Methods for description of the total field propagation in the irregular dielectric waveguides

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Abstract — **Numerical and semi-analytical methods useful for analysis of the total field propagation in dielectric waveguides with sharp and smooth discontinuities are reviewed. The efficiency of these methods to model the field propagation in such structures is discussed comparatively. As an example, spatial transient regime of the radiation field propagation in planar step-index waveguide excited by a Gaussian beam is simulated numerically by two different beam propagation methods.**

Keywords — *optical waveguides, numerical modelling, beam propagation methods.*

1. Introduction

The principle of operation of many fiber-optic devices is based on the spatial transformation of a guided mode by means of irregularities. If a fundamental mode occurs to be mismatched in the consecutive cross-sections of the guiding structure, spatial transient regime appears. In the smooth structures (tapers), the regime is continuous and depends on the taper adiabaticity. After a step-like discontinuity, radiation field leaks out of the fiber over some definite length and then only the fundamental steady-state mode is left. In the region of the spatial steady-state regime the electromagnetic field near the axis of the waveguide consists mainly of the guided modes. However, in any case the total field distribution is the result of the interference of the guided and the radiation parts.

In order to account for the radiation field and simulate the total field propagation in waveguiding devices, various methods have been elaborated. These are numerous realisations of the beam propagation method (BPM) which has been applied to a variety of waveguide problems. Meanwhile some semi-analytical methods have been proposed based mainly on solving the system of integro-differential equations resulting from the spectral expansion of the Maxwell equation solutions.

The purpose of the present paper is to review and classify the available techniques proved to be useful in the treatment of the total field propagation in irregular guiding structures. The attention will be paid to the single-core structures with smooth or sharp discontinuities of the core radius which are known to be the basic elements of the fiber-optic couplers, junctions, launching devices, etc. As an example, we have considered a planar step-index waveguide which can be treated as a sharp discontinuity when excited by a Gaussian beam. Radiation field propagation was simu-

lated numerically by two different beam propagation methods (FFT-BPM and FD-BPM). The results and comparative analysis are presented in the second part of this paper.

2. Review of the available techniques

2.1. Beam propagation methods

The most commonly used numerical method to solve the scalar wave equation is the split-step Fourier series method which is in fact an extension of the beam propagation method originally developed by Fleck, Morris and Feit [1, 2]. Physically the technique corresponds to replacing the continuous refractive index distribution by an infinitesimally thin lens emerged in a homogeneous reference medium of uniform refractive index.

Conventionally to utilise this method in solution of the wave equation an algorithm is employed which is based on the fast Fourier transformation (FFT). The so-called beam propagation method (FFT-BPM) proved to be an accurate and efficient tool for solving a variety of propagation problems in waveguide geometries involving one or two transverse Cartesian coordinates [3 ÷ 8]. However, in application of this method in cylindrical coordinates one has to cope with the increased storage and reduced efficiency.

A spectral method is described in [2] for solving the paraxial wave equation in cylindrical geometry that is based on expansion of the exponential evolution operator in a Tailor series and use of fast Fourier transforms to evaluate derivatives. The scheme avoids operator splitting altogether and should be useful in the geometries in which the operator splitting may not be possible. It allows the propagation calculation to be performed accurately in cylindrical coordinates on a uniform grid.

The accuracy and applicability of the FFT-BPM have been studied intensively [3, 4] resulting in the conclusion that the FFT-based algorithm becomes critical for high-contrast step-index profiles, i.e. the size of propagation steps must be reduced and the refractive index profile itself must be smoothed out. The main disadvantage of the method is in its structured solution since the field should be written in terms of a coupled set of equations that are solved numerically which require tedious matrix inversions. Further, the effects of nonlinearity and diffraction are assumed to be separated in space whereas in reality both act simultaneously.

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An alternate numerical scheme to solve the wave equation is to use a finite difference (FD) approximation [9, 10]. Following the finite-difference beam propagation method (FD-BPM) the wave equation is replaced by a finite-difference scheme. The resulting three-diagonal system of equations is solved by some iterative procedure. FD-BPM has been successfully applied mainly to the analysis of the field propagation in planar waveguides, although nonlinear propagation in a radially symmetric structure was simulated too [11].

The comparative analysis of FFT-BPM and FD-BPM was performed in [9]. FD-BPM occurred to be much more stable with respect to longitudinal step size and number of grid points variations. For comparable accuracy much smaller longitudinal step is necessary to use in FFT-BPM than in FD-BPM, especially in the analysis of the step-index waveguides. Furthermore, the computation time per propagation step for FD-BPM is $4 \div 6$ times less than that of FFT-BPM. However the wide-angle implementation of FD-BPM becomes less accurate if the angle of the structure relative to the propagation direction increases. In spite of the justified feasibility of the FD-BPM for the beam propagation simulations, in the case of the "diagonal dominance" [64] of the tridiagonal matrix coefficients, the algorithm fails. From this point of view the FFT-BPM seems to be more universal technique. Otherwise, there are few types of problems that cannot be analysed accurately using this method. There is a disadvantage consisting of the fact that FFT-BPM requires an uniform transverse grid spacing regardless of how fast are the index and field changes in the transverse direction. This leads to a fast increasing number of grid points for a sufficient calculation accuracy. In the contrary, when using FD scheme, one can choose freely the grid point local density and to increase it at the layer interfaces with high index contrast.

Another problem is the need to apply stair-case approximations to any material boundary of the irregular structure. The boundaries prove to be a non-physical source of numerical noise. The boundary condition used in the conventional BPM is the so-called absorbing boundary condition. The idea is to artificially place a lossy medium at the edges of the computational window to absorb the reflections at the boundary. The major disadvantage of the absorbing boundary condition is that for a specific structure, users have to choose different absorbing parameters. In addition, the computational window size has to be large enough. The transparent boundary condition [7] has distinguished advantages over the absorbing boundary one. First, this condition is automatically set in the numerical algorithm. Therefore, it is not problem dependent. Secondly, a relatively small computational window can be used to increase the efficiency.

Modified finite-difference schemes have been proposed for a dielectric step-like structure in order to stabilise the effect of staircasing on simulated results [12].

Recently the concept of structure related, non-orthogonal coordinate FD-BPM algorithms has been demonstrated to overcome both staircasing and wide-angled propagation problems [$13 \div 20$]. These algorithms adopt the structure related coordinate systems which naturally follow the local geometry of the structure. The algorithm was developed to describe the taper geometry [18] without introducing any approximations. This reduces the number of longitudinal steps that are necessary and the transverse mesh size required.

Early FD mode solvers were based on the scalar wave equation. However, the scalar approximation is accurate only when the refractive index difference is small enough that the system modes are quasi-degenerative. Many different techniques have been adopted for vectorial modelling [$21 \div 23$]. One of the schemes based on the Pade approximation [24, 25] demonstrates considerable improvement in accuracy over the paraxial BPM's.

The semivectorial FD method [26] has been proved to be effective but it does not provide a full vectorial description of the guided field. A novel iterative approach for solving finite-difference equations based on nonlinear iteration have been introduced in [27].

The BPM algorithm breaks down when applied to structures with high refractive index contrast. As an alternative a propagation algorithm based on a finite-difference scheme with a finite element (FE) algorithm is suggested in [28]. This approach is, in general, not restricted to the scalar wave equation for weakly guiding structures or to slow transverse variations in refractive index. The BPM algorithm consists of two separate steps and large step size causes its breakdown. The FE algorithm avoids this problem by not using two separate parts of the propagation. Therefore even large propagation steps are admissible. For the case of a Gaussian beam propagating in a homogeneous medium (for which analytical solutions are available), a much higher accuracy was obtained using the FE method rather than the BPM.

Common FE approaches utilise nodal elements. Nodal elements have been successfully demonstrated for the simulation of slab (two-dimensional) waveguides [29, 30]. Unfortunately, they cannot be applied to three-dimensional waveguides due to the nature of nodal elements as they do not prevent spurious solutions and due to the resulting nonunitary propagation scheme. The additional use of edge elements suppresses non-physical solutions [31]. The combination of edge and nodal elements is called a mixed element approach [32]. A full vectorial simulation of 3D-waveguide structures becomes possible, because spurious modes can be prevented and boundary conditions at interfaces are correctly described. Therefore, mixed elements offer the ability to model waveguides containing lossy media, dielectric and magnetic materials.

In the eigenmode propagation algorithm based on the method of lines (MoL) the eigenvectors of the transverse operator of the wave equation which is discretised by finite differences are used for the evaluation of the propagating field [33]. The method is very accurate, because it directly solves the Helmholtz equation and provides an analytical solution in propagation direction (along the lines). In con-

trast to BPM the field calculations include reflections and radiation from the waveguide discontinuities [34, 35]. Radiation modes are taken into account by using absorbing boundary conditions. In [36] the MoL has been applied for studying the propagation of TE polarised field in a nonlinear planar waveguide with saturable nonlinearity. At low input power there is no practical difference between FD BPM and MoL results. When the power is large enough the comparison shows that the paraxial approximation is not valid.

A new method for studying the propagation of a field through the general waveguiding structure was developed in [37, 38]. It is based on converting the scalar wave equation into a matrix total differential equation using the orthogonal collocation method which may be stated as follows. The partial differential equation is assumed to be satisfied exactly at some points along the radial coordinate. These points are known as the collocation points. The total field is expressed as a linear combination of a set of orthogonal functions. So, by applying the collocation principle, the scalar wave equation can be converted into a set of total differential equations, which can be solved then numerically using any standard procedure, such as the Runge-Kutta method.

2.2. Semi-analytical methods

Numerous techniques exist for analysing irregular optical waveguide systems in which some physical approximations are made since an exact analytical solution is not available. The local modes approach is based on the approximation of an irregular waveguide by the consequence of longitudinally uniform segments [39, 40]. The refractive index profile is assumed to be the one of a regular waveguide defined at the given longitudinal coordinate for every segment. The phase of a local mode is approximated by averaging the longitudinal propagation constant. The local modes have been proved to be suitable only in adiabatic approximation of the field propagation in the waveguides with slowly varying parameters. Since the total field propagating along the irregular waveguide can be expressed as a sum of the discrete set of the guided modes and the integral over the radiation modes, the local modes approach should be generalised on the non-adiabatic case. The total field is to be approximated by the system of the coupled-mode equations for the amplitude of every mode of the field expansion [41 ÷ 43].

The coupled-mode theory was initially used by Marcuse [44] for analysing wave propagation in dielectric waveguide transitions in terms of the guided and radiation modes of the waveguide. It has been used to analyse non-uniform slab waveguides [45] and optical fibers [41]. There are basically two approaches to solve the infinite set of coupled differential equations resulting from this method. On one hand, we can solve a fundamental set of equations numerically by discretising the problem.

Alternatively, we can obtain closed-form expressions using some physical approximations. Three basic approximations are made in coupled-mode theory [46]. First, it is assumed that the waveguide transition is gradual so that, at any point along the length of the transition, the coupling coefficients are small. Second, coupling to reflected guided modes and backward travelling radiation modes is assumed to be negligible. Third, coupling between the different radiation modes is ignored. By making these approximations, one obtains an infinite set of coupled differential equations for the guided mode and radiation-mode amplitudes that need to be solved. By making the assumption that the guided mode amplitude can be treated as a constant, it is possible to integrate directly the radiation-mode amplitudes. However, this approach does not always yield physically meaningful results [46].

To analyse waveguide tapers and step-like structures one can use the step-transition method [46] which was initially used by Marcuse to analyse low-loss tapers [47]. The technique was modified so that it could be used to analyse any taper and step-like structures [46]. It is based on the matching procedure of the transverse fields across the step. A similar mode-matching procedure have been developed that in addition to discrete set of guided modes in subparts of the cross-section includes also continuous spectra of radiation modes [48 ÷ 50]. The method starts by dividing the total cross-section into laterally uniform sections. For each section the complete set of modes is set up. By matching tangential field components at the interface to those of the complete set of modes in the neighbouring section a scattering matrix is constructed which relates the mode amplitudes which are excited in the neighbouring section and the amplitudes of the reflected modes in the section of incidence to the incident mode amplitudes. For the numerical evaluation of the integrals over continuous spectra of radiation modes in this formulation suitable discretisation and normalisation approximates the radiating modes by discrete set of modes.

The general approach to the diffraction problem solution applied to the irregular part of the dielectric waveguide is presented in [51]. It is based on the integral equations formalism [66]. The surface currents are presented as a sum of the uniform part (corresponding to the surface modes of the waveguide) and the non-uniform one. The latter appears due to the irregularities and has a complicated structure. To solve the system of integral equations the Galerkin method was applied. However the realisation of the method demands large computation time. Another technique based on the iterative specifications of the surface mode amplitudes occurred to be more efficient. The method was widely used to calculate and analyse the different types of discontinuities in planar dielectric waveguides [52, 53].

The problem of diffraction of a surface mode by the step-like irregularities of a thin anisotropic waveguide was studied in [54] by the spectral expansion of the total field. The integro-differential equation is transformed onto the Winer-Hopf one and is solved by the method of approximate factorisation [55]. The cases of the mode scattering by a small step in a diameter and by an open end of a semi-infinite

waveguide are analysed in detail. The problem is studied by the variational method in [56] and the both semi-analytical techniques are compared in [57].

In the terms of generalised variational approach a method for obtaining equations for moments of a light beam was proposed in [58]. The method is based on the use of the Galerkin criterion on the basis of a small number of flexible Gaussian modes. The system of ordinary differential equations, with the beam parameters obeying this equation, is easily solved numerically and can be qualitatively analysed [59, 60].

Quasi-analytical method for the analysis of rectangular waveguide structures with step discontinuities is proposed in [61]. It is one of the versions of the Galerkin method in which various weighted polynomials are used as the basis functions. Following this method, one reduces the set of integral equations to the set of linear algebraic equations. The basic point of the technique is evaluation of the matrix elements of the algebraic system, since the accuracy of their computation affects substantially the precision of the final solution. Rapid convergence and high accuracy of the numerical results is guarantied by the method proposed which is based on the separation of the static and dynamic parts of the matrix operator. The method can be applied for the design of different waveguide components: filters, phase shifters, polarisers, etc.

3. The application example

3.1. Comparative analysis of the FD-BPM and the FFT-BPM simulations of the radiation field propagation

Two different beam propagation methods have been applied to simulate the total field propagation in the step-index planar waveguide with infinite cladding excited by the Gaussian beam. One is the FFT-BPM used to solve the Helmholtz equation for the slowly varying amplitude of the total fie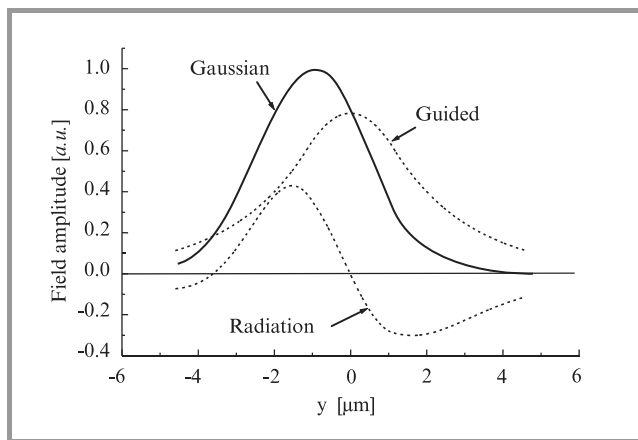ld [7]. The other is FD-BPM [62] applied to solve the paraxial parabolic wave equation, in assumption that the paraxial approximation is valid for a weakly-guiding waveguide. The main goal of the treatment is to evaluate the applicability of the paraxial approximation dealing with guiding structures with sharp contrast of the refraction coefficient. The FD-BPM algorithm is known to fail [64] being applied to solve the Helmholtz equation. Otherwise, the FFT-BPM is free of this disadvantage. On other hand, FFT-BPM is sensitive to refraction index difference in transverse direction. Therefore FFT-BPM breaks down when applied to structures with large index discontinuities.

Propagation of the radiation field excited by a Gaussian beam on an input endface of a single-mode planar waveguide was simulated numerically. The beam was assumed to be parallel to the waveguide layers. The excitation have been considered to be symmetric (beam axis coincides with the input endface center) as well as asymmetric (beam axis is shifted out of the input endface center). The transverse distribution of the incident radiation field was taken as
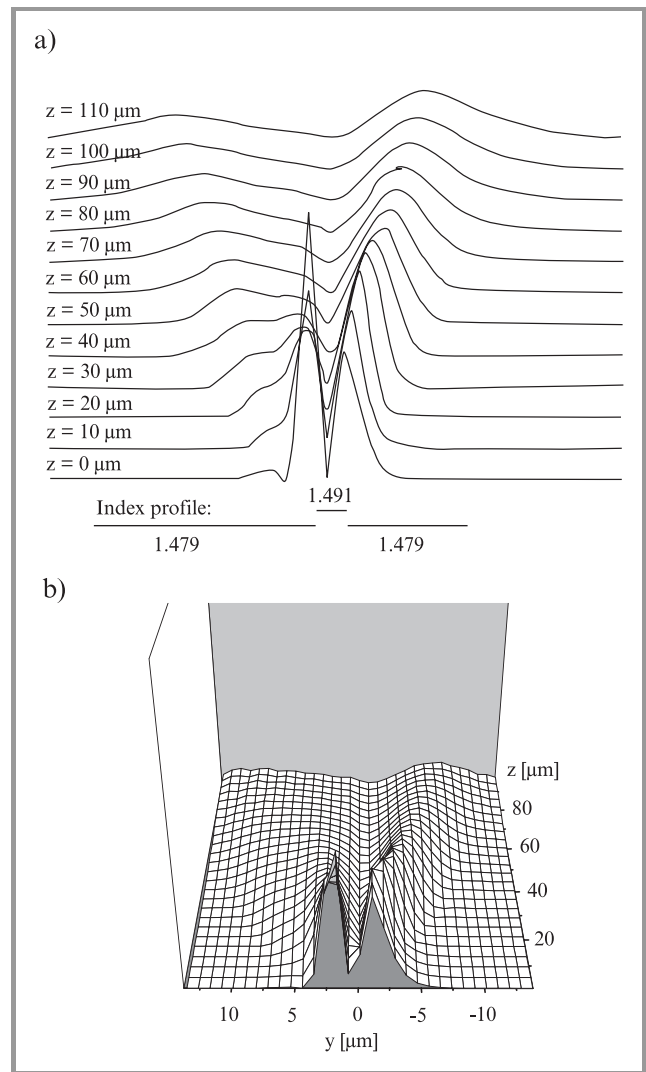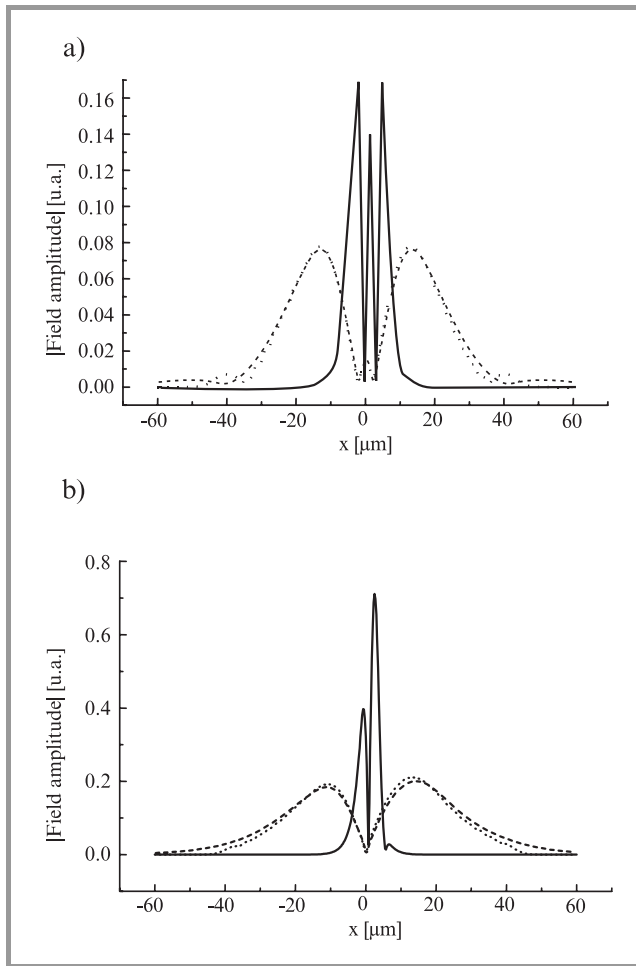

Fig. 1. Input Gaussian beam profile, its modal and radiation parts.

Fig. 2. Spatial transient process of the radiation field propagation simulated by the FFT-BPM (a) and by the FD-BPM (b).

Fig. 3. Transverse distributions of the radiation field at the input endface of the waveguide (solid line), and after the distance of 150 μm, simulated by the FFT-BPM (dashed line) and the FD-BPM (dotted line), respectively. (a) Symmetric excitation (shift = 0), (b) asymmetric excitation (shift = 2 μm).

a difference between the incident field distribution and the modal one [6]:

$$E_{rad} = E_{inc} - E_{\text{mod}}, \qquad (1)$$

where E_{mod} is proportional to a fundamental guided mode distribution Ψ_0 normalised to unit power flow, with a complex amplitude α:

$$E_{\text{mod}} = \alpha \Psi_0. \qquad (2)$$

The Gaussian beam profile and its modal and radiation parts are shown in Fig. 1.

The refractive indices chosen for the guiding and the cladding layers were $n_1 = 1.491$, $n_2 = 1.479$, respectively. The total width of the guiding layer was $D = 2.3$ μm and the wavelength was $\lambda = 1.53$ μm.

The FFT-BPM computational parameters were chosen in such a way that a part of the non-propagating (evanescent) spectrum and the whole propagating spectrum of the field were included in the k_y-space computational window in order to ensure exact modelling of the field propagation [6].
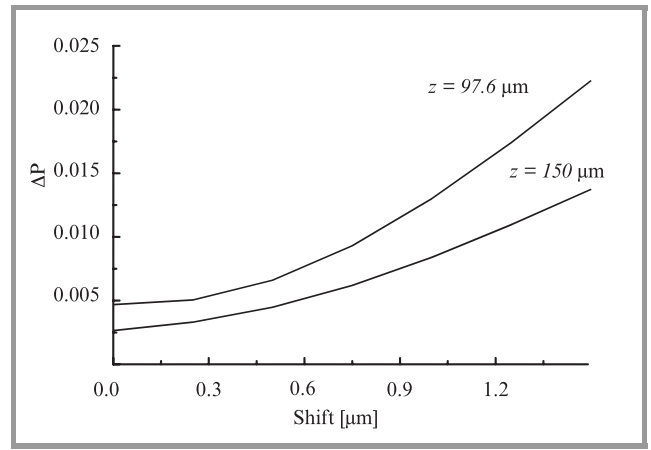

Fig. 4. Normalised power of the radiation field depending on the excitation conditions.

The mesh size was $\Delta x = 0.25$ μm and the longitudinal step size was $\Delta z = 0.5$ μm.

The FD-BPM method has been based on the Crank-Nicolson scheme and an iterative procedure proposed in [63]. The mesh size was $\Delta x = 0.25$ μm and the longitudinal step size was $\Delta z = 2.5$ μm.

Spatial transient process of the radiation field propagation is shown in Fig. 2 simulated by the FFT-BPM (a) and by the FD-BPM (b). The radiation field profiles at the input endface of the waveguide and the ones calculated after a propagation distance of 150 μm are presented in Fig. 3a,b. Additionally, the radiation field power averaged over the region $x = -30$ μm to 30 μm at the distances 96.7 μm and 150 μm was calculated by both techniques. The difference ($\Delta P = \left(P_{rad}^{FD-BPM} - P_{rad}^{FFT-BPM}\right) m / P_g$, P_g – guided mode power) is presented in Fig. 4. The results make evidence that the spreading of the paraxial beam (FD-BPM) is less than the wide-angle one (FFT-BPM). The discrepancy
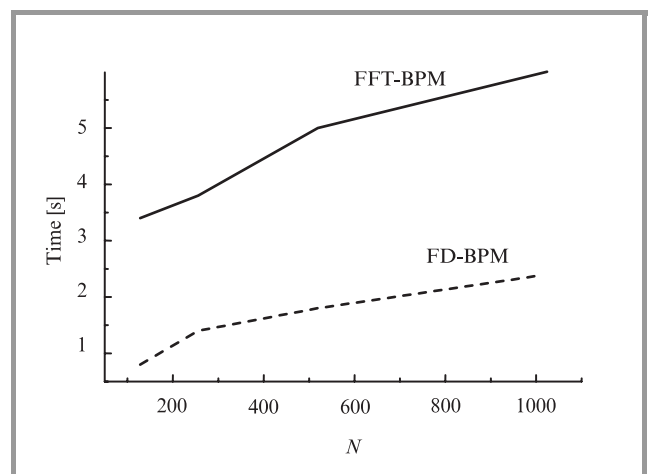
Fig. 5. Calculation time of the numerical simulations of the radiation field propagation (distance = 150 μm, $\Delta z = 0.5$ μm), using the FD-BPM and the FFT-BPM, depending on the number of grid points.

depends on the excitation conditions and grows with the shift of the input beam relative to the input endface center. Additionally, the calculation time required to simulate the radiation field propagation by both these methods was evaluated (Fig. 5). The computation efficiency of FD-BPM simulations is proportional to the number of grid points N while the one of FFT-BPM is of the order NlogN [17].

3.2. Discussion

The modelling of total field propagation by the FFT-BPM and FD-BPM is a clear example that shows some features of these numerical techniques. First, the difference between the results obtained by these methods depend on the radiation field power. If the input beam is strongly mismatched with the modal fields, a great part of the input power excites the radiation field. In this case the results obtained under the paraxial approximation (FD-BPM) differ significantly from the wide-angle ones (FFT-BPM).

Secondly, for comparable accuracy much smaller longitudinal step is to be used in the FFT-BPM than in the FD-BPM. This is one reason for the greater calculation time of the FFT-BPM to achieve the same accuracy. The other reason is that one has to use many FFT expansion terms to describe a complicated spectrum of the radiation field.

As a result, the FD-BPM seems to be more efficient technique comparing with the FFT-BPM being applied to low-contrast quasi-adiabatic structures with small discontinuities.

4. Conclusions

We have reviewed numerical and semi-analytical techniques useful to describe field propagation in all-dielectric guiding structures with smooth and sharp discontinuities. Widely used finite-difference beam propagation method is shown to be efficient tool to deal with weakly-guiding structures with small discontinuities if scalar wave propagation should be traced. Another restriction is that only one-direction field propagation can be simulated. Nevertheless, this method provides good qualitative approach for modelling propagation phenomena in different guiding structures. The review can be used as a reference directory to design fiber-optic elements such as tapers and junctions, as well as for modelling complicated physical phenomena in waveguides (gain, nonlinearity, etc.), when vectorial and wide-angle approaches have to be used.

Acknowledgements

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