_1 - \omega_2)$ amplitude reaches its maximum at $X_2 = 1.84$, when $J_1(1.84) = 0.581$. Because the maximum of function $J_1(X)$ is quite bigger than the maximum of $J_2(X)$, the conversion efficiency in this case is higher than the efficiency described in Sec. 3.2. Also it can be obtained at lower level of the heterodyne power (then at $X_2 = 3.05$, now at $X_2 = 1.84$). The heterodyne power at the optimal value of the conversion losses can be calculated with the use of the formulas $(5 \div 20)$.

5. Optical-microwave mixing with cascade connection of M-Z modulators

Another possibility to obtain the frequency conversion process is the use of the system shown in Fig. 6. Here, the optical signal generated by the laser is transmitted through two M-Z modulators.

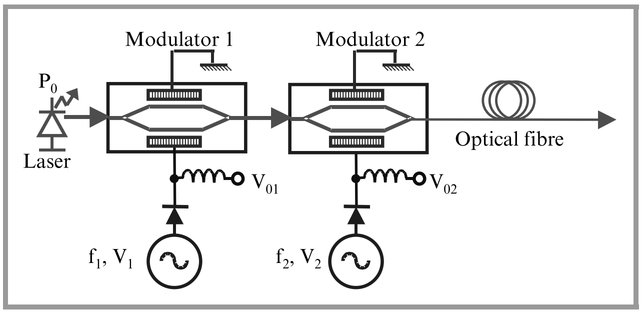

Fig. 6. System to perform optical-microwave mixing process with cascade connection of M-Z modulators

5.1. Products of optical-microwave mixing process- both modulators biased at inflexion points of transmission characteristics

In the case described here the modulators are pre-biased at the voltages V_{01} and V_{02}, which values are set to obtain the operation in the inflexion points of the transmission characteristics. The power transmission of the single modulator is described by the formula (9). Assuming that the modulators are identical, the modulating voltages are equal to: $V_1 \sin \omega_1 t$ and $V_2 \sin \omega_2 t$, respectively, and the values of the variables X_1 and X_2 are defined according to (13). The overall transmission T_{NC} can be expressed as follows:

$$T_{NC} = T_{NI} T_{N2} = \\ = \frac{1}{4}\left\{1 - \sin\left[X_1 \sin \omega_1 t\right]\right\}\left\{1 - \sin\left[X_2 \sin \omega_2 t\right]\right\} . \qquad (23)$$

With the use of the identity relations presented in the Appendix there is possible to expand the expression above into the Fourier series. Considering only the most important products at the frequencies ω_1, ω_2, $(\omega_1 - \omega_2)$ and $(\omega_1 + \omega_2)$ one can obtain:

$$T_{NC} = \frac{1}{4} - \frac{1}{2} J_1(X_1) \sin\left[\omega_1 t\right] - \frac{1}{2} J_1(X_2) \sin\left[\omega_2 t\right] + \\ -\frac{1}{2} J_1(X_1) J_1(X_2) \left\{ \sin\left[(\omega_1 - \omega_2)t\right] + \sin\left[(\omega_1 + \omega_2)t\right]\right\} . \qquad (24)$$

Assuming that the second modulator plays role of the heterodyne, and the first one is driven by the signal small enough to obtain $X_1 << 1$, the transmittance of the product corresponding to the frequency $(\omega_1 - \omega_2)$ can be written as:

$$T_{NC(\omega_1-\omega_2)} = -\frac{X_1}{4} J_1(X_2) \sin\left[(\omega_1 - \omega_2)\, t\right] . \qquad (25)$$

The results of calculations of some optical-microwave mixing products according to the formula (24) are shown in Fig. 7.

Also in this case the value of the lower sideband $(\omega_1 - \omega_2)$ amplitude reaches its maximum at $X_2 = 1.84$, when $J_1(1.84) = 0.581$, it means that the value of the conversion losses is the smallest and equal to 10.7 dB.

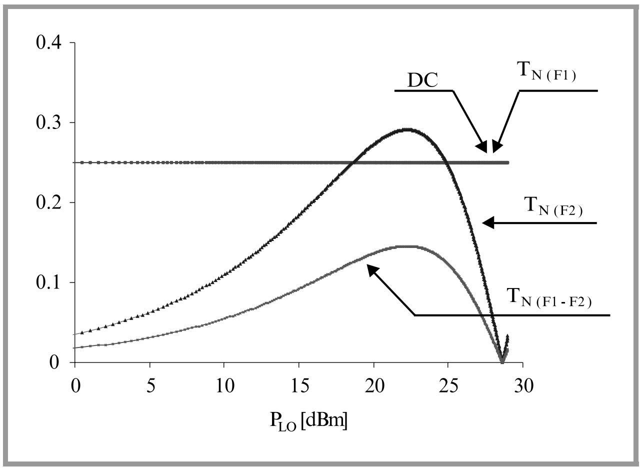

Fig. 7. Products of optical-microwave mixing for circuit shown in Fig. 6. Both modulators are biased at inflexion points. Calculations have been made for $Z_0 = 50\ \Omega$ and $V_\pi = 7$ V.

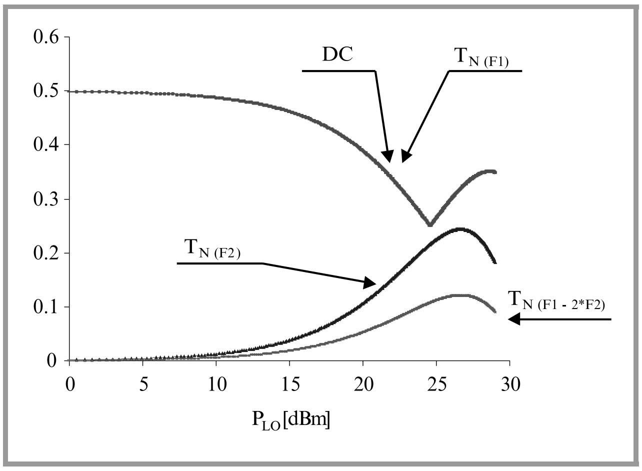

Fig. 8. Products of optical-microwave mixing for circuit shown in Fig. 6. Modulators are biased at different points. Calculations have been made for $Z_0 = 50\ \Omega$ and $V_\pi = 7$ V

5.2. Products of optical-microwave mixing process – modulator biases are different

Very interesting solution is the optical-microwave mixing process performed in the system where the first modulator, called the signal modulator, is biased at the inflexion point of the transmission characteristic, and the second one, the heterodyne modulator, is biased at the point of the maximal transmission. The transmission of the modulators is then described by the formulas (10) and (18). Let's assume that the modulators are identical, and the modulating voltages are: $V_1 \sin\omega_1 t$ and $V_2 \sin\omega_2 t$ respectively, and the values of the variables X_1 and X_2 are defined according to (13). The overall transmission of the cascading connection of the modulators is then equal to:

$$T_{NC} = \frac{1}{2}\Big\{1 - \sin\left[X_1 \sin\omega_1 t\right]\Big\} \times \times\Big\{1 + \cos\left[X_2 \sin\omega_2 t\right]\Big\} . \qquad (26)$$

The expression above can be expanded into the Fourier series with the use of the formulas given in the Appendix. Considering only the most important terms of the expansion one can obtain:

$$T_{NC} = \tfrac{1}{4}\left[1 + J_0(X_2)\right] - \tfrac{1}{2}J_1(X_1)\left[1 + J_0(X_2)\right]\sin\left[\omega_1 t\right] + + \tfrac{1}{2}J_2(X_2)\cos\left[2\omega_2 t\right] - \tfrac{1}{2}J_1(X_1)J_2(X_2) \times \times\Big\{\sin\left[(\omega_1 - 2\,\omega_2)t\right] + \sin\left[(\omega_1 + 2\omega_2)t\right]\Big\} . \qquad (27)$$

Assuming $X_1 << 1$, the transmittance related to the product at $(\omega_1 - 2\omega_2)$ can be written as:

$$T_{NC(\omega_1 - 2\omega_2)} = -\frac{X_1}{4} J_2(X_2)\sin\left[(\omega_1 - 2\omega_2)t\right] . \qquad (28)$$

The relationship above is almost identical with those obtained in previous cases. The conversion system operates as the harmonic mixer, it can be useful in some cases, the conversion efficiency is a little bit worse, because the maximum of the function $J_2(X_2)$ is smaller, in comparison to the maximum of the function $J_1(X_1)$ (Fig. 8).

6. Conclusions

The analysis presented above shows the possibility of performing the optical-microwave mixing process with the use of the nonlinear abilities of the electro-optical Mach-Zehnder modulators. The values of the microwave heterodyne power level for which the conversion losses reach their minimum are similar to the optimal heterodyne power levels for the quad-diode microwave mixers.

The relationships derived above are helpful and ready-to-use for calculations of the intermodulation distortions concerning the operation in the linear analogue modulation mode.

Appendix

The identity relations used for the calculations:

$$\sin(A + B) = \sin A\cos B + \cos A\sin B , \qquad (A1)$$

$$\sin(X\sin\Theta) = 2\sum_{k=0}^{\infty} J_{2k+1}(X)\sin\left[(2k+1)\,\Theta\right] , \qquad (A2)$$

$$\cos(X\sin\Theta) = J_0(X) + 2\sum_{k=1}^{\infty} J_{2k}(X)\cos 2k\Theta , \qquad (A3)$$

where $J_n(X)$ are Bessel functions.

For the small values of X it is usefully to expand the Bessel function into the series, according to the following formula:

$$J_n(X) = \sum_{k=0}^{\infty} \frac{(-1)^k}{k!(n+k)!}\left(\frac{X}{2}\right)^{n+2k} \quad n = 0,\ 1,\ 2, \ldots\ . \qquad (A4)$$

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