Paper

Optical Josephson effects using phase-conjugating mirrors: an analogy with superconductors

Miriam Blaauboer, Daan Lenstra, and Adri Lodder

Abstract — Motivated by the analogy between a phase-conjugating mirror (PCM) and a superconductor, we search for optical counterparts of the well-known DC and AC Josephson effects. We show that in a system consisting of two PCM's separated by vacuum an „optical supercurrent" arises as a function of an applied phase difference between the PCM's, which is the *optical analogue* of the DC supercurrent flowing in a superconducting weak link. The corresponding AC effect occurs when the two PCM's are pumped by light of a different frequency, causing the phase difference to oscillate in time with the frequency difference.

Keywords — Josephson effects, phase-conjugation mirrors, superconductors, four-wave mixing.

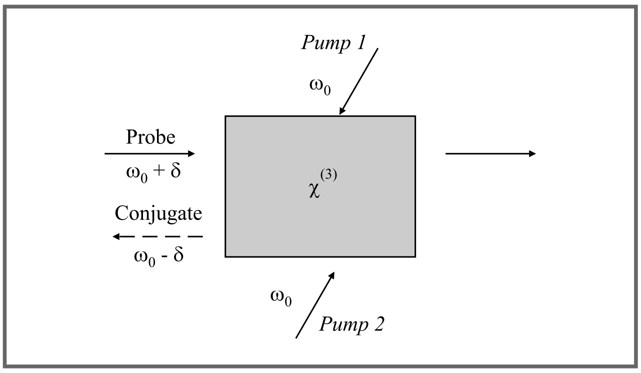

Fig. 1. A phase-conjugating mirror realized through a four-wave mixing process

Introduction

A phase-conjugating mirror (PCM) is a nonlinear optical device capable of reversing both the direction of propagation and the overall phase factor of an incident beam of light [1]. It consists of an optical medium with a large third-order susceptibility $\chi^{(3)}$ and can be realized through a four-wave mixing process, see Fig. 1. The medium is pumped by two intense counterpropagating laser beams of frequency ω_0. When a probe beam of frequency $\omega_0+\delta$ is incident on the material, a fourth beam will be generated due to the nonlinear polarization of the medium. The latter propagates with frequency $\omega_0-\delta$ in the opposite direction as the probe beam and is referred to as the conjugate beam [1]. The probe-to-conjugate reflection process at a PCM is the optical analogue of Andreev reflection, the electron-to-hole reflection which occurs at the interface between a normal metal (N) and a superconductor (S) [2]: just as the hole is sometimes called a „time-reversed" electron, the conjugate can be seen as the „time-reversed" of the probe wave. The role of the chemical potential μ in a superconductor is played by the pump frequency ω_0 in a PCM, and the energy gap Δ of the bulk superconductor corresponds to the coupling strength γ between probe and conjugate waves in the nonlinear optical medium, to be defined later (see below Eq. (1)).

This by now well-established analogy between a PCM and a superconductor [3, 4] can be extended to a system consisting of two PCM's separated by vacuum, which is then the analogue of a superconductor–normal-metal–superconductor (SNS) structure or weak link [5]. In these weak links the famous DC and AC Josephson effects occur, which were originally predicted for tunnel junctions [6]. The DC effect in a tunnel junction is the sinusoidal dependence of the supercurrent I_S on the phase difference $\Delta\phi$ between the pair potentials in the two superconductors, $I_S \sim \sin(\Delta\phi)$. In a point contact, i.e. a weak link consisting of a constriction in between two superconductors, it also occurs, but the current-phase dependence is then found to be $I_S \sim \sin(\Delta\phi/2)$ for $|\Delta\phi| < \pi$ [7, 8, 9]. The nonstationary (AC) Josephson effect arises when a voltage V is applied across a tunnel junction or weak link, causing the phase difference to change with time as $\dot{\Delta\phi} = 2eV/\hbar$.

The carriers of the supercurrent flow in weak links are the quasiparticle bound states [10], or „Andreev levels". These quasiparticle bound states are formed when an electron (or hole) with energy E above (below) the Fermi energy, but with E smaller than the gap energy Δ of the superconductor is present in the normal layer. The electron cannot enter the superconductor without pairing with another electron, which leaves a hole behind in the normal metal: the well-known process of Andreev reflection [2]. Conversely, a hole can break up a Cooper pair and produce an electron in the normal metal. The bound-state spectrum is found by calculating the condition for constructive interference of electron/hole waves in the middle layer after one roundtrip (corresponding to two Andreev reflections, electron-to-hole at one superconductor and hole-to-electron at the other superconductor). Here, we assume clean NS interfaces without any potential barriers, so that normal reflections can be neglected. Since each Andreev reflection is accompanied by a phase-shift which depends on the phase of the superconducting pair potential, these bound states produce a coupling between the phases of the order parameters of the two superconductors and are the carriers of the supercurrent.

Returning to the optical configuration of two PCM's separated by a layer of vacuum, it is known that just as bound states are formed in a SNS structure due to Andreev reflections at the NS interfaces, so-called axial modes[1] are formed in the vacuum region due to phase-conjugate reflections at the PCM's [4]. We will show here that like the Andreev bound-state levels are the „channels" for the supercurrent flow, the axial modes form the „channels" for an „optical supercurrent" flow: when the vacuum region is short and the two PCM's are pumped by light of the same frequency, but with a small phase difference $\Delta\phi$, a photonpair(„super")current arises as a function of this phase difference, similar to the supercurrent in a short superconducting weak link. The calculation and analysis of this DC optical Josephson effect forms the topic of section , after a discussion of the axial modes in section . In section the corresponding AC effect is discussed and we conclude in section with a suggestion for a possible experimental realization of these optical Josephson effects.

Axial modes

Consider the double PCM configuration depicted in Fig. 2. Just as the equilibrium state of a superconductor is described by the eigenfunctions of the Bogoliubov-de Gennes equations [11], each PCM-medium is described by the eigenfunctions of the matrix equation [3]

$$\begin{pmatrix} -\frac{c^2}{2\omega_0}\frac{\partial^2}{\partial x^2}-\frac{\omega_0}{2} & -\gamma \\ \gamma^* & \frac{c^2}{2\omega_0}\frac{\partial^2}{\partial x^2}+\frac{\omega_0}{2} \end{pmatrix}\begin{pmatrix}\mathcal{E}_p(x)\\ \mathcal{E}_c^*(x)\end{pmatrix} = i\frac{\partial}{\partial t}\begin{pmatrix}\mathcal{E}_p(x)\\ \mathcal{E}_c^*(x)\end{pmatrix}. \quad (1)$$

Here $(\mathcal{E}_p(x), \mathcal{E}_c^*(x))$, with $\mathcal{E}_p$ and $\mathcal{E}_c^*$ the electric field amplitude of the probe and conjugate waves respectively, represents an excitation in the pumped medium with a mixed probe/conjugate character: an „optical quasiparticle", in analogy with the mixed electronlike/holelike quasiparticles in a superconductor [11]. The off-diagonal parameter $\gamma = \gamma_0 e^{i\phi} = \frac{3\omega_0}{\varepsilon_0}\chi^{(3)}\mathcal{E}_1\mathcal{E}_2$ is the pumping induced coupling strength between the probe and conjugate wave in the PCM-medium, and ϕ denotes the phase of $\chi^{(3)}\mathcal{E}_1\mathcal{E}_2$, with $\mathcal{E}_1$, $\mathcal{E}_2$ the electric field amplitudes of the two pump beams. The PCM's in Fig. 2 are both pumped by two counterpropagating laser beams with the same frequency ω_0 but with a different phase, say ϕ_1 on the left and ϕ_2 on the right.

[1]By „axial" modes we mean longitudinal modes, to distinguish them from transverse modes. The latter are formed due to finite dimensions of the whole system in the transverse direction(s), e.g., when it is embedded in a waveguide. Here we consider a quasi-1D configuration, in which only one transverse mode is present.

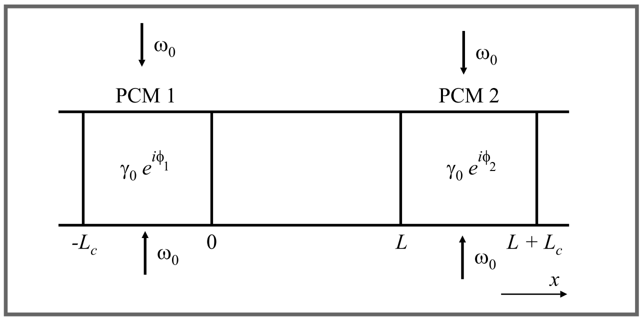

Fig. 2. Two PCM's of equal length L_c separated by a layer of vacuum of length L. The PCM's are each pumped with two laser beams of the same frequency ω_0, but such that the phases on the left and right are different, which leads to different phases of their coupling constants γ.

In the region between the PCM's, a probe beam incident on either mirror will be reflected as a conjugate beam, which again emerges as a probe beam after reflection at the other PCM. The condition for the formation of an axial mode is that the acquired phase shift on one round trip equals an integer multiple of 2π, and is given by [4]

$$2\frac{\delta}{c}L + 2\arctan\left(\frac{\delta}{\sqrt{\delta^2+\gamma_0^2}}\tan(\beta L_c)\right) \pm \Delta\phi = 2\pi n. \quad (2)$$

Here $\Delta\phi = \phi_1 - \phi_2$, $\beta = \frac{1}{c}\sqrt{\delta^2+\gamma_0^2}$ and the $\pm$-sign corresponds to a probe wave travelling with frequency $\omega_0 \pm \delta$ to the right and being reflected as a conjugate wave with frequency $\omega_0 \mp \delta$. Eq. (2) is the analogue of the bound-state spectrum (Andreev levels) of a SNS junction[2] [10]

$$k_F\frac{E}{E_F}L - 2\arccos(E/\Delta_0) \pm \Delta\phi = 2\pi n, \quad (3)$$

where k_F and E_F are the Fermi wavevector and energy respectively and $\Delta_0 = |\Delta|$. For a short weak link, in which the distance between the superconductors is much less than the superconducting coherence length $\xi_S = \frac{\hbar v_F}{\pi\Delta_0}$, the first term in (3) may be neglected, so that the mode spectrum simplifies to $E = \Delta_0\cos(\Delta\phi/2)$ [8, 9]. By analogy, we restrict ourselves here to the situation in which the vacuum region that separates the two PCM's is short compared to the „optical coherence length"[3] $\xi_0 = c/\gamma_0$. The axial mode spectrum (2) then reduces to

$$\frac{\delta}{\sqrt{\delta^2+\gamma_0^2}}\tan(\beta L_c) = -\tan\left(\frac{\Delta\phi}{2}\right), \quad (4)$$

[2]The first term on the left-hand side of Eq. (3) does not have a factor „2" as Eq. (2) has; this is due to the different (parabolic) dispersion relation of an electron in a normal metal compared with the linear dispersion relation of light in vacuum.

[3]ξ_0 is a a measure of the minimum spatial extent of the transition layer between a PCM and vacuum, just as the superconducting coherence length ξ_S is a measure of the minimum spatial extent of the transition layer between a normal metal and a superconductor, see e.g. [12].

independent of the mode-index n. For frequencies $\delta \ll \gamma_0$, Eq. (4) takes the simple form

$$\delta = -\gamma_0 \frac{1}{\tan\left(\frac{\gamma_0 L_c}{c}\right)} \tan\left(\frac{\Delta\phi}{2}\right)$$

$$\text{for} \quad \left| \frac{\tan\left(\frac{\Delta\phi}{2}\right)}{\tan\left(\frac{\gamma_0 L_c}{c}\right)} \right| \ll 1. \tag{5}$$

Now there is a *single* axial mode. This axial mode spectrum can support a stationary photon-current, which we now proceed to derive.

The stationary (DC) optical Josephson effect

The starting point of our search for a phase-dependent Josephsonlike photon-current in the double-PCM configuration is the fact that due to spontaneous emission and quantum fluctuations, photons with a range of frequencies are generated in each PCM, part of which are emitted into the region between the PCM's. Out of these, only the frequencies that satisfy the condition (4) will lead to the formation of axial modes, which can be the carriers of a phase-dependent current. Before analyzing this any further, we now first mention an important difference between Andreev reflection at a superconductor and phase-conjugate reflections at a PCM. Whereas the former always occurs with probability 1 at an ideal NS interface [13], i.e. one electron is reflected as one hole and particle conservation applies, the reflected conjugate beam at a PCM can be stronger than the incoming probe beam. Because the two pump beams continuously add energy to the medium, the PCM can act as a phase-conjugate amplifier. In fact, the probability for phase-conjugate reflection is given by [1]

$$R_c = \frac{\sin^2(\beta L_c)}{\cos^2(\beta L_c) + \left(\frac{\delta}{\gamma_0}\right)^2}, \tag{6}$$

$$R_c \approx \tan^2\left(\frac{\gamma_0 L_c}{c}\right) \quad \text{for } \delta \ll \gamma_0. \tag{7}$$

It can easily be seen from Eq. (6) that for frequencies $\delta \gg \gamma_0$ the phase-conjugate reflectance $R_c < 1$, so that the reflected beam is weaker than the incoming beam. In the opposite limit of $\delta \ll \gamma_0$, Eq. (7) applies, and then $R_c \geq 1$ for $|\gamma_0 L_c/c| \geq \pi/4$, in which case the reflected beam is stronger than the incoming one. According to (7) it may even become infinitely strong, for when $\gamma_0 L_c/c \rightarrow \pi/2$, $\tan(\gamma_0 L_c/c) \rightarrow \infty$. However, in reality the intensity of the reflected beam is limited by the intensity of the pump beams: if the former approaches the latter, pump depletion will set in and the expression (6) for R_c is no longer valid, since it was derived under the condition of undepleted pump beams [1].

The magnitude of R_c plays an important role in the formation of an equilibrium current in our double-PCM configuration. For if the phase-conjugate reflection is less than 100%, any current will decrease in time and eventually die. If, on the other hand, the phase-conjugate reflection is more than 100%, the wave is amplified at the expense of the pump beams upon each reflection at the PCM's. Then after a while pump depletion will set in, causing the reflected intensity and the current to decrease again, until a stable situation is reached in which there is neither gain nor loss. If the probability of phase-conjugate reflection is exactly 100%, the current in the region between the PCM's is „automatically" stable. This situation forms the closest analogy with the superconducting case.

Before calculating anything explicitly, we now thus already know that an equilibrium photon-current in the region between the PCM's can only form for frequencies which: (a) satisfy the axial-mode condition (4) and (b) are neither weakened nor amplified upon phase-conjugate reflection, i.e. for which a steady state exists or is established in the region between the PCM's. The important question is thus for which frequencies satisfying (4) a steady state is formed. The answer to this question requires a nonlinear analysis of the phase-conjugate reflection process which takes pump depletion into account. This is the topic of a separate paper *M. Blaauboer (to be published)*, and the main result is that a steady state is established for all frequencies δ that satisfy the condition $R_c \geq 1$, provided the PCM operates such that $|\tan(\gamma_0 L_c/c)| \geq 1$.

Here, we restrict ourselves to the case of $|\tan(\gamma_0 L_c/c)| = 1$, corresponding to $R_c = 1$ for $\delta \ll \gamma_0$, which most closely brings out the analogy with the superconductor (see for the general case *M. Blaauboer, submitted to Phys. Rev. Lett.*). The equilibrium current density in the region between the PCM's is obtained from the (normalized) eigenfunctions of Eq. (1) and given by

$$j = \frac{c^2}{\omega_0} \sum_{\delta} \mathrm{Re}\left(\bar{\mathcal{E}}_p^* i \nabla \bar{\mathcal{E}}_p + \bar{\mathcal{E}}_c^* i \nabla \bar{\mathcal{E}}_c\right), \tag{8}$$

where the sum is over all frequencies δ satisfying the axial-mode condition and $(\bar{\mathcal{E}}_p, \bar{\mathcal{E}}_c^*)$ is the eigenfunction corresponding to δ. The latter has been calculated (*M. Blaauboer*) for a short ($L \ll \xi_0$) PCM-vacuum-PCM junction, by adopting a WKB model for the propagation of each mode in the PCM-vacuum-PCM junction[4]. In essence, the WKB model requires that the length of the vacuum region $L \gg \lambda_0$ [with λ_0 the wavelength of the pump beams] and that the width of this region varies smoothly over L, leading to an x-dependent coupling constant $\gamma_0(x)$, phase $\phi(x)$ and wavevector $k_0(x)$ between $x = 0$ and $x = L$. The axial-mode eigenfunction is then given by

$$\begin{pmatrix} \bar{\mathcal{E}}_p(x) \\ \bar{\mathcal{E}}_c^*(x) \end{pmatrix} = \sqrt{\frac{\gamma_0}{2c}} \begin{pmatrix} e^{i\bar{\phi}/2} \\ e^{-i\bar{\phi}/2} \end{pmatrix} e^{\pm i \int_0^x dx' k(x')} \qquad \delta \ll \gamma_0, \tag{9}$$

[4]This model has also been used in Ref. [9] for a superconducting weak link.

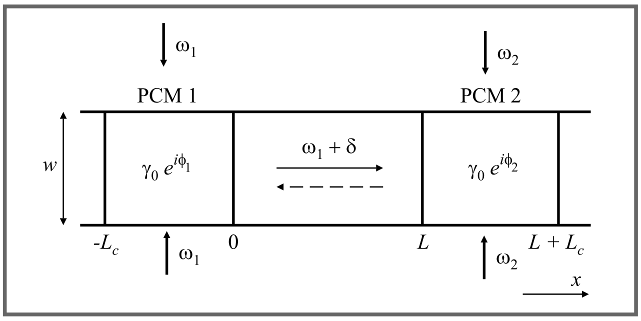

Fig. 3. Two PCM's separated by a distance L

for

with

$$\bar{\phi} = \phi(x) + i\,\mathrm{arcsinh}\,(\tan(\Delta\phi/2)) \tag{10}$$

$$k(x) = k_0 \left(1 + \frac{\gamma_0(x)}{\omega_0}\cos(\phi(x) - \bar{\phi})\right). \tag{11}$$

Here, we have used the axial-mode condition (5); the approximation $\delta \ll \gamma_0$ is justified because in our case of $R_c = 1$ only for these frequencies a steady state is established (*M. Blaauboer*)[5]. Substituting (9) into (8) yields

$$I = \gamma_0 \tan\left(\frac{\Delta\phi}{2}\right) \quad \text{for } \tan(\Delta\phi/2) \ll 1. \tag{12}$$

This is the *optical analogue* of the DC supercurrent $I_S = \frac{e\Delta_0}{\hbar}\sin\left(\frac{\Delta\phi}{2}\right)$ for $|\Delta\phi| < \pi$ flowing through a quasi-1D superconducting weak link. Note, however, that Eq. (12) is only valid for small angles $\Delta\phi$, in order to be consistent with the condition $\delta \ll \gamma_0$. For larger phase-differences no steady-state, and hence no equilibrium current exists. If there were N propagating modes in the intermirror region, Eq. (12) would be multiplied by N and so for a given $\Delta\phi$ each mode would contribute γ_0 to the resulting current *(M. Blaauboer)*.

The nonstationary (AC) optical Josephson effect

When a voltage V is applied across a superconducting tunnel junction, or weak link, the phase difference between the pair potentials changes in time as $\mathrm{d}(\Delta\phi)/\mathrm{d}t = 2eV/\hbar$. The Josephson current is then an oscillating function in time, $I_S(t) \sim \sin(\Delta\phi) = \sin(\frac{2eV}{\hbar}t + (\Delta\phi)_{t=0})$, and this is called the nonstationary (AC) Josephson effect.

The optical analogue of the Fermi energy in a superconductor is the pump frequency of a PCM. The analogue of a voltage difference V across the two superconductors, which is in fact a difference in Fermi energies, is then a frequency difference between the two pump beams on the left and right. In view of this analogy one might wonder whether such a frequency difference gives rise to an optical AC Josephson effect. It is straightforward to show that this is indeed the case.

First we examine the axial modes for the situation depicted in Fig. 3. The only difference with the PCM-system considered before (Fig. 2) is that the nonlinear media on the left and right are now pumped with different frequencies, ω_1 and ω_2 respectively. As a result of this frequency difference a probe (or conjugate) wave propagating in the region between the two PCM's will have a different frequency after each round-trip. Consider e.g. a probe beam of frequency $\omega_1 + \delta$ travelling to the right (see Fig. 3). The detuning of this beam with respect to ω_2 is $\omega_1 - \omega_2 + \delta$, so after phase-conjugate reflection at the PCM on the right a conjugate beam will travel to the left with frequency $2\omega_2 - \omega_1 - \delta$. Reflection at PCM 1 leads to a probe beam with frequency $3\omega_1 - 2\omega_2 + \delta$. The frequency change is thus $2(\omega_1 - \omega_2)$ per round-trip, or, equivalently, $\omega_1 - \omega_2$ per unit of time L/c. In the same time, during propagation from 0 to L, the phase ϕ of the coupling constant γ changes from ϕ_1 to ϕ_2. Stable axial modes then only occur if the frequency change causes a change in time of the phase difference

$$\frac{\mathrm{d}(\Delta\phi)}{\mathrm{d}t} = \pm(\omega_2 - \omega_1). \tag{13}$$

This is the optical analogue of the AC Josephson effect. If the frequency difference $\omega_2 - \omega_1$ is much smaller than the inverse response time of the PCM's (for Kerr-type materials typically GHz to THz) the system will adiabatically follow and produce an alternating optical current with the pump-frequency difference as its fundamental frequency.

Summary and experimental outlook

In conclusion, we predict a new analogy between optics and micro-electronics [14], which exploits the known analogy between quasiparticle excitations in a superconductor and optical phase conjugation by four-wave mixing in a nonlinear optical Kerr-type material, and consists of the optical analogue of the DC and AC Josephson effects that occur in superconducting weak links.

A PCM is typically pumped with frequency $\omega_0 \approx 10^{15}$ rad s^{-1}, has length L_c usually several millimeters and coupling strength $\gamma_0 \approx 10^9 \div 10^{10}$ s^{-1} [15]. One can thus arrange $\gamma_0 L_c/c \approx \pi/4$, so phase-conjugate reflection at the PCM occurs with probablity 1. Since $\lambda_0 = c/\omega_0 (\approx 10^{-7}$ m) and the coherence length $\xi_0 \approx 10^{-2} \div 10^{-1}$ m, an intermirror distance $L \approx 10^{-3}$ m satisfies the condition $\lambda_0 \ll L \ll \xi_0$, corresponding to a short PCM-vacuum-PCM junction. The optical Josephson current which can then be observed by varying the phase of the coupling constants in the two PCM's with the respective pump beams, through e.g. letting the path lengths of the pump beams differ on the left

[5]Strictly speaking, in the case of $R_c = 1$ a steady state is established only for $\delta = 0$. As $|\delta| > 0$, we have $R_c < 1$ and the resulting current wil slowly decay. For frequencies $\delta \ll \gamma_0$, however, this occurs on a timescale of $\geq \mu$s (corresponding to a decay to a value 1/e of the original amplitude), long enough to be observable.

and right, and has frequency $\gamma_0 \approx$ GHz-THz. The AC optical Josephson effect would be observable for a frequency-difference between the pump waves on the left and right of $10^9 \div 10^{12}$ rad s^{-1}. The alternating current will then oscillate on a nano- to picosecond time scale.

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