Paper

The simple analysis method of nonlinear frequency distortions in FMCW radar

Krzysztof S. Kulpa, Andrzej Wojtkiewicz, Marek Nałęcz, and Jacek Misiurewicz

Abstract — The paper presents a simple method for estimating nonlinear frequency distortions of linear frequency modulated (LFM) signals used in FMCW radars. This method, derived from the polynomial model of the nonlinear FM signal phase, is based on finding the maximum of two-dimensional chirp-like transform of the IF video signal. The IF signal is obtained by mixing transmitted FM signal with its delayed copy. Using suggested transform we show that the presented method is able to detect and classify signal distortions.

Keywords — radar, nonlinear frequency distortions, linear frequency modulation.

1. Introduction

In many applications such as radars, sonars, biomedical engineering etc. the constant amplitude complex harmonic signals

$$s(t) = S_0 \exp(j\phi_M t) \tag{1}$$

with varying frequency are used. The signal phase $\phi_M(t)$ can be modeled by the M-th order polynomial

$$\phi_M(t) = \sum_{m=0}^{M} a_m t^m \tag{2}$$

with coefficients a_m. For example the chirp signal corresponds to the second order phase polynomial ($M = 2$) and so-called quadratic frequency modulated (FM) signal corresponds to the third order phase polynomial ($M = 3$). To estimate unknown parameters of frequency modulated (FM) signal, well-known time-frequency analysis, both linear (spectrogram, scalogram) and bilinear (such us Wigner-Ville distribution) are commonly used.

These tools are, however, inefficient for nonlinear frequency modulation. The recent works $[6-8]$ on generalization of Wigner-Ville distribution are very useful for analyzing the signal (1) for $M > 2$ and for estimating instantaneous signal frequency $f(t) = \frac{1}{2\pi}\frac{d\phi_M(t)}{dt}$.

One of the most important practical problems is to estimate phase coefficients a_M of unknown signal contaminated by white Gaussian noise. Such estimation allows identification and classification of polynomial phase signals $[2-4]$. These methods, however, are not well suited to such problem.

The paper presents a simple method for analyzing nonlinear distortions of LFM signals used in FMCW radars [1]. In theory, the radar transmitter generates continuous wave $s(t)$ with sawtooth frequency modulation ($M = 2$) of period T. The return echo reflected from a stationary target at distance R from the radar can be considered as the delayed and attenuated copy of the transmitted signal with time delay equal to $\tau = 2R/c$, where c is the light velocity. This received signal is mixed with transmitted signal and at the output of a homodyne receiver the pure harmonic video signal is obtained. The video signal frequency f_R is equal to $\alpha\tau$, where $\alpha = \Delta f/T$ is the slope of frequency modulation and Δf is signal frequency deviation. In FMCW radar the unknown frequency f_R is estimated by finding the maximum of signal $y(t) = s(t)s^*(t-\tau)$ spectrum (* denotes complex conjugation). In practice, the transmitted signal $s(t)$ has nonlinear frequency modulation. Its phase can be modeled by the polynomial (2) of higher order ($M > 2$) with non-zero coefficient a_M. In the paper we restrict analysis to the third order ($M = 3$), however our method can be easily extended for higher order ($M > 3$) effects.

2. Statement of the problem

Let us assume that the signal transmitted by the FMCW radar is given by:

$$s(t) = S_0 \exp\left(j\phi_3(t)\right), \tag{3}$$

where $\phi_3(t)$ is periodical function with the period T, and for time interval $0 \le t < T$

$$\phi_3(t) = a_0 + a_1 t + a_2 t^2 + a_3\left(t - \frac{T}{2}\right)^3 \tag{4}$$

is the third order polynomial (for convenience, we use slightly different notation than (2)). The instantaneous angular frequency $\omega(t) = d\phi_3(t)/dt$ of the signal (3) (for $0 < t < T$) is equal to

$$\omega(t) = a_1 + 2a_2 t + 3a_3\left(t - \frac{T}{2}\right)^2, \tag{5}$$

where $a_1/(2\pi)$ is the carrier frequency f_0, $2a_2/(2\pi)$ is the slope of frequency modulation α, and the third term in Eq. (5) is responsible for quadratic modulation distortion. After mixing the signal $s(t)$ with the return echo $s^*(t-\tau)$ we obtain (for $\tau < t < T$)

$$y(t) = s(t)s^*(t-\tau) = Y_0 \exp\left(j\phi(t)\right). \tag{6}$$

The differential phase $\phi(t) = \phi_3(t) - \phi_3(t-\tau)$ is given by:

$$\phi(t) = b_0 + b_1 t + b_2 \left(t - \frac{T}{2}\right)^2, \tag{7}$$

where

$$b_0 = a_1\tau - a_2\tau^2 + a_3\tau^3 + \frac{3}{2}a_3\tau^2 T, \tag{8}$$

$$b_1 = 2a_2\tau - 3a_3\tau^2, \tag{9}$$

$$b_2 = 3a_3\tau. \tag{10}$$

To simplify further considerations we have assumed unit amplitude of the signal ($S_0 = 1$) and we do not take into consideration initial phase b_0. To measure nonlinear effects of modulation, the unknown parameters b_1 and b_2 of the signal

$$x(t) = \exp\left(j\left[b_1 t + b_2\left(t - \frac{T}{2}\right)^2\right]\right) \tag{11}$$

have to be estimated. For small nonlinear distortions the second term in Eq. (9) is negligible and we can assume $b_1 = 2a_2\tau$. The parameter b_1 is the unknown angular frequency of the signal, proportional to time delay τ (range to the target), and $b_2 = 3a_3\tau$ is the unknown quadratic modulation distortion. It must be noted that due to distortions the signal (11) is an LFM signal instead of a pure harmonic signal.

3. Estimation of unknown parameters

In the modern digital FMCW radar receivers the video signal (11) is sampled with sampling period $\Delta T = T/N$, where N is a natural number (typically $N = 1024$, 2048, ...). A discrete-time version of Eq. (11) has the form

$$x(n) = \exp\left(j\left[n\theta_R + \gamma\theta_R(n - N/2)^2\right]\right), \tag{12}$$

where $\theta_R = \omega_R \Delta T$ is the unknown range angular frequency normalized with respect to sampling frequency $f_s = \frac{1}{\Delta T}$ and the parameter

$$\gamma = \frac{3a_3 T}{2a_2 N} \tag{13}$$

is an unknown coefficient of nonlinear distortions.

Let us apply the idea of the matched filter to the estimation of unknown parameters θ_R and γ of a deterministic signal (12) contaminated by additive white Gaussian noise. If the impulse response $h(n)$ of the deterministic $N-1$ order FIR filter fulfills the condition

$$h(n) = x^*(N-1-n) \tag{14}$$

then the filter is matched to the signal $x(n)$ and its response

$$z(n) = \sum_{k=0}^{N-1} x(n-k)\,h(k) = \sum_{k=0}^{N-1} x(n-k)\,x^*(N-1-k) \tag{15}$$

for $n = N-1$ is equal to the signal energy E

$$z(N-1) = \sum_{k=0}^{N-1} \left|x(k)\right|^2 = E. \tag{16}$$

The energy E is obtained only when estimated values $\hat{\theta}_R$ and $\hat{\gamma}$ of matched filter impulse response are equal to actual values θ_R and γ of the analyzed signal. In the estimation process it is necessary to search for values $\hat{\theta}_R$ and $\hat{\gamma}$ that maximize (15) for $n = N-1$, or equivalently to maximize absolute value of the two-dimensional transform

$$X(\theta_R, \gamma) = \\ = \sum_{k=0}^{N-1} x(N-1-k)\exp\left(-j\left[(N-1-k)\theta_R + \right.\right. \\ \left.\left. + \gamma\theta_R\left(\tfrac{N}{2} - 1 - k\right)^2\right]\right). \tag{17}$$

For $\gamma = 0$ Eq. (17) becomes the classical discrete Fourier transform (DFT) of the signal $x(n)$, often used for estimating range frequency $\hat{\theta}_R$ in FMCW radars with linear frequency modulation. The second order term in Eq. (17) (for $\gamma \neq 0$) can be interpreted as the correction of nonlinear effects. In general, Eq. (17) can be treated as the extended, generalized chirp transform, which allows detection and estimation of modulation distortions. It is worth to notice that transform (17) is a special case of the so-called generalized chirp transform (GCT) and the estimated values $\hat{\theta}_R$ and $\hat{\gamma}$ are the maximum likelihood estimates of θ_R and γ parameters [5].

4. Measurement results and conclusions

To verify the ability of the transform (17) to detect and estimate nonlinear distortions in FMCW radar signal, the recorded and simulated nonlinear modulated FMCW signals were tested. In Fig. 1 the recorded signal transform is presented. It is easy to notice that the transform (17) of recorded signal reaches its maximum for γ approximately equal to -1%.

For comparison purposes in Fig. 2 the transform of simulated signal with -1% third order component is presented. These two figures are very similar, but it can be easily seen that recorded signal transform is less symmetrical than the simulated one. This is due to the presence of higher order nonlinear distortions in the recorded signal. Further investigations show that the fourth order component causes vertical asymmetry, and the fifth order term causes horizontal asymmetry. This example proves that the presented method may be used successfully to estimate third order nonlinear coefficients and to detect higher order effects.

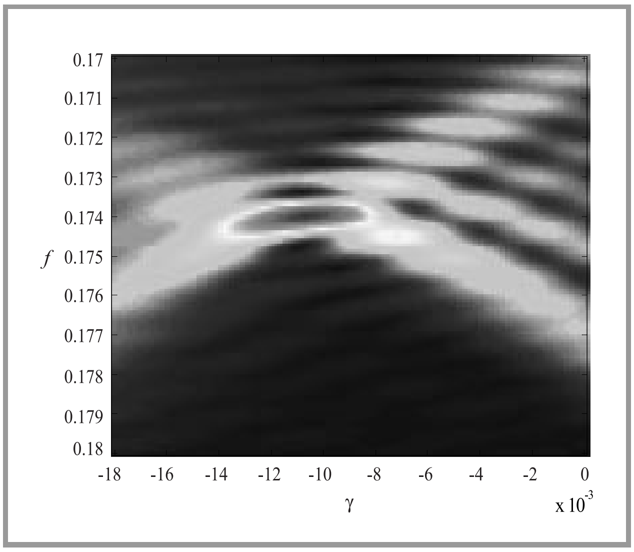

Fig. 1. Recorded signal transform (17) magnitude; $f-\gamma$ plane ($f=\theta_R/2\pi$).

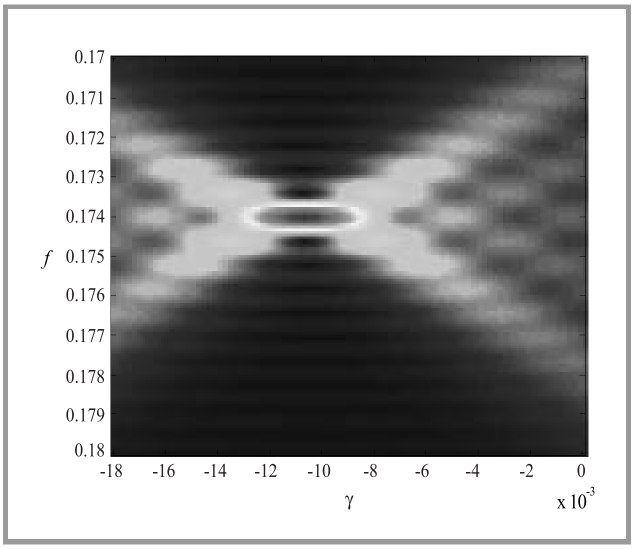

Fig. 2. Simulated signal transform (17) magnitude.

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Krzysztof S. Kulpa was born in Warsaw, Poland on April 13, 1958. He received the M.Sc. and Ph.D. degrees in electronic engineering from Warsaw University of Technology in 1982 and 1987, respectively. Since 1982 he has been in the Institute of Electronics Fundamentals at Warsaw University of Technology on the posts of a Teaching Assistant and Assistant Professor. His teaching activities included the areas of measurements and digital signal processing. In the years 1988–1990 he was also at the Technical University of Bialystok, Bialystok, Poland. His professional experience includes training visits in IBM Spain or Elmer-Perkins in England, as well as being a technical consultant in CN-PEP RADWAR in Poland. He was involved with several research and development projects, granted from the Polish radar industry, in the area of digital processing of radar signals. His interests include Kalman filtering and object tracking problems, adaptive MTI filtering, and continuous wave radars.

e-mail: k.kulpa@ise.pw.edu.pl
Institute of Electronic Systems
Warsaw University of Technology
Nowowiejska st 15/19
00-665 Warsaw, Poland

Andrzej Wojtkiewicz was born in Nowogrodek (Poland) in 1938. He received the M.Sc. and Ph.D. degrees from the Gdansk University of Technology (Poland) in 1961 and 1971, respectively, both in electronic engineering. Since 1974 he has been with the Warsaw University of Technology, where he currently holds the position of Professor. His fields of interest concentrate on analysis and design of digital filters, adaptive signal processing and parameter estimation, applied primarily in digital processing of radar signals. He has authored and co-authored more than 100 research papers and technical reports in his areas of interest. He is also author of the book, *Introduction do Digital Filter Synthesis* (1984). He has been a member of the IEEE.

e-mail: a.wojtkiewicz@ise.pw.edu.pl
Institute of Electronic Systems
Warsaw University of Technology
Nowowiejska st 15/19
00-665 Warsaw, Poland

Marek Nałęcz was born in Warsaw (Poland) in 1961. He received the M.Sc. degree in 1984 and the Ph.D. degree in 1992 from the Warsaw University of Technology, both in electronic engineering and both conferred with honors. Since 1984 he has been with the Warsaw University of

Technology, where he currently holds the position of Assistant Professor. His early works were devoted to the analysis of switched-capacitor circuits. Now his fields of interest concentrate on digital processing of radar signals, applications of digital signal processors, adaptive filters, modern spectral estimation methods and robust algorithm of system identification. He has authored and co-authored more than 50 research papers, conference papers and technical reports in his areas of interest. He has been a member of the IEEE.

e-mail: m.nalecz@ise.pw.edu.pl
Institute of Electronic Systems
Warsaw University of Technology
Nowowiejska st 15/19
00-665 Warsaw, Poland

Jacek Misiurewicz was born in Warsaw, Poland on March 26, 1965. He received the M.Sc. and Ph.D. degrees in electronic engineering from Warsaw University of Technology in 1988 and 1996, respectively. Since 1988 he has been in the Institute of Electronics Fundamentals at Warsaw University of Technology on the posts of Teaching Assistant and Assistant Professor in the area of digital signal processing. He was involved with several research and development projects, granted from the Polish radar industry, in the area of digital processing of radar signals. His interests include non-uniform sampling problems, MTI filtering and filter design, as well as programmable logic applications and real-time programming. He was a member of an Organizing Committee of National Circuit Theory conferences in 1991 and 1992. He is a member of the IEEE since 1993, and IEEE Poland Section Newsletter Editor since 1998.

e-mail: j.misiurewicz@ise.pw.edu.pl
Institute of Electronic Systems
Warsaw University of Technology
Nowowiejska st 15/19
00-665 Warsaw, Poland