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Preface

The Fifth International Conference on Decision Support for Telecommunications and Information Society DSTIS-2005, organized by the National Institute of Telecommunications in Warsaw (1st – 2nd September, 2005), was an opportunity to bring together researchers working on optimization and decision support for telecommunications industry and information society.

The first paper presents a reflection on the role of technology and, in particular, information technology in the era of knowledge civilization. Diverse perceptions of this era, the concepts of three civilization eras versus three waves, of a cultural platform versus an episteme of a civilization era, of a big change at the end of industrial civilization era are outlined.

The next two papers are devoted to network planning. The first paper presents a variable neighborhood search (VNS) metaheuristic that is applied to solve the provisioning and routing problem in WDM networks. An integer flow formulation is modeled in AMPL and solved by CPLEX in order to obtain optimal solutions as a reference for the heuristic. The second paper shows how the concepts of multiple criteria equitable optimization can be effectively used to generate various fair and efficient allocation schemes. The authors introduce a multiple criteria model equivalent to equitable optimization and they develop a corresponding reference point procedure for fair and efficient network dimensioning for elastic flows. The procedure was tested on a sample network dimensioning problem for elastic traffic and its abilities to model various preferences are demonstrated.

In various types telecommunication networks, namely mobile ad hoc networks, WDM networks and MPLS networks, there is the necessity of calculating disjoint paths for given node to node connections in order to increase the reliability of the services supported by these networks. This leads to the problem of calculating a pair of disjoint paths (or a set of disjoint paths) which optimises some measure of performance in those networks. In the next paper an algorithm, designated as OptDP, for obtaining the most reliable pair of disjoint paths based on the loopless version of MPS, a very efficient k-shortest path algorithm, and on Dijkstra algorithm is presented. In the next elaboration the authors propose some extensions of the minimum labelling spanning tree problem. The main focus is on the minimum labelling Steiner tree problem: given a graph G with a color (label) assigned to each edge, and a subset Q of the nodes of G (basic vertices), we look for a connected subgraph of G with the minimum number of different colors covering all the basic vertices. The problem has several applications in telecommunication networks.

We have included the paper that reviews current algorithms for distributed, asynchronous control of networks when the customer is guaranteed to get some predetermined (e.g., as a part of a service level agreement – SLA) values of flow. Two cases are considered – both with single and multiple commodity. It is assumed, that the flow cost functions are convex with special attention devoted to linear and strictly convex cases.

In designing and analysis of telecommunication networks important role have various type of simulators. The paper presented in this volume addresses issues associated with the application of federations of parallel/distributed simulators to large scale networks simulation. Two principal paradigms for constructing simulations today are discussed. Particular attention is paid to an approach for federating parallel/distributed simulators. The authors describe the design and performance of frame relay network simulator (FR/ASimJava) implemented based on a Java-based library for distributed simulation – ASimJava.

The last five papers are focusing on data analysis and advanced systems. In the first paper the authors are interested in the question of coherence of radial implicative fuzzy systems with nominal consequents. Implicative fuzzy systems are fuzzy systems employing residuated fuzzy implications for representation of IF-THEN structure of their rules. Radial fuzzy systems are fuzzy systems exhibiting the radial property in antecedents of their rules. The property simplifies computational model of radial systems and makes the investigation of their properties more tractable.

In four Maghreb countries a considerable improvement of the situation of the telecommunication operators has been noticed during the nineties. The evolution in these countries was very different depending on their economic policies, their effort of reorganization of their telecommunication sector and their technological change. Theses differences were analysed by comparing the operators' performances over a decade (1992–2001) by using the Malmquist DEA TFP index for measuring the total factors productivity change, decomposed into technical efficiency change and technological changes and by using the Promethee II method.

Today many different software tools for decision support exist; the same is true for data mining which can be seen as a particularly challenging sub-area of decision support. Choosing the most suitable tool for a particular industrial data mining application is becoming difficult, especially for industrial decision makers whose expertise is in a different field. The next contribution provides a conceptual analysis of crucial features of current data mining software tools, by establishing an abstract view on typical processes in data mining. Thus a common terminology is given which simplifies the comparison of tools. Based on this analysis, objective decisions for the application of decision supporting software tools in industrial practice can be made.

The next paper describes a framework for modeling and management of complex systems. There are various approaches for modeling of these systems. One of the approaches is events driven modeling and management of complex system. Such approach is needed in information systems that provide information in real-time. Most of the existing modeling approaches use only information about type of event and the time when an event occurs. However, in the databases we can store and then we can use much richer information about events. This information might be structured as well as unstructured. There are new challenges in algorithms development in case of description of event by various attributes.

The last paper is not directly related to telecommunications. However, the developed method might be applied also in telecommunications industry. Stochastic version of DEMATEL was developed as a revised DEMATEL (Decision Making Trial and Evaluation Laboratory) to extract structural models of a complex problems composed of many factors under uncertainty.

Janusz Granat
Guest Editor

Paper

Technology and change: the role of information technology in knowledge civilization

Andrzej P. Wierzbicki

Abstract— The paper presents a reflection on the role of technology and, in particular, information technology in the era of knowledge civilization. Diverse perceptions of this era, the concepts of three civilization eras versus three waves, of a cultural platform versus an episteme of a civilization era, of a big change at the end of industrial civilization era are outlined. The first principle of cultural anthropology and the concept of cultural imperialism are recalled. The contemporary philosophy of technology is shortly reviewed. An interpretation of *Die Technik und die Kehre* of M. Heidegger from a technological point of view is given. It is shown that we should distinguish technology proper from the system of its socio-economic applications, and that the relation of technology proper to hard science and to socio-economic applications of technology forms two positive feedback loops; the one of socio-economic applications might be more dangerous in cases of social infatuation with technological possibilities or other misapplications of technology. It is shown that the technology of knowledge civilization era will differ from that of industrial era in proposing boundless number of diversified technological possibilities; thus, the Heideggerian warning against social infatuation with technological possibilities must be not only repeated, but also modified and strengthened.

Keywords—knowledge civilization era, philosophy of technology, definition of technology, technology proper versus its system of social applications, relation between technology, hard sciences, soft social sciences and humanities.

1. Introduction

As long as fifty years ago, there was no doubt, see, e.g., [1] that humanity developed because of tool-making, thus technology is an intrinsic human trait; that many old civilizations collapsed because their political leaders (pharaohs, kings, head priests) used the tool-making and the technological abilities of their people for too ambitious goals; that technology is a way of mastering nature but nature often punishes those human civilizations which use their technological abilities too ambitiously. All this simple, basic truth has been, however, questioned during last fifty years, while social science and humanities started to look at technology as an autonomous, dehumanizing and enslaving force that in itself leads to an excessive use of its own. Despite these accusations, technology has brought about the information revolution that includes also the dematerialization of work: automation, computerization and robotization relieved humans from most of heavy work and created conditions for an actual realization of the equality of women. This prepared a new civilization era that can be called global knowledge civilization (or simply *knowledge civilization*, since it will last many decades yet before this type of civilization becomes truly global). This development solves many old problems and brings many hopes, but also brings new problems and many dangers.

Thus, it is necessary to reflect what will be the future role of technology in the starting era of knowledge civilization. Having almost fifty years experience in developing information technology and over twenty years in assessing its future developments and impacts, the author of this paper intended to write an article on such future technology assessment. However, the basic character of temporary civilization changes has induced the author to check also the philosophy of technology; and the state of contemporary philosophy of technology appeared to him both deeply disturbing and frightening. Disturbing, because the writers in this field seem not to be able even to arrive at a consensus how to define consistently what technology is; moreover, they propose definitions and interpretations of technology not acceptable to a technologist. Frightening, because we need a basic philosophic reflection on the future role of technology in knowledge civilization; but if philosophy is not even willing to listen to the opinion of technologists what they truly do, then it will not be able to understand this apparently distinct human culture. This can have disastrous results for the entire human civilization on global scale, because the historical too ambitious uses of technology by political leaders seem to be based on similar misunderstandings.

Therefore, we must first reflect what has happened during the last fifty years, when three different cultural spheres apparently separated themselves: of social sciences with humanities, versus hard sciences, versus technologists; how these cultures view each other; how does this influence the philosophy of technology; what is and what is not the definition of technology acceptable to its practitioners. First upon clearing this background we can discuss the future role of technology in knowledge civilization, its promises and chances versus its problems and dangers. We must start, however, with a review of some basic features of the starting era of knowledge civilization.

2. The era of knowledge civilization

2.1. Diverse perceptions of a new era

There is a voluminous literature on the subject of *information society* and current *information revolution*, see [2–9]. In this voluminous literature, there are diverse views, diagnoses, prognoses, judgments, prescriptions – and a universally accepted, slowly evolving core. There is an universal agreement that we are living in times of information revolution and this revolution leads to a new civilization era, in which knowledge plays even more important role than just information, thus the new epoch might be called knowledge civilization era. However, many other aspects of this development are uncertain.

Concerning the date marking the beginning of new era, we shall follow the method given by historians, in particular F. Braudel [10]. Braudel defined the preindustrial era of the beginnings of capitalism, of print and geographic discoveries, as starting in 1440 with the improvement of printing technology by Gutenberg, who promoted broad applications of printing press, and ending in 1760 with the improvement of steam engine technology by Watt, who made possible broad applications of steam machine; this started the next, industrial civilization era. Similarly, we can take the date 1980, related to the improvements of computer technology (personal computers) and network technology (broad applications of new protocols of computer networks), which made possible broad social use of information technology, as the beginning date of the era of information and knowledge civilization.

2.2. Three civilization eras versus three waves

In such a way, instead of speaking broadly about *three waves* of *agricultural, industrial, information civilization* such as discussed in [5], we concentrate more precisely on three recent (they are not the first, nor they will be the last) civilization eras that are marking the slow globalization of mankind civilization. These are the eras of:

- *preindustrial civilization*: print, banking and geographic discoveries;
- *industrial civilization*: steam, electricity and mobile transportation;
- *information and knowledge civilization*: networks and mobile communication, knowledge engineering.

For a more detailed discussion, see, e.g., [11].

2.3. The cultural platform and the episteme of a civilization era

It is important to note here, however, that each of these eras started basing on a definite cultural platform of new concepts and ideas formed even before the beginning of the era, see [6], which after some time was followed by the formation of an episteme characteristic for the era, see M. Foucault in [12]. While Foucault rightly stresses that the way of constructing knowledge in a given era is very specific and emerges some time after the beginning of the era (he dates the emergence of the *preindustrial episteme* at least a century after Gutenberg and the emergence of the *industrial episteme* at least half a century after Watt), he does not pay much attention to the origins of an *episteme*. But before Gutenberg we had the beginnings of Renaissance, before Watt we had Newton and French encyclopedists; the episteme of knowledge civilization is not formed yet, but the destruction of the *industrial episteme* and the construction of a new *conceptual platform* started with relativism of Einstein, indeterminism of Heisenberg, with the concept of feedback and that of deterministic chaos, of order emerging out of chaos, finally – with the emergence principle.

The last point deserves an explanation, because its significance is not universally perceived yet, particularly in philosophy. Mathematical modeling of dynamic nonlinear systems was highly developed already fifty years ago, with diverse applications but especially in control engineering, see, e.g., [11] for a more detailed discussion. But such modeling has lead to the concepts of deterministic chaos and of order emerging out of chaos, see, e.g., [13].

Thus, the study of mathematical models of nonlinear dynamic systems resulted in *a change of the reduction paradigm to an emergence paradigm, in a rational justification of the emergence principle: of new systemic properties emerging on new levels of complexity, independent of and irreducible to the properties and parts on lower levels*. Parallel, this emergence principle was justified empirically by biology in its concept of punctuated evolution, see, e.g., [14]; but the rational justification was important because it has shown the emptiness of diverse ideological attacks on the concept of evolution. This change of perception was additionally supported by a pragmatic justification given by technology, in particular telecommunications and information science. An example is the ISO/OSI (International Organization for Standardization/Open Systems Interconnection) model of seven layers of a computer network. This model stresses that the functions of such complex network not only cannot be explained by, but are also fully independent of the functions of its lowest *physical layer*, by the way electronic switching elements work, repeat and process signals. On each higher layer, new functions and properties of the network emerge. The functions of the highest *application layer*, are responsible for application software and are absolutely independent from the way the lowest layer works; they would be the same even if the switching in the lowest layer would be fully optical or even quantum mechanics driven.

The ISO/OSI model was used to unify the functions of various network protocols from TCP/IP (transmission control protocol/Internet Protocol) family (IP, TCP, UDP, etc.) that enabled the information revolution and brought digital information processing potentially to every home on our globe. The authors of the ISO/OSI model were not

necessarily aware of changing the reduction paradigm to emergence paradigm. They simply wanted to conquer the complexity of the modern telecommunication networks and needed to assume the emergence of new properties of the system on higher layers because otherwise they would be lost in details. They were probably also unaware of the fact that the theory of hierarchical systems, including the theory of systems with many layers of qualitatively different functions, irreducible to the functions of lower layers, was developed some time earlier by control system theorists, see, e.g., [15].

The *industrial episteme* believed in *reduction principle – the reduction of the behavior of a complex system to the behavior of its parts* – which is valid only if the level of complexity of the system is rather low.

With very complex systems today, mathematical modeling, technical and information sciences adhere rather to emergence principle – the emergence of new properties of a system with increased level of complexity, qualitatively different than and irreducible to the properties of its parts.

It should be noted that the emergence principle is the essence of complexity (essence in the Heideggerian sense which will be discussed later) and means much more than the *principle of synergy or holism* (that the *whole is more than the sum of its parts*) which was noted already by [16, 17] but without stressing the irreducibility of holistic properties, see also [11] for a more detailed discussion.

2.4. What happened at the end of industrial civilization era

The technology of industrial civilization era was developed to such a degree that, for the first time in the history of human civilizations, on one hand it promised the possibility of freeing people from hard work, on the other hand has shown also the possibility of a total destruction of life on Earth. Fast and inexpensive travel, mass media and mobile communication, robotics and automation, landing on the Moon on one hand were counterbalanced by the specter of atomic bombs and nuclear death. Additionally, the overambitious uses of technology by political leaders mentioned in the introduction were aggravated by the fact that entire societies or social systems have became blinded by their seemingly unlimited power over nature given to them by the industrial technology, what has led to the large overexploitation of natural resources and severe degradation of natural environment. This has occurred especially in the communist system, where the official ideology stressed the social power of transforming the nature; this is occurring even today in the capitalist system, where the official ideology stresses that free market should determine the use of technology (e.g., in the issue of climate changes) as if the historical evidence of nature punishing too ambitious uses of technological abilities counted for nothing. In face of such controversies, it is no wonder that the ideological and intellectual crisis at the end of industrial civilization era has been very deep indeed.

This crisis, by the way, was even deepened by the erosion and then the fall of communism. The industrial civilization era had its basic great conflict. No matter what our ideological position, it must be objectively admitted that the big conflict of industrial civilization concerned the property of the fundamental productive resources of this era – the industrial assets. As soon as the industrial civilization era ended, the conflict became obsolete, which is what ended the importance of communist ideology. In other words, the mentioned above trend of dematerialization of work made obsolete the importance of the proletariat, which took away communism's legitimacy. Even if many intellectuals were disillusioned with communism, most were involved ideologically in this basic great conflict and the end of its importance deepened the intellectual crisis.

In epistemology, the beginning of the end of the *industrial era episteme* was marked already in 1953 by the seminal paper of W. V. Quine [18] which has shown that the logical empiricism is logically inconsistent itself, that all human knowledge is constructed. However, Quine insisted that this constructed knowledge should be evolutionary useful and thus should have *limited objectivity*, should *touch reality at least by its edges*. For diverse reasons, but possibly mostly because of the controversies and the crisis mentioned above, social science and humanities went much further to maintain that all knowledge is *intersubjective* – results from a discourse, is constructed, negotiated, relativist. This general belief has many variations. One thesis took the form of radical biological constructivism – see, e.g., [19, 20]: if all of knowledge is constructed by the human mind as a result of biological evolution, then the concept of truth is not necessary. This radical constructivism was in a sense supported by radical relativism, starting with radical sociology, mainly by the strong program of the Edinburgh school, see, e.g. [21, 22], but also by *post-existentialism* and *postmodernism* of, e.g., [12, 23, 24]. Opposite was a further development of humanistic rationalism: H.-G. Gadamer [25] stressed the value of truth as an essence of human self-realisation. However, humanistic sociology soon took an anti-rationalist and anti-technological position, initiated by H. Marcuse [26] with his concept of the single-dimensional man enslaved by the autonomous, dehumanizing force of technology; this position was followed essentially by all social scientists, including, e.g., J. Habermas [27].

In all these disputes, the emergence principle was essentially unnoticed and disregarded, while clearly reductionist arguments were used to deconstruct the concepts of truth and objectivity, trying to explain or even to deny the importance of such more complex concepts by the analysis of more primitive ones, such as money and power. But seen from the perspective of the emergence principle, truth and objectivity are concepts of a different layer of systemic complexity; they might be unattainable, but serve very clear purposes as ideals. Without trying to pursue objectivity, technology could not be successful, e.g., when trying to

increase the reliability of transportation vehicles. Thus, these reductionist deconstruction attempts were in a sense *signs* of the end of a civilization era, when a general uncertainty of values results in a universal, playful anarchy.

The reader might infer that the above judgment is just an opinion of a technologist – but this already would indicate that a deep cultural rift has emerged between social sciences and technology towards the end of industrial civilization era. But we can quote here also the opinion of H. Kozakiewicz – a known Polish philosopher of sociology – who diagnoses [28] a crisis in sociology. She states that sociology is often called "the most general of social sciences". But she asks: *in what sense sociology is a science*? It is a science by tradition, since it started from positivistic beliefs of Comte that society can be described using methods similar to hard science. However, sociology itself revised these beliefs; the formulation that somebody uses "scientific methodology" means a strongly negative epithet for a sociologist today. A branch of sociology, sociology of science, including known trends of the second half of 20th century – the strong program of Edinburgh school [21, 22] with its stress of *interests*, the micro-constructivism (see, e.g., [29]) with its self-description of knowledge development, translation sociology (see, e.g., [30]) – all deny the possibility of objective epistemological explanations of science, and treat science only as a social discourse. What happens if we apply this approach to sociology itself? A paradox: *sociology is a social discourse about itself*.

3. The three separate cultures of technology, hard science and social science with humanities

3.1. Why separate cultural spheres?

We have indicated above that the cultural sphere of social sciences with humanities is different from the cultural sphere of technology, because they adhere to different values, have different *episteme*, use different concepts and language. But the same obviously concerns also social sciences and humanities versus basic, hard sciences. Less obvious is the fact that the same distinction concerns hard sciences versus technology. Some (social science) writers speak about *technoscience*; however, it is a great error, one of many signs of not fully understanding technology – while science and technology are obviously related, they differ essentially in their values and *episteme*. We shall discuss this difference later in more detail, but indicate its essence already here: while science ideals are true theories, technology ideals relate to the art of solving practical problems, even if the corresponding theory does not exist yet.

The anthropology of 20th century created a very useful principle of dealing with separate cultures: *you should never judge a foreign culture without trying to understand it well – otherwise you are just a cultural imperialist*. But then, what does postmodern sociology of science? By telling a hard scientist that he does not value truth, only power and money, it behaves like a communist activist coming to a priest and telling him that he does not value God, only power and money. By telling a technologist that his products enslave people, it behaves like telling an artist that his religious paintings enslave people – *essentially*, it behaves like a cultural imperialist. Thus, the *episteme* of hard sciences should be discussed, internally criticized and further developed by hard sciences themselves; the same concerns technology. The same concerns social sciences and humanities: until they overcome their own internal crisis, they should not expect that their opinions about hard science and about technology will be seriously attended to.

3.2. The dominant episteme of a cultural sphere and its limitations

If we adhered too closely to the principle described above, these three cultural spheres would become completely separated, which is neither possible nor desirable. Therefore, intercultural understanding should be promoted; with this aim, we present here notes about the dominant episteme of each culture. In order to foster intercultural understanding, but also to indicate the limitations of each episteme, we shall also use metaphors when describing the differences between these cultural spheres.

Even if a hard scientist knows that all knowledge is constructed and there are no absolute truth and objectivity, he believes that scientific theories are *laws of nature discovered by humans* rather than *models of knowledge created by humans*. He values truth and objectivity as ultimate ideals; metaphorically, *hard scientist resembles a priest*. However, a modern hard scientist does not value tradition very much; he is willing to abandon old theories, if new theories are closer to the ideals of truth and objectivity.

A technologist is much more relativist in his episteme, he readily agrees that scientific theories are models of knowledge – because he uses such theories in solving practical problems, and if he has several competing theories, he simply compares their usefulness. If he does not have scientific theories to rely upon, he will not agree to wait until such theories are created[1], but will try to solve the problem anyway using his own creativity. Metaphorically, a *technologist resembles an artist*. He also values tradition like an artist does: an old car is beautiful and, if well cared about, can become a classic.

A postmodern social scientist or a soft scientist (e.g., historian[2]) believes that all knowledge is *intersubjective*, results from a social discourse, is constructed, negotiated, relativist. There are traps in such episteme, it would not stand up against a serious internal, Kantian-type critique, as indicated by the examples given by Kozakiewicz; but this is

[1] This corresponds also to personal experiences of the author of this text, see, e.g., [11].

[2] Again from personal experience of his family and friends, the author knows well that not all historians are postmodernist and relativist.

a sign of an internal crisis that must be overcome by social and soft sciences themselves. Metaphorically, a *postmodern social or soft scientist resembles a journalist*: anything goes as long it is interesting.

3.3. Is a serious philosophy of technology possible without consulting technologists?

If technology corresponds today to a different cultural sphere, we must give a strongly negative answer to such a question. This not only results from the principles of cultural anthropology; it is simply a common sense. It is just too dangerous not to understand technology, if it gives us today not only the power to transform totally our lives, but also to destroy life on Earth – not only by an inappropriate use of nuclear energy, but also, e.g., by an inappropriate use of genetic engineering, or even robotic technology. Postmodern social and soft sciences will not able to understand technology until they overcome their internal crisis, achieving a synthesis of intersubjectivity and objectivity. Hard sciences will continue to see technology as a mere application of their theories. All this creates an extremely dangerous situation; the perception of this terrifying danger only deepens when we study the contemporary philosophy of technology.

4. The views of philosophy of technology

4.1. The general impression of a technologist

The general impression from reading contemporary publications on philosophy of technology is that they do not understand technology, even do not actually investigate technology – they present only slightly modified views on philosophy of science, treating technology as a mere application of science – and often represent anti-technological attitudes, by propagating the mistaken opinion about technology as an autonomous, dehumanizing, enslaving force. For example, an excellent – at least, in its breadth – review of old and current writings on philosophy of technology [31] includes 55 papers, of which 14 at the beginning the volume are on philosophy of science and the first of papers starting the actual discussion on philosophy of technology [32] is based on the assumption that technology is just an application of the theories of hard science. The most basic question of ethics of technology is addressed by a paper [33] that counterposes technology and ethics: technology is seen as not only an autonomous and dehumanizing force, but also an unethical force. This type of anti-technological flavor can be seen in most of remaining papers; of the final seven papers, only one [34] is free of such flavor, but it is immediately followed by a paper criticizing the previous one and presenting technology as the opiate of intellectuals [35]. And in all 55 papers, *there is no paper written by a technologist*.

4.2. A few acceptable views

Nevertheless, few papers present views that are acceptable to technologists; notably, they are the ones most discussed or criticized by other papers.

The most close to the perception of a technologist what he truly does is the fundamental analysis of M. Heidegger in *Die Technik und die Kehre* [36], repeated in [31] in somewhat unfortunate translation *The Question Concerning Technology*; thus, we use somewhat more adequate translation as a part of the title of this paper. *The Question Concerning Technology* is commented upon in [31] by a number of other papers, all trying to show either that Heidegger perceived technology as an autonomous, dangerous force or that he was not critical enough of technology; neither of this papers interprets Heidegger in a way that a technologist would. The problem of the difficulty and diversity of interpretations relates to the fact that Heidegger was a poet at heart, playing with words to achieve empathy and *essential truth* as opposed to a *correct understanding.* Possibly because of that, he empathically understood the artistic nature of technology; we comment on this in more detail later.

There are few other papers in [31] that indicate an understanding of the (Heideggerian) *essence* of technology; an important one by E. G. Mesthene [34] is devoted to the social impact of technological change. We quote here some of his sentences important for further analysis.

At its best, then, technology is nothing if not liberating. Yet many fear it increasingly as enslaving, degrading, and destructive of man's most cherished values. It is important to note that this is so, and to try to understand why.

Unfortunately, further analysis given by Mesthene is not conclusive, because he does not make a clear enough distinction between technology proper and the socio-economic system exploiting technology, which we shall also discuss in more detail later.

There are also other writings on philosophy of technology – curiously, not represented in [31] – that show a (Heideggerian) correct understanding of technology; they are dealing mostly with the question whether the concept of a Kuhnian revolution in science is applicable also to technology[3], see [37], and define technology as a practical problem-solving activity, which is certainly correct if still not fully essential.

4.3. The dangers of mistaken diagnosis

There is, however, a grave danger in the mistaken diagnosis that technology is an autonomous, enslaving and degrading force: *a wrong diagnosis cannot help to cure the illness.* Technologists perceive the diagnosis as a sign of misunderstanding, thus disregard it; social scientists have found

[3]It is interesting that these writings question the applicability of the Kuhnian concepts to technology, which is consistent with the perception of the author of this text that technology is closer to the Popperian concept of *falsification* than to the Kuhnian concept of *paradigm*.

a scapegoat to put the blame on, thus do not reflect on their own responsibility. But, as we shall show later, both sides should feel responsible.

We should note that technologists perceive the misunderstanding by social sciences also in other cases. In systems research, there is the example of debate between soft systems thinking and hard systems thinking, in particular, the issue of soft systems methodology (SSM) [38]. SSM stresses listing diverse perspectives, including so-called *Weltanschauungen*, *problem owners*, and following open debate representing these diverse perspectives. Actually, when seen from a different perspective, that of hard mathematical model building, SSM (if limited to its systemic core) must be also evaluated as an excellent approach, consistent with the lessons derived from the art of engineering system modeling even much earlier. More doubts arise when we consider not the systemic core, but the paradigmatic motivation of SSM. The SSM is presented by P. Checkland in [38] as a general method, applicable in interdisciplinary situations; but a sign of misunderstanding is the opinion that *soft systems thinking* is broader and includes *hard system thinking* as defined there. But then, should not SSM be also applicable to itself? It includes two *Weltanschauungen: hard and soft*; thus the problem owners of hard Weltanschauung should have the right to define their own perspective. However, hard systems practitioners never agreed with the definition of hard systems thinking given by Checkland. He defines hard systems thinking as the belief in the statement of [39] that *all problems ultimately reduce to the evaluation of the efficiency of alternative means for a designated set of objectives*. On the other hand, hard system technological practitioners say *no, they are hard because they use hard mathematical modelling and computations, but for diverse aims, including technology creation, when they often do not know what objectives they will achieve.* As a result of such differences in episteme, hard and soft systems researchers simply do not understand each other.

5. What technology is and what it is not

5.1. The definition of technology by Heidegger as understood by a technologist

M. Heidegger came closest to the *essence* of technology by stressing several essential facts:

- technology is obviously means of transforming nature and also obviously a human activity;
- *technology is an art of solving practical problems*, not an application of abstract theory;
- in its *essence*, the technological act of creation is an act of *revealing the truth* out of many possibilities offered by nature.

We can thus interpret Heidegger that *humans cannot escape creating technology*, similarly as a child cannot escape playing with blocks. It is thus our basic, even defining characteristics, an intrinsic human trait.

No matter how we define humanity, we would stop to be human if we stopped technology creation.

5.2. The warnings of Heidegger as understood by a technologist

M. Heidegger also perceived that technology in industrial civilization changed qualitatively when compared to technology of older times by offering humans almost complete control over nature. However, such control, when exercised without reflection and restraint, might threaten the very essence of human being. This warning was correct, we learned later at much cost that our control over nature is never complete and that unrestrained control over nature is very dangerous for us.

But Heidegger never condemned technology in itself as an autonomous, alienating and enslaving force; this condemnation came later, started in social sciences by Marcuse [26]. Heidegger writes (about the results of perception of a complete control over nature) explicitly: *Meanwhile ... man exalts himself and postures as the lord of the earth*. Thus, though Heidegger did not make a precise distinction here, *his warning concerns not technology proper, but the social use of technology* – and, assuming that Marcuse has read and understood Heidegger, his condemnation of technology must be read as shifting the blame.

Nevertheless, a technologist must read a lesson for himself out of these controversies: *he must be careful what technologies he puts at social disposal, because the socio-economic system might use them without restraints and the blame will be put later not on the system and social scientists apparently responsible for such systems, only on technology*.

5.3. The sovereign though not autonomous position of technology

We start with a definition of technology acceptable to technologists, distinguishing *technology proper* from *the system of socio-economic applications of technology*.

Technology proper is a basic human trait that concentrates on the creation of artifacts needed for humanity in dealing with nature. It presupposes some human intervention in nature, but can also serve the goal of limiting such intervention to the necessary scale. It is essentially a truth revealing, creative activity, thus it is similar to arts. It is also, for the most part, a problem solving activity, concentrating on solving practical problems.

Thus, it uses the results of basic sciences, if they are available; if they are not, technology proposes its own solutions, often promoting this way quite new concepts assimilated later by basic or social sciences. It is not an autonomous force, because it depends on all other human activities and influences them in return. It is, how-

ever, sovereign, in a similar sense as arts are sovereign human activities. Autonomous forces can be found in the socio-economic system of applications of technology proper.

The second part of this definition requires some discussion which will be given in the next sections.

5.4. The reverse relation of science and technology

It happens actually very often that technological solutions precede the developments of science.

The first obvious example is the technological development of a wheel. The mathematical concepts of a circle and that of actual infinity stem from this technological development: a wheelwright constructs a wheel as a polygonal structure, slowly increasing the number of sides of the polygon by cutting consecutive angles, until an approximate circle and an (approximately) smooth wheel is achieved[4]. Some philosophers of mathematics [40, 41] show that most of ancient mathematics before Greek times was technology-oriented and used not the concept of a formal proof, only that of pragmatic demonstration.

Another example, well known in the philosophy of science [37] is the impact of the technological development of a telescope on astronomy and Galileo's findings.

But there are also modern examples. The improvement of a steam engine by Watt was a mechanical control engineering feedback system for stabilizing the rotational speed of the engine (before Watt, the rotational speed was unstable and steam engines tended to explode). This not only started the industrial civilization era, it also motivated several lines of scientific enquiry. One was that of stability of dynamic systems, started by such great minds as W. Kelvin-Thomson and J. C. Maxwell, see [42, 43], leading eventually to diverse aspects of nonlinear systems dynamics and to the theory of deterministic chaos, thus finally to the emergence principle, see [11, 13, 44]. Another was the extremely important concept of feedback, upon which we comment later, attributed incorrectly by social scientists – see, e.g., [45] – to N. Wiener [46]; actually developed much earlier in telecommunications by H. Nyquist [47], V. Bush (the creator of the first analog computer, earlier than digital computers) [48], and many others. Equally important was actually the concept of a system[5], attributed by social science first to Comte, then – when Comtian positivism came under critique – to Wiener and Bertallanfy [16]; but practical systems engineering developed in technology much earlier, since Watt, and has lead eventually to the most developed technological systems today – to computer networks.

[4]A turning lathe making the wheel really smooth was invented much later.

[5]The concept of a system is extremely important too, but possibly not more important than the concept of feedback; without feedback we would not have robotics, without robotics we could not transit to knowledge-based economy where human labor as a basic productive input is substituted by human knowledge.

Less known is the example of a quasi-random number generator in digital computers. Developed already in 1950s, preceding the development of the theory of deterministic chaos starting in 1960s, the principle of such a generator exemplifies in fact the basic principles of a strange attractor: take a dynamic system with strong nonlinearity and include in it a sufficiently strong negative feedback to bring it close to instability. In the quasi-random generator, we use recourse, repetition instead of dynamics and feedback and add a strong nonlinearity. The simplest example is: take a digital number, square it, cut a quarter of its highest bits and a quarter of its lowest bits, and repeat the procedure. The resulting sequence of digital numbers is in fact periodic, but with a very long period and behaving meanwhile as if it were random. Thus, technological "applications" of deterministic chaos theory appeared earlier than the theory.

There are many other such examples in the recent history of information science and technology. The development of *data warehousing* in early 1990s was caused by economic and technological necessities, independent from existing theories; but it in a sense surprised information science specialists that concentrated before on relational data bases, and is leading today to new advancements in information science, etc.

5.5. Two positive feedback loops

Thus, how do hard, basic science and technology depend on each other? As in many questions of human development, they influence each other through the intellectual heritage of humanity, the *third world* of K. Popper, see [11, 49]. But this influence forms a *positive feedback loop*, see Fig. 1; technological development stimulates basic science, scientific theories are applied technologically.

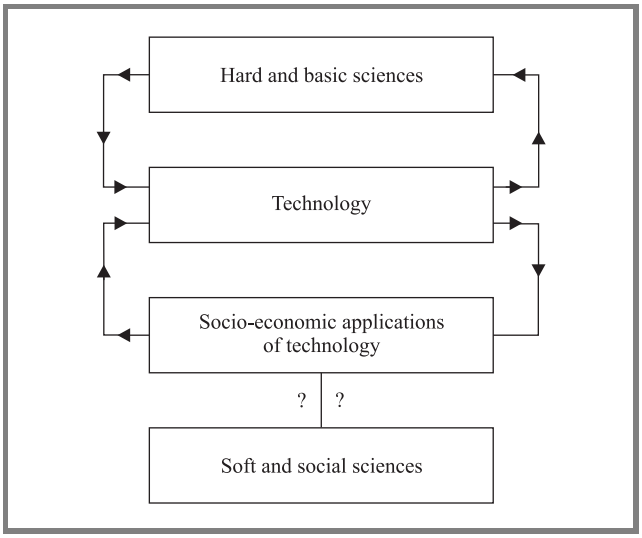

Fig. 1. Two positive feedback loops.

We must recall that *feedback* – the circular impact of the time-stream of results of an action on its time-stream of causes – was used by Watt in a *negative feedback loop*.

Feedback can be of two types: *positive feedback* when the results circularly support their causes, which results in a fast development, like a growing avalanche, and *negative feedback* when the results circularly counteract their causes, which results in an actually positive effect of stabilization (for example, the stabilization of human body temperature is based on negative feedback). The concept of feedback essentially changed our understanding of the cause and effect relationship, resolving paradoxes of circular arguments in logic, though it must be understood that such paradoxes can be resolved only by dynamic, not static reasoning and models.

But the positive feedback loop between technology and science works relatively slowly: technological stimulations are analyzed by science with much delay, technology also does not reply instantly to new scientific theories.

The second positive feedback loop is between technology and the systems of its socio-economic applications. The distinction between technology proper and its socio-economic applications is not stressed sufficiently by social sciences, in particular by postmodern philosophy of technology, though it should be obvious for at least two reasons. The first is that technologists often work on a technological problem quite long (e.g., almost fifty years in the case of digital television) before their results are broadly socially applied. The second is simple: *technologists do not make much money, technology brokers do*, similarly as art brokers make more money than artists. By *technology brokers* we understand here entrepreneurs, managers, bankers, etc., all our socio-economic systems turn around applications of technology. If a technological product or service, such as mobile telephony, produces much revenue, then more money is available for its further technological development; this leads to truly avalanche-like processes of social adoption of technological hits.

But these processes have also strange dynamic properties, socio-economic acceptance of novelties is slow, there is usually a large delay between purely technological possibility and the start of an avalanche of its broad socio-economic applications (not only in the case of digital television; this delay time amounted also to almost 50 years in the case of cellular telephony). This delay has many causes; one is the necessity to develop such technological versions that are inexpensive enough for an average customer; another is an initial social distrust turning into a blind social fascination once a technological hit becomes fashionable. For this reason, once it starts to work, the second positive feedback loop is much stronger and faster than the first one. *This blind social fascination is actually the autonomous force incorrectly attributed by social philosophy to technology proper*, it is precisely the source of the Heideggerian danger that *man exalts himself and postures as the lord of the earth*. For example: how many people are aware that mobile telephony makes it very difficult to practice radio-astronomy from Earth surface, that it is the reason of moving radio-telescopes into cosmic space? And this is a relatively modest adverse effect; what if an avalanche-like adoption of a technological hit would result in truly disastrous effects? After all, a nuclear power station is also based on avalanche-like processes that must be carefully controlled – by negative feedback systems of control engineering – to be safe; but if such systems fail (or are tampered with for fun by irresponsible people, like in the Chernobyl case), the disaster can have no limits.

The answer to the question of Mesthene: why it is so that many people perceive technology as an alienating force, enslaving, degrading, and destructive of man's most cherished values, might be the following: the essential reason of it is the intuitive perception of such danger of a social infatuation with technology leading to avalanche-like process of social adoption of technological hits with diverse resulting threats and possible catastrophic results.

Being intuitive, the perception needs not to be rationally correct and the diagnosis can be wrong, see the discussion of a rational theory of intuition in [11]; in order to obtain correct answers and useful diagnosis, we must analyze it critically. Thus, we encounter crucial questions here:

1. What mechanisms limit and stabilize the avalanche-like processes of socio-economic adoption of technological hits?
2. Who is responsible for overseeing that these mechanisms work effectively?

The one mechanism that at least safely prevents any economic excesses is the market economy; people tried to replace market by human intervention in the communist system without success. However, it is only a robust mechanism, it does not solve many problems and creates some new ones. For example, because knowledge-based economy sharply decreases marginal production costs, prices on high technology markets have today no relation to (actually, are over hundred times higher than) marginal production costs; therefore, an ideal, free market simply does not work in knowledge-based economy, and a monopolistic or oligopolistic behavior is typical, see, e.g., [50]. Who will watch over such global oligopolistic markets?

As to the responsibility, obviously it should be borne first by the *technology brokers*. However, to be effective on the market, they must be motivated by profit, let us only hope that the motivation will be tempered by ethics. Ethics results from education; *who educates technology brokers*? Not technologists proper, but social and soft scientists. They should not only educate well technology brokers ethically, but also help them to understand their future jobs by analyzing in detail the mechanisms of social demand for technology, of infatuation with technological hits, together with their dangers.

Thus, the ultimate responsibility for socio-economic applications of technology, for overseeing the effective limitations of blind social fascination with technological hits lies at soft and social sciences.

Unfortunately, they do not perform well in this respect, prefer to put the blame on technology proper, undistinguished

from the system of its socio-economic applications and deplored by them as a technocratic tool of enslavement by promoting the functionalist view of the world. This is indicated by the question marks in Fig. 1: while the role of hard, basic sciences and technology proper versus its socio-economic applications is clear, soft and social sciences do not seem to fullfil nor even understand their role.

This does not mean that technology proper is not co-responsible and should not at least try to work together with social scientists on limiting such dangers. However, a technologist is usually very conscious of ethical dangers, carefully considers possible future impacts of technology developed by him – and even if it is not the case, he must be careful because he knows that the blame for any possible misfortunes and misapplications will be put on him. On the other hand, we cannot expect that the responsibility of technologists will prevent all misapplications of technology. One reason is that human creativity of misapplications is boundless (*against stupidity, the gods themselves contend in vain*). Another is more serious: the very nature of knowledge-based economy will give human societies almost boundless possibilities to choose from diverse technological options.

6. What will be the technology of the knowledge era (postmodern technology?)

6.1. The character of technology in the knowledge era

We must ask today a renewed version of Heidegger's question about *die Kehre* (the change of the character of technology, in his case in the industrial era as compared to earlier times). The question is: *in what qualitative aspect will the technology of knowledge civilization era differ from the technology of industrial civilization era*? A tentative answer proposed as the main conclusion of the paper is:

The technology of knowledge civilization era will differ in complexity, by proposing an unlimited number of diversified technological possibilities, oriented toward not only products, but also services, including such services as creativity support, and only a small part of these possibilities will be actually accepted for economic and social use.

We could call it *postmodern technology*, but the change will be deeper than the intellectual fad of postmodernism indicating the end of industrial civilization. We shall illustrate this answer by some examples.

6.2. Some examples of technology of the knowledge era

One of the most important possibilities brought by the technology of the knowledge era will *be the change of the character of recording of the intellectual heritage of humanity.* In the last two civilization eras – the preindustrial and the industrial – the dominant medium of recording the human heritage were printed books. Information technology will make soon possible full multimedia recording of human heritage; in other words, instead of a book we will have an electronic record including film, music, interactive exercises and virtual laboratories. Imagine today the possibility of listening to a lecture of Kant or Einstein recorded in such a way; but the change goes beyond such possibilities. This change will have impacts exceeding the impacts of Gutenberg printing technology; the nature of our civilization will change, multimedia recording will stronger support the intergenerational transmission of intuitive knowledge and of humanity intuitive heritage, will enable more effective distant and electronic education, see [11] for more detailed discussions.

Another possibility concerns *ambient intelligence*, called also AmI in Europe, either *ubiquitous* (omnipresent) *computing* or *wireless sensor network* in the United States, *intelligent home* or *building* or *yaoyorozu*[6] in Japan. There is no doubt that the number of possible ways of helping people by using computer intelligence dispersed in sensors and processors in our ambient habitat – at homes, in offices, in shops, in vehicles, etc. – is endless and that people will buy such technology once it is truly ubiquitous and inexpensive. However, there are also grave social threats related to this technology – not immanent in the technology, but in the way people might use it. Ambient intelligence requires electronic identification of a person, say, when entering his room. What would prevent overzealous police from using this technology as a way of realizing the concept of a *Big Brother*? Ambient intelligence means also ubiquitous robotization; what would constrain too inventive criminals from using robotic squads to break into banks or as invincible bodyguards?

We will mention here only one more of the endless possibilities of future technology of the knowledge civilization era. *Computerized decision support*, developed towards the end of industrial civilization, can be developed further into *computerized creativity support*, helping in the creation of knowledge and technology. For this purpose, we must understand better knowledge creation processes – not on grand historical scale, such as in the theories of T. Kuhn [51] and many philosophers following his example, but on a micro scale, for today and tomorrow. There are many such micro-theories of knowledge creation emerging in the last decade of the 20th century and in the first decade of the 21st; the book *Creative Space* [11] was motivated precisely by the need of integrating such theories.

6.3. New warnings: what we must be careful about

In all these possibilities, complexity and diversity, there is also a general danger and we must thus also repeat a renewed version of Heidegger's warning. As already perceived by Heidegger, the danger lies not in technology proper, but in us, humans fascinated by the possibilities

[6]Eight million Shinto gods, implying omnipresence.

of technology and not fully understanding the threats of such fascination.

In particular, the seemingly unbounded technological possibilities might suggest to people – particularly to *technology brokers* – that human intellectual heritage is rich and boundless enough to privatize it without restraint. Already today we observe many attempts of knowledge privatization. However, similarly as the unbounded privatization of natural resources in the industrial civilization era has led to grave pollution of natural environment, unbounded privatization of intellectual heritage will lead to pollution of this heritage – what we already observe, e.g., on drug markets. Thus, the modified Heideggerian warning is:

In the industrial civilization era, people became blinded by their seemingly unlimited power over nature given to them by the industrial technology, what has led to the large overexploitation of natural resources and frequent degradation of natural environment. We must take care in the knowledge civilization era not to become blinded by the seemingly unlimited possibilities of products and services offered by technology, in particular – we must take care to preserve our intellectual environment, the intellectual heritage of humanity.

7. Conclusions

There is no doubt that *technology contributed essentially to the change of civilization eras, from the industrial to information and knowledge civilization* observed now. The change has a social character, but *resulted from technology, from computer networks making possible the wide social use of information technology*. Technology brought also the dematerialization of work: automation, computerization and robotization have relieved humans from most of heavy work and created realistic conditions for the equality of women.

This was desired by many social thinkers, but, ironically, they usually – starting with [26] – condemned technology as an autonomous, alienating, de-humanizing force, as a technocratic tool of enslavement or functionalist view of the world. Similarly, technological objectivism was condemned as an outdated form of positivistic thinking; this paradigmatic attitude was strongest in postmodernist and constructivist approaches, but it has been paradigmatically upheld by sociology in general. This condemnation is still the prevailing reason for the lack of understanding of technology by sociology.

Technology, on the other hand, is motivated by the joy of creation (as observed by Heidegger, the old Greek word *techne* meant creative arts and crafts). To be successful in such creation, technology requires *informed objectivity*. Technologists understand that there is no absolute knowledge and truth, nor absolute measurement precision – but they must try to be as objective as possible, must not overlook inconvenient or unpopular information, since such neglect can result in a technical failure of systems they construct.

Technological *informed objectivism* is not the positivistic belief that ultimate truth exists based on empirical facts, since many technologists admit that we create knowledge and cannot attain absolute truth, but it is the conviction that objectivity and closeness to empirical facts are useful goals that have always helped in the successful construction of technological artifacts, even if these goals are ultimately unattainable. Social science seems not to be able to understand this distinct culture of technologists and condemns it without understanding, which is equivalent to cultural imperialism.

Even more pronounced is the misunderstanding of technology in postmodern social philosophy. Philosophy could not come to a synthesis of opinions about the role of technology, even if a very deep analysis of the essence of technology was given by Heidegger. However, we need an acceptable definition of technology at the beginning of knowledge civilization era and should agree that such *definition of technology for a general reflection must come from problem owners*, i.e., *technologists*, in particular from technologically oriented systems science.

Such a definition is proposed in this paper; it stresses that technology is a basic human trait that concentrates on the creation of artifacts needed for humanity in dealing with nature. We cannot stop being technologically creative without stopping being human. Technology presupposes some human intervention in nature, but can also serve the goal of limiting such intervention to the necessary scale. As suggested by Heidegger, *technology is, in its essence, a truth revealing, creative activity, thus it is similar to arts*. It is also, for the most part, *a problem solving activity*, concentrating on solving practical problems – although recently, like basic science, it is involved also in *searching for new perspectives*.

The relation of *technology and basic science forms a positive feedback loop*: technology poses new problems and concepts for basic science, basic science produces results that might be later applied in technology – but *technology is sovereign in this loop*, proceeds to find solutions even without having the input of basic sciences. In this sense, the technological development of the wheel motivated the development of mathematics together with the concept of actual infinity, which in turn helped in further development of technology. There are many other examples of such reverse relationship between hard science and technology.

Even more important is the *second positive feedback loop between technology proper and the system of its socio-economic applications*. These applications are managed by *technology brokers*, i.e., entrepreneurs, managers, bankers, etc., all our socio-economic systems turn around applications of technology. This second feedback loop brings about most social and economic results of technology, but at the same time it may create grave dangers. This is because processes of socio-economic adoption of technological novelties in this feedback loop are avalanche-like; such processes are known, e.g., in nuclear reactors, where they must be controlled and stabilized by additional neg-

ative feedbacks. If this additional stabilization does not work properly, disasters might occur. An intuitive perception of the threat of such disasters is the essential reason for condemnation of technology by social sciences and humanities. But this intuitive perception does not give a correct diagnosis.

In socio-economic adoption of technology, the stabilization of avalanche-like processes is achieved by market mechanism, but this mechanism on high technology markets does not function ideally, has a tendency to promote oligopolies and monopolies. Moreover, market obviously does not resolve ethical issues of technology adoption. Since technology brokers are educated mostly by soft and social sciences, the ultimate responsibility for socio-economic applications of technology, for overseeing the effective limitations of blind social fascination with technology lies also at soft and social sciences.

We are repeating in this paper, in a sense and in new conditions, the analysis presented by Heidegger in *Die Technik und die Kehre*, coming to the main conclusion that the technology of knowledge civilization era will differ from that of industrial era in complexity, by proposing an unlimited number of diversified technological possibilities, oriented toward not only products, but also services, including such services as creativity support.

We also are repeating and strengthening the Heideggerian warning about human fascination with technological possibilities: *we must take care in the knowledge civilization era not to become blinded by the seemingly unlimited possibilities of products and services offered by technology*, in particular – we must take care to preserve our intellectual environment, the intellectual heritage of humanity.

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Paper

Designing WDM networks by a variable neighborhood search

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Abstract— With the ever-rising data volume that is demanded by the market, network planning in order to minimize the necessary investment while meeting the demands is constantly an important task for the network providers. Synchronous digital hierarchy (SDH) and wavelength division multiplex (WDM) form the core of many current backbone networks. In order to solve the provisioning and routing problem in such WDM networks, we develop a variable neighborhood search (VNS) metaheuristic. VNS is a metaheuristic that combines series of random and improving local searches based on systematically changed neighborhoods. An integer flow formulation is modeled in AMPL and solved by CPLEX in order to obtain optimal solutions as a reference for the heuristic.

Keywords— network design, WDM, SDH, variable neighborhood search.

1. Introduction

With the ever-rising data volume that is demanded by the market, network planning in order to minimize the necessary investment while meeting customer demands is constantly an important task for the network providers. Synchronous digital hierarchy (SDH) and wavelength division multiplex (WDM) form the core of many current backbone networks. When we speak of SDH, this also applies to its American counterpart synchronous optical network (SONET). Though there are some differences, this does not affect network planning and optimization in general. Many of these networks, especially in Europe, have a general mesh topology. Thus, we do not consider special ring topologies but an arbitrary mesh of fiber lines (links) that connects the locations of the network provider (nodes) where the actual traffic demands arise. WDM systems are only used point-to-point, as in most current commercial networks. Therefore, the routing and wavelength assignment problem (RWA) does not arise. Common line-speeds for SDH/WDM networks range from 622 Mbit/s up to 40 Gbit/s per channel. Thus, with state-of-the-art multiplexers that provide 80 or even 160 channels, WDM is currently the fastest commercially available transmission technology for long-range networks. A good overview of the SDH and WDM technology can be found in [6].

While SDH requires a dedicated pair of fibers for each transmission, WDM multiplexes several optical signals on a single pair of fibers. The costs for several discrete fibers for SDH compared to the multiplexer costs and only a single fiber pair for WDM make WDM suitable for longer distances and high bandwidth demands while discrete SDH lines are cost effective for short lines with limited bandwidth demand. However, the overall optimization problem includes additional equipment like cross-connects, port-cards, amplifiers and regenerators and is much more versatile. Apart from the equipment, also building costs for the installation of new fiber lines might be included. The resulting optimization problem is to find a cost-minimal combination of the equipment and the routing for a given static demand matrix (i.e., traffic forecast). Static demand is a valid approximation for such kind of networks because most of the demands are leased lines over a certain period. Even if they carry originally bursty Internet traffic it is so highly aggregated that an IP provider can lease a fixed capacity from the operator of the long-haul network. The general topology of the network is bounded by the available links, but of course not all of these links have to be used.

In this paper, we present a variable neighborhood search (VNS) metaheuristic and an integer programming formulation as a reference for the described network-planning problem. Section 2 explains the general problem and the integer model, Section 3 contains the detailed description of the VNS. Computational results for various problem instances are given in Section 4; Section 5 summarizes the results and provides an outlook on future research.

2. Problem description and integer model

The optimization problem at hand deals with a set of demands to be routed through an optical network. Associated with each demand is an origin node, a destination node and a size expressed in 2.5 Gbit/s units. Each demand can be routed either entirely on one or more discrete fiber pairs or over one or more channels of a WDM system. The demands can be switched from one system to another at the intermediate locations through digital cross-connects (DXC). Optical fibers joining pairs of nodes are used to carry the demands through the network. These connections from one node to another are called links or edges. The costs of an edge depend on the length, the required bandwidth and the transmission technology. SDH requires a pair of fibers and maybe additional amplifiers or regenerators for long ranges. WDM links are basically composed of a pair of multiplexer terminals, the fiber pair and possibly also amplifiers. Depending on the number of channels that are actually used, transponder pairs are needed. The number of transponders

can be any number between one and the maximum capacity of the multiplexer. In real WDM systems, the transponders are sometimes added in blocks, e.g., ten at a time, but this detail has been omitted in the model. It is important that the costs of the fiber and of the amplification do not depend on the number of channels that are actually used, they are always per pair of terminals; this is a huge advantage of WDM for long-haul transmissions. An SDH line and a single channel of a WDM system each alike occupy one port in the DXC and thus each need one port-card at both ends. The goal of the network planner is to minimize the total cost of the additional fibers and the SDH/WDM equipment.

The basic model is derived from [7]. It considers demands of a specific granularity, e.g., 2.5 Gbit/s, and the entire demand between a pair of nodes has to be routed on the same path. All cross-connects have the same number of ports; if a higher capacity is required, several cross-connects can be stacked. The given infrastructure is composed of a set of nodes N that represents the switching locations of the provider. A set of undirected edges E connecting these nodes represents the fiber links. A set E' containing all arcs of a complete graph with node set N is derived from E. Furthermore, a set of demands D is given which contains the number of units for each single demand d_{st} from node s to node t.

The cost input consists of the following data:

C_e^{FS}	costs of an SDH line on edge e (fiber, amplifiers, regenerators)
C_e^{FW}	costs of a WDM line on edge e (fiber, amplifiers)
C^W	costs of the WDM multiplexer terminals on edge e
C^O	cost of a basic DXC system
C^C	cost of a WDM channel (a pair of transponders)
C^P	cost of a DXC port (port-card)

The capacity of the systems is defined as follows:

M^W	capacity of a WDM system (number of wavelengths)
M^O	capacity of a DXC (number of ports)

Decision variables:

f_e	number of SDH systems on edge e
w_e	number of WDM systems on edge e
v_e	number of channels used in the WDM systems on edge e
y_n	number of DXCs used in node n
z_{ij}^{st}	1 if demand (s,t) is routed along edge (i,j); 0 otherwise

Objective value:

$$\text{Minimize} \quad \sum_{e\in E}\Big((C_e^{FS}+2C^P)f_e+(C_e^{FW}+C^W)w_e+(C^C+2C^P)v_e\Big)+\sum_{n\in N}C^O y_n$$

s.t.:

$$\sum_{j\in N} z_{ji}^{st}-\sum_{j\in N} z_{ij}^{st}=\left\{\begin{array}{rl} -1 & i=s \\ 0 & \forall i\neq s,t \\ +1 & i=t \end{array}\right\} \quad \forall (s,t)\in D, \quad (1)$$

$$\sum_{(s,t)\in D} d_{st}(z_{ij}^{st}+z_{ji}^{st})\leq v_e+f_e \quad \forall e\in E \text{ with } i \text{ and } j \text{ adjacent to } e, \quad (2)$$

$$v_e\leq M^W w_e \quad \forall e\in E, \quad (3)$$

$$\sum_{e \text{ adjacent to } n}(v_e+f_e)\leq M^O y_n \quad \forall n\in N, \quad (4)$$

$$\sum_{(i,j)\in E'}(z_{ij}^{st}+z_{ji}^{st})\leq H \quad \forall (s,t)\in D, \quad (5)$$

$$y_n\geq 0 \text{ and integer} \quad \forall n\in N, \quad (6)$$

$$f_e,w_e,v_e\geq 0 \text{ and integer} \quad \forall e\in E, \quad (7)$$

$$z_{ij}^{st}\in\{0,1\} \quad \forall (i,j)\in E',(s,t)\in D. \quad (8)$$

Constraints (1) guarantee the flow-conservation. The origin and the destination nodes of each demand both have one adjacent edge that is used; all other nodes have either none or two. Constraints (2) ensure that the demands that each edge carries are less or equal than the total capacity of the installed SDH and WDM systems. Constraints (3) match the number of available WDM channels with the maximum capacity of the installed WDM multiplexers. Constraints (4) adjust the capacity of the cross connects of each node to the capacity of its adjacent edges. Constraints (5) impose a hop limit for each demand. With sufficiently high values for H (between 8 and 15, depending on the size of the network), this set of constraints speeds up the computation times without sacrificing solution quality. Constraints (6) to (8) ensure the integrality and non-negativity of the decision variables. A more detailed description of a closely related model and many possible extensions are discussed in [4].

3. Variable neighborhood search

Variable neighborhood search [1–3] is a recent metaheuristic for solving combinatorial and global optimization problems based on a simple principle: systematic changes of neighborhoods within the search. Its development has been rapid, with a lot of papers already published and its applications have been numerous and successful. Many extensions have been made, mainly to be able to solve large

problem instances. However, the main idea behind variable neighborhood search is to keep the simplicity of the basic scheme.

Let $\mathcal{N}_k$, $(k = 1,\ldots,k_{\max})$ be a finite set of neighborhood structures, and $\mathcal{N}_k(s)$ the set of solutions in the kth neighborhood of a solution s. Neighborhoods $\mathcal{N}_k$ may be induced from metric functions introduced into a solution space S. If $d(.,.)$ is this distance, then take increasing values d_k, $k = 1,...,k_{\max}$ and set $N_k(s) = \{s' \in S : d(s,s') \leq d_k\}$.

Usually, a series of nested neighborhoods is obtained from a single neighborhood by taking $\mathcal{N}_1(s) = \mathcal{N}(s)$ and $\mathcal{N}_{k+1}(s) = \mathcal{N}(\mathcal{N}_k(s))$, for every solution s. This means that a move to the kth neighborhood is performed by repeating k times a move into the original neighborhood. A solution $s' \in S$ is a *local minimum* with respect to $\mathcal{N}_k$ if there is no solution $s \in \mathcal{N}_k(s') \subseteq S$ better than s' (i.e., such that $f(s) < f(s')$, where f is the objective function of the problem).

The variable neighborhood search metaheuristic is based on three simple facts:

1) a local minimum with respect to one neighborhood structure is not necessarily a local minimum with respect to another;

2) a global minimum is a local minimum with respect to all possible neighborhood structures;

3) local minima with respect to one or several neighborhood structures are usually similar to each other.

By applying the VNS principles to an improving local search we get the variable neighborhood descent (VND). The method consists of changing the neighborhoods systematically within a local search. The basic VND is presented in Fig. 1.

The final solution should be a local minimum with respect to all $k_{\max}$ neighborhoods, and thus the probability of reaching the global minimum is higher than by using a single structure. Beside this *sequential* order of neighborhood structures in VND, one can develop a *random* strategy by choosing the successive values for k at random.

Most local search heuristics use a single or sometimes two neighborhoods ($k_{\max} \leq 2$) in their descents. A usual strategy with two neighborhoods consists of performing local searches for the first neighborhood from points s' that belong to the second neighborhood of the current solution (i.e., $s' \in \mathcal{N}_2(s)$). The perturbation strategy consists of applying a local search using the first neighborhood and then perturbing the current solution by choosing a random solution in the second neighborhood to perform a new local search. The basic variable neighborhood search (BVNS) method uses deterministic changes in the neighborhood structure for perturbation or shaking. Its steps are given in Fig. 2.

The stopping condition may be the maximum CPU time allowed, the maximum number of iterations, or the maximum number of iterations between two improvements. The reduced variable neighborhood search (RVNS) method is

Initialization.

Select the set of neighborhood structures $\mathcal{N}_k$, for $k = 1,\ldots,k_{\max}$, that will be used in the descent.

Find an initial solution s.

Iterations.

Repeat the following sequence until no more improvement is obtained:

(1) Set $k \leftarrow 1$.

(2) Repeat the following steps until $k = k_{\max}$:

(a) *Exploration of neighborhood.*
Find the best neighbor s' of s ($s' \in \mathcal{N}_k(s)$).

(b) *Move or not.*
If the solution thus obtained s' is better than s, set $s \leftarrow s'$ and $k \leftarrow 1$; otherwise, set $k \leftarrow k+1$.

Fig. 1. Variable neighborhood descent.

Initialization.

Select the set of neighborhood structures $\mathcal{N}_k$, for $k = 1,\ldots,k_{\max}$.

Find an initial solution s.

Choose a stopping condition.

Iterations.

Repeat the following sequence until the stopping condition is met:

(1) Set $k \leftarrow 1$.

(2) Repeat the following steps until $k = k_{\max}$:

(a) *Shaking.*
Generate a point s' at random from the kth neighborhood of s ($s' \in \mathcal{N}_k(s)$).

(b) *Local search.*
Apply some local search method with s' as initial solution; denote the so obtained local optimum with s''.

(c) *Move or not.*
If this local optimum is better than the incumbent, move there ($s \leftarrow s''$), and continue the search with $\mathcal{N}_1$ ($k \leftarrow 1$); otherwise, set $k \leftarrow k+1$.

Fig. 2. Basic variable neighborhood search.

obtained if random points are selected from $\mathcal{N}_k(s)$, without being followed by descent. It is useful for very large instances for which local search is costly.

The local search Step 2b in the basic VNS may be replaced by VND. This gives the general variable neighborhood search (GVNS) that is the version with the most recent success. Its steps are shown in Fig. 3.

Initialization.

Select the set of neighborhood structures $\mathcal{N}_k$, for $k = 1, \ldots, k_{\max}$ for the shaking.

Select the set of neighborhood structures $\mathcal{N}'_j$, for $j = 1, \ldots, j_{\max}$ for the descent.

Find an initial solution s.

Choose a stopping condition.

Iterations.

Repeat the following sequence until the stopping condition is met:

(1) Set $k \leftarrow 1$.

(2) Repeat the following steps until $k = k_{\max}$:

(a) *Shaking*.
Generate a point s' at random from the kth shaking neighborhood of s ($s' \in \mathcal{N}_k(s)$).

(b) *Descent*.
Apply to s' the VND with $\mathcal{N}'_j, j = 1, \ldots, j_{\max}$ as neighborhoods to get a new solution s''.

(c) *Move or not*.
If $f(s'') < f(s)$ set $s \leftarrow s''$ and $k \leftarrow 1$; otherwise, set $k \leftarrow k + 1$.

Fig. 3. General variable neighborhood search.

Several extensions of the VNS have also been proposed. The basic VNS is a first improvement descent method with randomization. It is transformed into a descent-ascent method if Step 2c sets also $s \leftarrow s''$ with some probability even if the solution is worse than the incumbent (or the best solution found so far). It is changed into a best improvement method by making a move to the best neighborhood k^* among all $k_{\max}$ of them.

3.1. Application to the design of WDM networks

In order to solve the problem of designing WDM networks, we propose a basic variable neighborhood search metaheuristic.

The mixed integer programming (MIP) formulation described above considers any path between an origin and a destination of a demand and does not limit the number of intermediate nodes. Based on that it may be used to find optimal solutions or at least to provide lower bounds of the problem. On the other hand, the variable neighborhood search developed in this work restricts the number of paths available to route each demand through the network to five reaching an upper bound of the problem.

The construction of a solution starts with the selection of a path for each demand requirement. Once each demand is assigned to a path, the cost of the resulting design is calculated. The cost is associated with the equipment that is required to satisfy the demands using the chosen paths. A solution is fully determined by a data structure that stores the path assignments and the equipment required in each element of the original network.

- *Initialization.* In order to perform the initialization step of the BVNS metaheuristic, the procedure that generates the initial solution and the neighborhood structure must be defined.
  - *Initial solution.* The initial solution is constructed by using the constructive procedure proposed in [8] that attempts to assign demands to paths in order to efficiently utilize the spare capacity in the original base network. The rationale behind this initialization is that spare capacity for channels in the final network design should be zero except for channels on WDM systems covering a segment without slack. The strategy acknowledges that spare capacity in the original network simply accounts for existing network infrastructure.
  - *Initial solution.* The kth neighborhood of the solution s, $\mathcal{N}_k(s)$, consists of all the solutions that can be reached from s by changing the paths assigned to k different demands.
- *Shaking.* This procedure generates a solution S' at random from the kth neighborhood of s ($s' \in \mathcal{N}_k(s)$).
- *Improvement method and move decision.* The improvement method is the local search procedure proposed in [8]. In order to run the improvement method from a given solution, the demands are ordered according to their unit cost. The demand ordering is important because the improvement method, which is based on changing one demand from its current path to another, starts with the demand that has the largest unit cost. The first candidate move is then to reassign the demand that is at the top of the unit cost list. If reassigning this demand leads to an improving move, the move is executed to change the current solution. If an improving move that involves reassigning the first demand in the list cannot be found, then the second demand is considered. The process continues until a demand is found for which a reassignment of paths leads to an improving move. If all the demands are examined and no improving move is found, the local search is abandoned.

4. Computational results

The (mixed) integer flow formulation is modeled in a modeling language for mathematical programming (AMPL) and solved by CPLEX in order to obtain optimal solutions as a reference for the heuristic. The flow formulation does not limit the number of paths for each demand; it searches the entire solution space thus providing lower bounds of the problem. For the larger problem instances, where CPLEX

Table 1
Overview of the computational results

Set	|N|	|E|	|D|	VNS		CPLEX			% deviation of VNS from CPLEX
				cost	time [s]	cost	time [s]	bound	
Extant0D	12	17	15	3.69	0.01	3.69	0.4	opt	0
			21	6.21	0.46	6.21	2	opt	0
			44	14.68	0.05	14.36	11	opt	2.17
			66	13.20	0.05	11.83	22	opt	10.37
	12	33	15	3.69	0.09	3.69	7	opt	0
			21	7.51	1.32	6.03	67	opt	19.70
			44	14.45	0.05	13.66	4228	opt	5.46
			66	15.48	0.58	11.76	9024	opt	24.03
		46	15	3.69	0.02	3.69	12	opt	0
			21	7.99	0.01	6.03	165	opt	24.53
			44	14.79	0.04	13.16	6148	opt	11.02
			66	13.03	1.49	11.77	11194	opt	9.66
Example2D	17	26	27	23.85	1.33	22.47	3	opt	5.78
			36	81.84	8.32	81.84	31	opt	0
			81	97.47	36.62	96.65	470	opt	0.84
			135	173.53	97.19	170.15	29255	opt	1.94
		68	27	24.95	0.06	19.27	110	opt	22.76
			36	64.72	10.54	63.75	28841	opt	1.49
			81	80.75	30.01	76.66	216806	75.21	5.06
			135	145.07	53.96	138.40	81098	130.98	4.59
NationalD	50	63	45	38.14	2.61	34.07	854	opt	10.67
			65	50.91	18.72	48.65	77493	45.56	4.43
			91	60.11	58.08	55.64	95755	53.19	7.43
			112	42.88	0.27	42.88	39544	38.67	0

is not able to find optimal solutions, it can at least provide good bounds. The total provisioning costs provided by the VNS when solving instances whose number of nodes range from 12 up to 50 were compared with the results provided by CPLEX.

All the experiments were carried out on a Pentium4 with 2.4 GHz and the CPLEX calculations were performed with CPLEX version 8.1. The size of the branch and bound tree was limited to 400 MB, which was also the termination criterion for the computation. Table 1 shows a comparison of the objective values and runtimes for the heuristic and the CPLEX calculations. The bound is either the optimal solution of a linear relaxation or a CPLEX internal bound taken from the node log. The CPLEX runtimes include just the final integer run. In most cases, a linear relaxation has been computed beforehand and served as a starting solution for the integer calculation. The computation times of the relaxation were usually far below the times of the integer model.

For the smaller problem instances the computations were simply performed with the universal AMPL model. For the larger networks, some custom modifications to cut off suboptimal parts of the solution space had to be used in order to achieve better results. This includes upper bounds on certain variables, e.g., the number of DXCs in a node. Also an a priori prohibition for SDH on certain very long links where link costs compared to WDM are prohibitive even for a single channel could be applied on some problem instances.

The computational results reported in Table 1 corroborate that the variable neighborhood search metaheuristic is able to obtain solutions which are within a 10% of the lower bounds provided by CPLEX for many of the instances. The VNS results are promising since the solutions for the bigger instances are reached in a few seconds while CPLEX takes minutes and even an hour for one of the problems. Anyway, additional work to remove the number of paths constraint in the variable neighborhood search is required.

5. Summary and conclusions

In this paper, we have addressed the important and current problem in the telecommunications industry of designing WDM networks. We have described a node-arc formulation as the framework to develop a variable neighborhood search procedure and as a means for finding lower and upper bounds.

Our experiments with real and randomly generated data show the value of our proposed solution procedure when compared to the bounds generated by solving the MIP for-

mulation with CPLEX. Our experiments show that small instances are better solved to optimality by way of a commercial MIP solver. However, for bigger instances the metaheuristic procedure is able to reach upper bounds which are within 10% of the lower bounds given by CPLEX in a few seconds. With regard to the usually much larger uncertainty in the demand forecast of the network operator, 10% is already a practical gap for real world applications.
Although the variable neighborhood search reaches acceptable solutions by restricting the number of paths available for the demands, an extension of our work will include solving the problem without limiting the number of paths. The rationale behind this is to carry out a real comparative analysis with the lower bounds obtained solving the MIP described in this paper. Other possible extensions are 1+1 protected demands and problem instances with existing spare capacities (capacity expansion problem). A comparison with other metaheuristics applied to similar problems, like the greedy randomized search procedure used in [5], on the basis of the same problem instances, is also planned for the future.

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Paper

Fair and efficient network dimensioning with the reference point methodology

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Abstract— The dimensioning of telecommunication networks that carry elastic traffic requires the fulfillment of two conflicting goals: maximizing the total network throughput and providing fairness to all flows. Fairness in telecommunication network design is usually provided using the so-called max-min fairness (MMF) approach. However, this approach maximizes the performance of the worst (most expensive) flows which may cause a large worsening of the overall throughput of the network. In this paper we show how the concepts of multiple criteria equitable optimization can be effectively used to generate various fair and efficient allocation schemes. We introduce a multiple criteria model equivalent to equitable optimization and we develop a corresponding reference point procedure for fair and efficient network dimensioning for elastic flows. The procedure is tested on a sample network dimensioning problem for elastic traffic and its abilities to model various preferences are demonstrated.

Keywords— multiple criteria optimization, efficiency, fairness, equity, reference point method, telecommunications, network design, elastic traffic.

1. Introduction

The problem of fairness in the allocation of resources occurs in many contexts, from economics and law to engineering. In all cases, a scarce or constrained resource must be divided among many users in a way that respects fairness and does not ignore efficiency [9, 13]. In the area of telecommunication and computer networks, fair resource allocation usually concerns the allocation of bandwidth to users, services or flows. This problem may be dynamic and solved by adaptive protocols like transmission control protocol (TCP) [3], or it may concern the design or configuration of the network [16, 20]. This paper deals with the problem of fair and efficient network dimensioning.

Telecommunication network design is usually based on a set of estimated traffic demands. The task is then to design the cheapest networks that can satisfy the demands. The estimation of traffic demands is usually possible in networks that are mainly used to communicate voice (like the public switched telephone network – PSTN), since voice communication uses a fixed amount of bandwidth. In data networks, traffic is much more variable and hard to predict; also, data communications does not have quality of service (QoS) requirements that need a fixed bandwidth share. Data traffic is usually carried by the TCP protocol that adapts its throughput to the amount of available bandwidth. Such traffic, called *elastic traffic*, is capable to use the entire available bandwidth, but it will also be able to reduce its throughput in the presence of contending traffic. Nowadays, the network management often faces the problem of designing networks that carry elastic traffic. These network design problems are, essentially, network dimensioning problems as they can be reduced to a decision about link capacities. Flow sizes are outcomes of the design problem, since the flows adapt to given network resources on a chosen path.

Network management must stay within a budget constraint on link bandwidth to expand network capacities. They want to achieve a high throughput of the IP network, to increase the multiplexing gains (due to the use of packet switching by the Internet Protocol – IP). This traffic is offered only a best-effort service, and therefore network management is not concerned with offering guaranteed levels of bandwidth to the traffic. A straightforward network dimensioning with elastic traffic could be thought of as a search for such network flows that will maximize the aggregate network throughput while staying within a budget constraint for the costs of link bandwidth. However, maximizing aggregate throughput can result in extremely unfair solutions allowing even for starvation of flows for certain services. On the other extreme, while looking at the problem from the perspective of a network user, the network flows between different nodes should be treated as fairly as possible [2]. The so-called max-min fairness (MMF) [1, 4] is widely considered as such ideal fairness criteria. Indeed, the lexicographic max-min optimization used in the MMF approach generalizes equal sharing at a single link bandwidth to any network while maintaining the Pareto optimality. Certainly, allocating the bandwidth to optimize the worst performances may cause a large worsening of the overall throughput of the network. Therefore, network management must consider two goals: increasing throughput and providing fairness. These two goals are clearly conflicting, if the budget constraint has to be satisfied.

The purpose of this work is to show that it is possible to balance the two conflicting goals of increasing the total network throughput and providing fairness to all flows. The tradeoff between these two goals can be controlled using a multiple criteria model that allows to represent the overall efficiency and fairness goals. The network manager can choose among many compromise solutions by specifying his preferences using the so-called quasi-satisficing approach to multiple criteria decision problems [22]. The best

formalization of the quasi-satisficing approach to multiple criteria optimization was proposed and developed mainly by Wierzbicki [21] as the reference point method. The reference point method (RPM) is an interactive technique where the decision maker (DM) specifies preferences in terms of aspiration levels (reference point), i.e., by introducing desired (acceptable) levels for several criteria. This allows the DM to simultaneously learn about the problem during the process of expressing his (possibly evolving) preferences. Our methods also enable the DM to choose solutions obtained by methods developed in previous work, that depend on maximization of the sum of the flows evaluated with some (concave) utility function. In particular, the so-called *proportional fairness* (PF) approach [5] maximizes the sum of logarithms of the flows. This approach has been further extended to a parametric class of concave utility functions [11]. However, the methods developed in this paper are more general and allow the DM to choose among many solutions, including solutions that would be obtained by other methods.

The paper is organized as follows. In the next section we formalize the network dimensioning problem, we consider. In Section 3, basic fair solution concepts for resource allocation are related to the multiple criteria equitable optimization theory. In Section 4, the reference point methodology is applied to the multiple criteria allowing us to model various fair and efficient allocation schemes with simple control parameters. Finally, in Section 5, we present some results of our initial computational experience with this new approach.

2. The network dimensioning problem

The problem of network dimensioning with elastic traffic can be formulated basically as a linear programming (LP) based resource allocation model as follows [16]. Given a network topology $G =< V,E >$, consider a set of pairs of nodes as the set $I = \{1,2,\ldots,m\}$ of services representing the elastic flow from source v_i^s to destination v_i^d. For each service, we have given the set P_i of possible routing paths in the network from the source to the destination. We describe them with binary coefficients $\delta_{eip} = 1$ if link e belongs to the routing path $p \in P_i$ (connecting v_i^s with v_i^d) and $\delta_{eip} = 0$ otherwise.

For each service $i \in I$, the elastic flow from source v_i^s to destination v_i^d is a variable representing the model outcome and it will be denoted by x_i. This flow may be realized along various paths $p \in P_i$. The flow may be either split among several paths or a single path must be finally selected to serve the entire flow. Actually, the latter case of nonbifurcated flows is more commonly required. Both bifurcated or nonbifurcated flows may be modeled as $x_i = \sum_{p\in P_i} x_{ip}$ where x_{ip} (for $p \in P_i$) are nonnegative variables representing the elastic flow from source v_i^s to destination v_i^d along the routing p. Although, the single-path model requires additional multiple choice constraints to enforce nonbifurcated flows. This can be implemented with additional binary (flow assignment) variables u_{ip} equal 1 if path $p \in P_i$ is assigned to serve flow x_i and 0 otherwise. Assuming existence of some constant M upper bounding the largest possible total flows x_i, the assignment variables u_{ip} can easily be used to limit the number of positive flows x_{ip} with the following constraints:

$$0 \le x_{ip} \le M u_{ip},\ u_{ip} \in \{0,1\} \qquad \forall\, i \in I;\ p \in P_i, \quad (1)$$

$$\sum_{p\in P_i} u_{ip} = 1 \qquad \forall\, i \in I. \quad (2)$$

The network dimensioning problem depends on allocating the bandwidth to several links in order to maximize flows of all the services (demands). Typically, the network is already operated and some bandwidth is already allocated (installed) and decisions are rather related to the network expansion. Therefore, we assume that each link $e \in E$ has already capacity a_e while decision variables ξ_e represent the bandwidth newly allocated to link $e \in E$ thus expanding the link capacity to $a_e + \xi_e$. Certainly, all the decision variables must be nonnegative: $\xi_e \ge 0$ for all $e \in E$ and there are usually some bounds (upper limits) on possible expansion of the links capacities: $\xi_e \le \bar{a}_e$ for all $e \in E$. Finally, the following constraints must be fulfilled:

$$\sum_{i\in I} \sum_{p\in P_i} \delta_{eip} x_{ip} \le a_e + \xi_e \qquad \forall e \in E, \quad (3)$$

$$0 \le \xi_e \le \bar{a}_e \qquad \forall e \in E, \quad (4)$$

$$\sum_{p\in P_i} x_{ip} = x_i \qquad \forall i \in I, \quad (5)$$

where Eq. (5) define the total service flows, while Eq. (3) establish the relation between service flows and links bandwidth. The quantity $y_e = \sum_{i\in I} \sum_{p\in P_i} \delta_{eip} x_{ip}$ is the load of link e and it cannot exceed the available link capacity.

Further, for each link $e \in E$, the cost of allocated bandwidth is defined. In the basic model of network dimensioning it is assumed that any real amount of bandwidth may be installed and marginal costs c_e of link bandwidth is given. Hence, the corresponding link dimensioning function expressing amount of capacity (bandwidth) necessary to meet a required link load [16] is then a linear function. While allocating the bandwidth to several links in the network dimensioning process the decisions must keep the cost within available budget B for all link bandwidths. Hence the following constraint must be satisfied:

$$\sum_{e\in E} c_e \xi_e \le B. \quad (6)$$

The model constraints (3)–(6) together with respective nonnegativity requirements define a linear programming feasible set. They turn into mixed integer LP (MILP), however, if nonbifurcated flows are enforced with discrete constraints (1) and (2).

In the simplified problem with linear link dimensioning function and dimensioning of a completely new network

($a_e = 0$ for all links), the cost of the entire path p for service i can be directly expressed by the formula:

$$\kappa_{ip} = \sum_{e \in E} c_e \delta_{eip} \quad \text{for } i = 1, \ldots, m, \ p \in P_i . \tag{7}$$

The cheapest path for each service can then easily be identified and preselected. Having preselected routing path for each demand ($|P_i| = 1$) one may consider variable x_i directly as flow along the corresponding path ($x_i = x_{i1}$). Constraints (6) and (3) may be then treated as equations and together with formula (7) they allow one to eliminate variables ξ_e, thus formulating the problem as a simplified resource allocation model with only one constraint:

$$\sum_{i=1}^{m} \kappa_i x_i = B , \qquad \text{where } \kappa_i = \kappa_{i1} \quad \forall \ i \in I \tag{8}$$

and variables x_i representing directly the decisions. Note that one cannot define directly any cost κ_{ip} of the path $p \in P_i$ (similar to formula (7)) when some capacity is already available ($a_e > 0$ for some $e \in E$). In other words in the problem, we consider, the cost of available link capacity is actually nonlinear (piecewise linear) and this results in the lack of direct formula for the path cost since it depends on possible sharing with other paths of the preinstalled bandwidth (free capacity a_e).

The network dimensioning model can be considered with various objective functions, depending on the chosen goal. One may consider two extreme approaches. The first extreme is the maximization of the total throughput (the sum of flows) $\sum_{i \in I} x_i$. On the other extreme, the network flows between different nodes should be treated as fairly as possible which leads to the maximization of the smallest flow or rather to the lexicographically expanded max-min optimization (the so-called max-min ordering) allowing also to maximize the second smallest flows provided that the smallest remain optimal, the third smallest, etc. This approach is widely recognized in networking as the so-called max-min fairness [1, 4] and it is consistent with the Rawlsian theory of justice [17].

Note that in the simplified dimensioning model (with preselected paths and continuous bandwidth), due to possible alternative formulation Eq. (8), the throughput maximization approach apparently would choose one variable x_{i^o} which has the smallest marginal cost $\kappa_{i^o} = \min_{i \in I} \kappa_i$ and make that flow maximal within the budget limit ($x_{i^o} = B/\kappa_{i^o}$), while eliminating all other flows (lowering them to zero). On the other hand, the MMF concept applied to the simplified dimensioning model (resulting in Eq. (8)) would lead us to a solution with equal values for all the flows: $x_i = B/\sum_{i \in I} \kappa_i$ for $i \in I$. Such allocating the resources to optimize the worst performances may cause a large worsening of the overall (mean) performances as the MMF throughput ($mB/\sum_{i \in I} \kappa_i$) might be considerably smaller than the maximal throughput ($B/\min_{i \in I} \kappa_i$). In more realistic dimensioning models assuming nonlinearities in link dimensioning function (like the existence of a free capacity a_e of preinstalled bandwidth) and nonbifurcation requirements a direct formula for a path cost is not available and the corresponding solutions are not so clear. Nevertheless, the main weaknesses of the above solutions remain valid. The throughput maximization can always result in extremely unfair solutions allowing even for starvation of certain flows while the MMF solution may cause a large worsening of the throughput of the network. In an example built on the backbone network of a Polish Internet service provider (ISP), it turned out that the throughput in a perfectly fair solution could be less than 50% of the maximal throughput [14].

Network management may be interested in seeking a compromise between the two extreme approaches discussed above. One of possible solutions depends on maximization of the sum of the flows evaluated with some (concave) utility function $\sum_{i \in I} U_i(x_i)$. In particular, for $U_i(x_i) = \log(x_i)$ one gets the proportional fairness approach [5]. However, every such approach requires to build (or to guess) a utility function prior to the analysis and later it gives only one possible compromise solution. It is very difficult to identify and formalize the preferences at the beginning of the decision process. Moreover, apart from the trivial case of throughput maximization all the utility functions that really take into account any fairness preferences are nonlinear. Nonlinear objective functions applied to the MILP models we consider results in computationally hard optimization problems. In the following, we shall describe an approach that allows to search for such compromise solutions with multiple linear criteria rather than the use nonlinear objective functions.

3. Fairness and equitable efficiency

The network dimensioning problem, we consider, may be viewed as a special case of general resource allocation problem where a set I of m services is considered and for each service $i \in I$, its measure of realization x_i is a function $x_i = f_i(\xi)$ of the allocation pattern $\xi \in A$. This function, representing the outcome (effect) of the allocation pattern for service i we call the individual objective function. In the network dimensioning problem the measure expresses the service flow and a larger value of the outcome means a better effect. This leads us to a vector maximization problem:

$$\max \ \{(x_1, x_2, \ldots, x_m) \ : \quad \mathbf{x} \in Q\} , \tag{9}$$

where $Q = \{(x_1, \ldots, x_m) : x_i = f_i(\xi) \text{ for } i \in I, \ \xi \in A\}$ denotes the attainable set for outcome vectors $\mathbf{x}$. For the network dimensioning problems, we consider, the set Q is an MILP feasible set defined by basic constraints (1)–(6).

Multiple criteria model (9) only states that for any outcome x_i ($i \in I$) larger value is preferred. In order to make it operational, one needs to assume some solution concept specifying what it means to maximize multiple outcomes. The commonly used concept of the Pareto-optimal solutions, as feasible solutions for which one cannot improve any outcome without worsening another, depends on the rational dominance $\succeq_r$ which may be expressed in terms of the vector inequality: $\mathbf{x}' \succeq_r \mathbf{x}''$ iff $x'_i \geq x''_i$ for all $i \in I$.

The concept of fairness has been studied in various areas beginning from political economics problems of fair allocation of consumption bundles to abstract mathematical formulation [18]. In order to ensure fairness in a system, all system entities have to be equally well provided with the system's services. This leads to concepts of fairness expressed by the equitable rational preferences [6, 12]. The fairness requires impartiality of evaluation, thus focusing on the distribution of outcome values while ignoring their ordering, i.e., in the multiple criteria problem (9) one is interested in a set of outcome values without taking into account which outcome is taking a specific value. Hence, we assume that the preference model is impartial (anonymous, symmetric) thus the preference relation must fulfill the following axiom

$$(x_{\tau(1)}, x_{\tau(2)}, \ldots, x_{\tau(m)}) \cong (x_1, x_2, \ldots, x_m) \qquad (10)$$

for any permutation τ of I. Fairness requires also equitability of outcomes which is formalized in the requirement that the preference model must satisfy the (Pigou–Dalton) principle of transfers, i.e., a transfer of any small amount from an outcome to any other relatively worse–off outcome results in a more preferred outcome vector. As a property of the preference relation, the principle of transfers takes the form of the following axiom: for any $x_{i'} > x_{i''}$

$$\mathbf{x} - \varepsilon \mathbf{e}_{i'} + \varepsilon \mathbf{e}_{i''} \succ \mathbf{x} \quad \text{for } 0 < \varepsilon < x_{i'} - x_{i''}, \qquad (11)$$

where $\mathbf{e}_i$ denotes the ith unit vector. The rational preference relations satisfying additionally axioms (10) and (11) are called hereafter *fair (equitable) rational preference relations*. We say that outcome vector $\mathbf{x}'$ *fairly dominates* $\mathbf{x}''$ ($\mathbf{x}' \succ_e \mathbf{x}''$), iff $\mathbf{x}' \succ \mathbf{x}''$ for all fair rational preference relations $\succeq$. An allocation pattern $\xi \in A$ is called *fairly (equitably) efficient* if $\mathbf{x} = \mathbf{f}(\xi)$ is fairly nondominated. Note that each fairly efficient solution is also Pareto-efficient, but not vice verse.

The relation of fair (equitable) dominance can be expressed in terms of a vector inequality on the cumulative ordered outcomes [6]. This can be formalized as follows. First, introduce the ordering map $\Theta : R^m \to R^m$ such that $\Theta(\mathbf{x}) = (\theta_1(\mathbf{x}), \theta_2(\mathbf{x}), \ldots, \theta_m(\mathbf{x}))$, where $\theta_1(\mathbf{x}) \le \theta_2(\mathbf{x}) \le \cdots \le \theta_m(\mathbf{x})$ and there exists a permutation τ of set I such that $\theta_i(\mathbf{x}) = x_{\tau(i)}$ for $i = 1, \ldots, m$. Next, apply to ordered outcomes $\Theta(\mathbf{x})$, a linear cumulative map thus resulting in the *cumulative ordering map* $\bar{\Theta}(\mathbf{x}) = (\bar{\theta}_1(\mathbf{x}), \bar{\theta}_2(\mathbf{x}), \ldots, \bar{\theta}_m(\mathbf{x}))$ defined as

$$\bar{\theta}_i(\mathbf{x}) = \sum_{j=1}^{i} \theta_j(\mathbf{x}) \quad \text{for } i = 1, \ldots, m. \qquad (12)$$

Quantities $\bar{\theta}_i(\mathbf{x})$ $(i = 1, \ldots, m)$ express, respectively: the smallest outcome, the total of the two smallest outcomes, the total of the three smallest outcomes, etc. The theory of majorization [10] includes the results which allow us to derive the following theorem [6].

Theorem 1: Outcome vector $\mathbf{x}'$ fairly dominates $\mathbf{x}''$, if and only if $\bar{\theta}_i(\mathbf{x}') \ge \bar{\theta}_i(\mathbf{x}'')$ for all $i \in I$ where at least one strict inequality holds.

Theorem 1 permits one to express fair solutions of problem (9) as Pareto-efficient solutions to the multiple criteria problem with cumulated ordered objectives

$$\max \{(\eta_1, \ldots, \eta_m) : \quad \eta_k = \bar{\theta}_k(\mathbf{x}) \ \forall\ k \in I,\ \mathbf{x} \in Q\}. \qquad (13)$$

Alternatively one may consider problem (13) with normalized objective functions $\mu_k(\mathbf{x}) = \bar{\theta}_k(\mathbf{x})/k$ thus representing for each k the mean of the k smallest outcomes, called the worst conditional mean [13]. Note that the last (mth) objective in (13) represents the sum of outcomes thus corresponding to throughput maximization. Standard maximin optimization corresponds to maximization of the first objective in (13). The complete MMF solution concept represents the lexicographic approach to multiple criteria in (13):

$$\text{lexmax } \{(\eta_1, \ldots, \eta_m) : \quad \eta_k = \bar{\theta}_k(\mathbf{x}) \ \forall\ k \in I,\ \mathbf{x} \in Q\}.$$

Hence, the MMF is only a specific (extreme) solution concept while the entire multiple criteria problem (13) may serve as a source of various fairly efficient allocation schemes. Although the definitions of quantities $\bar{\theta}_k(\mathbf{x})$ are very complicated, they can be modeled with simple auxiliary constraints. Note that for any given vector $\mathbf{x}$, the quantity $\bar{\theta}_k(\mathbf{x})$ is defined by the following LP problem:

$$\begin{array}{rcll} \bar{\theta}_k(\mathbf{x}) & = & \min \sum_{i \in I} x_i u_{ki} & \\ & & \text{s.t.} \quad \sum_{i \in I} u_{ki} = k,\ 0 \le u_{ki} \le 1 \ \forall\ i \in I. & \end{array} \qquad (14)$$

Exactly, the above problem is an LP for a given outcome vector $\mathbf{x}$ while it begins nonlinear for a variable $\mathbf{x}$. This difficulty can be overcome by taking advantages of the LP dual to Eq. (14):

$$\begin{array}{rcll} \bar{\theta}_k(\mathbf{x}) & = & \max\ kt - \sum_{i \in I} d_i & \\ & & \text{s.t.} \quad t - x_i \le d_i,\ d_i \ge 0 \ \forall\ i \in I, & \end{array} \qquad (15)$$

where t is an unrestricted variable while nonnegative variables d_i represent, for several outcome values x_i, their downside deviations from the value of t [15].

Formula (15) allows us to formulate the multiple criteria problem (13) as follows:

$$\begin{array}{lll} \max & (\eta_1, \ldots, \eta_m) \quad \text{s.t.} \quad \mathbf{x} \in Q & \\ & \eta_k = kt_k - \sum_{i \in I} d_{ik} & \forall\ k \in I \\ & t_k - d_{ik} \le x_i, \quad d_{ik} \ge 0 & \forall\ i,k \in I. \end{array} \qquad (16)$$

The problem (16) adds only linear constraints to the original attainable set Q. Hence, for the basic network dimensioning problems with the set Q defined by constraints (1)–(6), the resulting formulation (16) remains in the class of (multiple criteria) MILP. For the simplified LP model (3)–(6) with flows bifurcation allowed and continuous bandwidth the multiple criteria formulation (16) remains in the class of (multiple criteria) LP.

The expanded model (16) introduces m^2 additional variables and constraints. Although the constraints are simple

linear inequalities they may cause a serious computational burden for real-life network dimensioning problems. Note that the number of services (traffic demands) corresponds to the number of ordered pairs of network nodes which is already square of the number of nodes $|V|$. Thus, finally the expanded multiple criteria model introduces $|V|^4$ variables and constraints which means polynomial but fast growth and can be not acceptable for larger networks. For instance, rather small backbone network of Polish ISP [14], we analyze in Section 5, consists of 12 nodes which leads to 132 elastic flows ($m = 132$) resulting in 17 424 constraints and the same number of deviational variables d_{ik}. In order to reduce the problem size we will restrict the number of criteria in the problem (13).

Consider a sequence of indices $K = \{k_1, k_2, \ldots, k_q\}$, where $1 = k_1 < k_2 < \ldots < k_{q-1} < k_q = m$, and the corresponding restricted form of the multiple criteria model (13):

$$\max \left\{(\eta_{k_1}, \ldots, \eta_{k_q}) : \quad \eta_k = \bar{\theta}_k(\mathbf{x}) \ \forall\ k \in K, \quad \mathbf{x} \in Q\right\} \quad (17)$$

with only $q < m$ criteria. According to Theorem 1, the full multiple criteria model (13) allows us to generate any fairly efficient solution of problem (9). When limiting the number of criteria we restrict these capabilities but still one may generate reasonable compromise solutions as stated in the following theorem.

Theorem 2: If $\mathbf{x}^o$ is an efficient solution of the restricted problem (17), then it is an efficient (Pareto-optimal) solution of the multiple criteria problem (9) and it can be fairly dominated only by another efficient solution $\mathbf{x}'$ of (17) with exactly the same values of criteria: $\bar{\theta}_k(\mathbf{x}') = \bar{\theta}_k(\mathbf{x}^o)$ for all $k \in K$.

Proof: Suppose, there exists $\mathbf{x}' \in Q$ which dominates $\mathbf{x}^o$, i.e., $x'_i \geq x^o_i$ for all $i \in I$ with at least one inequality strict. Hence, $\bar{\theta}_k(\mathbf{x}') \geq \bar{\theta}_k(\mathbf{x}^o)$ for all $k \in K$ and $\bar{\theta}_{k_q}(\mathbf{x}') > \bar{\theta}_{k_q}(\mathbf{x}^o)$ which contradicts efficiency of $\mathbf{x}^o$ in the restricted problem (17).

Suppose now that $\mathbf{x}' \in Q$ fairly dominates $\mathbf{x}^o$. Due to Theorem 1, this means that $\bar{\theta}_i(\mathbf{x}') \geq \bar{\theta}_i(\mathbf{x}^o)$ for all $i \in I$ with at least one inequality strict. Hence, $\bar{\theta}_k(\mathbf{x}') \geq \bar{\theta}_k(\mathbf{x}^o)$ for all $k \in K$ and any strict inequality would contradict efficiency of $\mathbf{x}^o$ within the restricted problem (17). Thus, $\bar{\theta}_k(\mathbf{x}') = \bar{\theta}_k(\mathbf{x}^o)$ for all $k \in K$ which completes the proof. □

According to Theorem 2 while solving the restricted multiple criteria model (17) we can essentially still expect reasonably fair efficient solution and only *unfairness* may be related to the distribution of flows within classes of skipped criteria. In other words, we have guaranteed some rough fairness while it can be possibly improved by redistribution of flows within the intervals $(\theta_{k_j}(\mathbf{x}), \theta_{k_{j+1}}(\mathbf{x})]$ for $j = 1, 2, \ldots, q-1$. Since the fairness preferences are usually very sensitive for the smallest flows, one may introduce a grid of criteria $1 = k_1 < k_2 < \ldots < k_{q-1} < k_q = m$ which is dense for smaller indices while sparser for lager indices and expect solution offering some reasonable compromise between fairness and throughput maximization. In our computational analysis on the network with 132 elastic flows (Section 5) we have preselected 24 criteria including 12 the smallest flows. Note that any restricted model (17) contains criteria $\bar{\theta}_1(\mathbf{x}) = \min_{i \in I} x_i$ and $\bar{\theta}_m(\mathbf{x}) = \sum_{i \in I} x_i$ among others. Hence, it provides more detailed fairness modeling than any bicriteria combination of max-min and throughput maximization.

4. Reference point approach

Taking adavantages of model (17) and Theorem 2 we may generate various fairly efficient network dimensioning patterns as efficient solutions of the multiple criteria problem:

$$\begin{aligned} \max \quad & (\eta_k)_{k \in K} \quad \text{s.t.} \quad \mathbf{x} \in Q \\ & \eta_k = kt_k - \sum_{i \in I} d_{ik} \qquad \forall\ k \in K \\ & t_k - d_{ik} \leq x_i, \quad d_{ik} \geq 0 \quad \forall\ i \in I,\ k \in K, \end{aligned} \quad (18)$$

where $K \subseteq I$ and the attainable set Q is defined by constraints (1)–(6). Actually, in the case of the complete multiple criteria model ($K = I$), according to Theorem 1, all fairly efficient allocations can be found as efficient solutions to (18) while in the case of restricted set of criteria $K \subset I$ some minor unfairness related to the distribution of flows within classes of skipped criteria may occur (Theorem 2).

The simplest way to generate various fairly efficient dimensioning patterns may depend on the use some combinations of criteria $(\eta_k)_{k \in K}$. In particular, for the weighted sum with weights $w_k > 0$

$$\sum_{k \in K} w_k \eta_k = \sum_{k \in K} w_k \bar{\theta}_k(\mathbf{x}) = \sum_{i \in I} \Big(\sum_{k \in K : k \geq i} w_k \Big) \theta_i(\mathbf{x})$$

one apparently gets the so-called ordered weighted averaging (OWA) [23] with weights $v_i = \sum_{k \in K : k \geq i} w_k$ $(i \in I)$. If the weights v_i are strictly decreasing, i.e., in the case of full model ($K = I$), each optimal solution corresponding to the OWA maximization is a fair (fairly efficient) solution of (9) while the fairness among flows within classes of equal weights v_i (of skipped criteria) may be sometimes improved. Moreover, in the case of LP models, as the simplified network dimensioning (3)–(6), every fairly efficient allocation scheme can be identified as an OWA optimal solution with appropriate strictly monotonic weights [6]. Several decreasing sequences of weights provide us with various aggregations. Indeed, our earlier experience with application of the OWA criterion to the simplified problem of network dimensioning with elastic traffic [14] showed that we were able to generate easily allocations representing the classical fairness models. On the other hand, in order to find a larger variety of new compromise solutions we needed to incorporate some scaling techniques originating from the reference point methodology. Better controllability and the complete parameterization of nondominated solutions even for non-convex, discrete problems can be achieved with the direct use of the reference point methodology.

The reference point method was introduced by Wierzbicki [21] and later extended leading to efficient

implementations of the so-called aspiration/reservation based decision support (ARBDS) approach with many successful applications [8, 22]. The approach is an interactive technique allowing the DM to specify the requirements in terms of aspiration and reservation levels, i.e., by introducing acceptable and required values for several criteria. Depending on the specified aspiration and reservation levels, a special scalarizing achievement function is built which generates an efficient solution to the multiple criteria problem when maximized. The generated solution is accepted by the DM or some modifications of the aspiration and reservation levels are introduced to continue the search for a better solution. The ARBDS approach provides a complete parameterization of the efficient set to multi-criteria optimization. Hence, when applying the ARBDS methodology to the ordered cumulated criteria in (13), one may generate any (fairly) equitably efficient solution to the original problem (9).

In order to guarantee that for any individual outcome η_k more is preferred to less (maximization), the scalarizing achievement function must be strictly increasing with respect to each outcome. A solution with all individual outcomes η_k satisfying the corresponding reservation levels is preferred to any solution with at least one individual outcome worse (smaller) than its reservation level. Next, provided that all the reservation levels are satisfied, a solution with all individual outcomes η_k equal to the corresponding aspiration levels is preferred to any solution with at least one individual outcome worse (smaller) than its aspiration level. That means, the scalarizing achievement function maximization must enforce reaching the reservation levels prior to further improving of criteria. In other words, the reservation levels represent some soft lower bounds on the maximized criteria. When all these lower bounds are satisfied, then the optimization process attempts to reach the aspiration levels.

The basic scalarizing achievement function takes the following form [21]:

$$\sigma(\eta) = \min_{k \in K}\{\sigma_k(\eta_k)\} + \varepsilon \sum_{k \in K} \sigma_k(\eta_k), \qquad (19)$$

where ε is an arbitrary small positive number and σ_k, for $k \in K$, are the partial achievement functions measuring actual achievement of the individual outcome η_k with respect to the corresponding aspiration and reservation levels (η_k^a and η_k^r, respectively). Thus the scalarizing achievement function is, essentially, defined by the worst partial (individual) achievement but additionally regularized with the sum of all partial achievements. The regularization term is introduced only to guarantee the solution efficiency in the case when the maximization of the main term (the worst partial achievement) results in a non-unique optimal solution.

The partial achievement function σ_k can be understood as a measure of the DM's satisfaction with the current value (outcome) of the kth criterion. It is a strictly increasing function of outcome η_k with value $\sigma_k = 1$ if $\eta_k = \eta_k^a$, and $\sigma_k = 0$ for $\eta_k = \eta_k^r$. Thus the partial achievement functions map the outcomes values onto a normalized scale of the DM's satisfaction. Various functions can be built meeting those requirements [22]. We use the piecewise linear partial achievement function introduced in [12] as

$$\sigma_k(\eta_k) = \begin{cases} \gamma\lambda_k(\eta_k - \eta_k^r) & \text{for } \eta_k \le \eta_k^r, \\ \lambda_k(\eta_k - \eta_k^r) & \text{for } \eta_k^r < \eta_k < \eta_k^a, \\ \beta\lambda_k(\eta_k - \eta_k^a) + 1 & \text{for } \eta_k \ge \eta_k^a, \end{cases}$$

where $\lambda_k = 1/(\eta_k^a - \eta_k^r)$ while β and γ are arbitrarily defined parameters satisfying $0 < \beta < 1 < \gamma$. This partial achievement function is strictly increasing and concave which guarantees its LP computability with respect to outcomes η_k.

In our network dimensioning model (18) outcomes η_k represent cumulative ordered flows x_i, i.e., $\eta_k = \sum_{i=1}^{k} \theta_i(\mathbf{x})$. Therefore, the reference vectors (aspiration and reservation) represent, in fact, some reference distributions of outcomes (flows). Moreover, due to the cumulation of outcomes, while considering equal flows ϕ as the reference (aspiration or reservation) distribution, one needs to set the corresponding levels as $\eta_k = k\phi$. Certainly, one may specify any desired reference distribution in terms of the ordered values of the flows (quantiles in the probability language) $\phi_1 \le \phi_2 \le \ldots \le \phi_m$ and cumulating them automatically get the reference values for the outcomes η_k representing the cumulated ordered flows. However, such rich modeling technique may be too complicated to control effectively the search for a compromise solution. Therefore, we rather consider to begin the search with a simplified approaches based on the reference flow distribution given as a linear sequence $\phi_k = \phi_1(1 + (k-1)r)$ with the (relative) slope coefficient r thus leading to the cumulated reference levels increasing quadratically $\bar{\theta}_k(\phi) = \phi_1 k(2 + (k-1)r)/2$. Although, special meaning of the last (throughput) criterion should be rather operated independently from the others. Such an approach to control the search for a compromise fair and efficient network dimensioning has been confirmed by the computational experiments.

5. Computational analysis

The reference distribution approach has been tested on a sample network dimensioning problem with elastic traffic. The outcome of the network dimensioning procedure (using elastic traffic) are the capacities of links in a given network, because the flows will adapt to the bandwidth available on the links in the designed network. The data to a network dimensioning problem with elastic traffic consists of a network topology, of pairs of nodes that specify sources and destinations of flows, of sets of network paths that could be used for each flow, and of optional constraints on the capacities of links or on flow sizes. Moreover, there are also given pricesof a unit of link capacity (possibly different for each link, c_e in (6)), and the budget amount for purchasing link capacity (B in (6)). The given network topology may contain information about preinstalled link capacities (a_e in (3)): the budget is then spent on additional link capacities that extend the present capacity of a link.

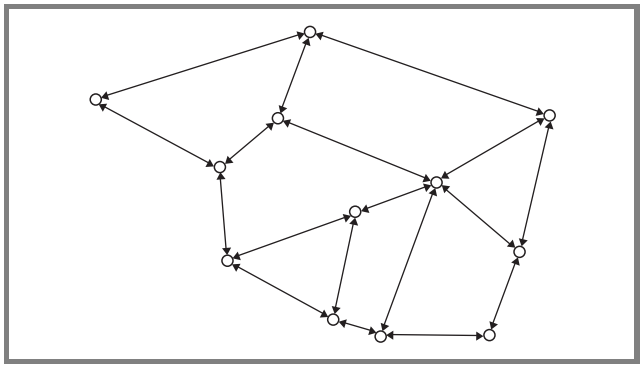

Fig. 1. Sample network topology patterned after the backbone network of Polish ISP.

For our computational analysis we have used the network (Fig. 1) patterned after the network topology of the backbone network of Polish ISP [14]. The network consists of 12 nodes and 18 links. Flows between any pair of different nodes have been considered (i.e., $144 - 12 = 132$ flows). In real networks flows are usually realized on small number of paths. Therefore, we have used lists with only 2 alternative paths for one flow. We have used a single-path formulation (nonbifurcation formulation (1) and (2)), meaning that the entire flow had to be switched to the alternative path. Flows could not be split, which is consistent with several traffic engineering technologies used today.

We set all unit costs $c_e = 1$, and the total budget amount $B = 1000$. For certain links, free link capacity was set to values from 5 to 20, and the upper limit on the capacity of certain links was set to 20. Due to the presence of free link capacity and upper limits on link capacity, the MILP solver found solutions where certain flows had to use alternative paths rather than shortest paths. These flows were more expensive than other flows that were allowed to use their shortest paths.

A simplified LP model for network dimensioning problem without additional constraints on link capacity, with a limitation that flows could only use the shortest path has been studied in [14]. For such a problem it is simple to calculate the solution obtained by the MMF and PF methods. Indeed, in [14] we have calculated these solutions and have shown the appropriate OWA aggregations allows us to obtain similar results. Additionally, using the OWA criterion, it was possible to obtain alternative solutions. Here, we focus on extensions of the problem studied in [14] that make the studied models more practical and realistic. Our extension allowed flows to choose one of two paths for transport (1) and (2), added constraints that limited the capacity of certain links from above and added free link capacity for certain links (3). The intention behind the modification has been to model a situation when the network operator wishes to extend the capacity of an existing network. In this network, certain links cannot be upgraded beyond a certain values to the use of legacy technologies, due to prohibitive costs or administrative reasons (for instance, it may be cheap to use already installed fiber that has not been in use before, but it may be prohibitively expensive to install additional fiber). The existence of free link capacity and of link capacity constraints may be the reason for choosing alternative paths for certain flows. The extended model we consider is too complex for a simple application of MMF and PF methods. To apply either of these methods to the discussed problem extensions, it would be necessary to solve a nonlinear optimization problem or a sequence of MILP problems with changing constraints.

In our analysis while using the RPM methodology we do not have used all 132 criteria η_k as in [14]. Instead, we have selected only 24 criteria by choosing the indices $1, 2, 3, \ldots, 10, 11, 12, 18, 24, 30, 36, 48, 60, 72, \ldots, 120, 132$. As a result, the computation time has dropped from around one hour for each problem to the order of seconds. At the same time, the ability to control the outcomes using the reservation levels has not deteriorated; we were able to obtain similar results with the reduced set of criteria as with the full set. For our approach the final input to the model consisted of the reservation and aspiration levels for the sums of ordered criteria. For simplicity, all aspiration levels were set close to the optimum values of the criteria, and only reservation levels were used to control the outcome flows. One of the most significant parameters was the reservation level for the sum of all criteria (the network throughput). This value denoted by η_m^r was selected (varying) separately from the other reservation levels. All the other reservation levels were formed following the linearly increasing sequence of the ordered values with slope (step) r and where the reservation level for minimal flow was taken $\phi_1 = 1$. Hence, for the final criteria $\eta_k = \bar{\theta}_k(\mathbf{x})$ representing the sums of ordered outcomes in model (16), the sequence of reservation levels increased quadratically (except from the last one). Thus, the three parameters have been used to define the reference distribution but we have managed to identify various fair and efficient allocation patterns by varying only two parameters: the reservation level η_m^r for the total throughput and the slope r for the linearly increasing sequence.

In the experiment, we have searched for various compromise solutions that traded off fairness against efficiency while controlling the process by the throughput reservation level η_m^r and the slope r. The throughput reservation has been varied between 500 and 1100. The linear increase of the other reservation levels was controlled by the slope parameter r. In the experiment this parameter have set to values of: 0.02, 0.03 and 0.04. The results of the experiment are shown in Figs. 2–4 with the corresponding absolute Lorenz curves [7]. The figures present plots of cumulated ordered flows $\bar{\theta}_k(\mathbf{x})$ versus number k (rank of a flow in ordering according to flow throughput) which means that the normalizing factor $1/m = 1/132$ has been ignored (for both the axes). The total network throughput is represented in the figures by the altitude of the right end of the curve ($\bar{\theta}_{132}(\mathbf{x})$). A perfectly equal distributions of flows would be graphically represented by an ascending line of constant slope.

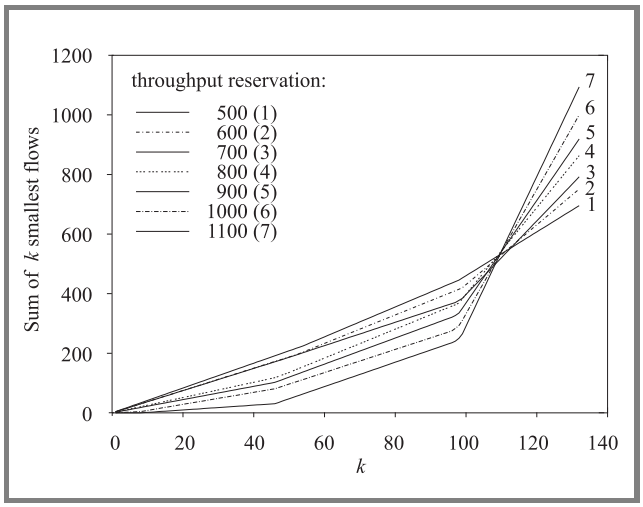

Fig. 2. Flows distributions for varying throughput reservation with $r = 0.02$.

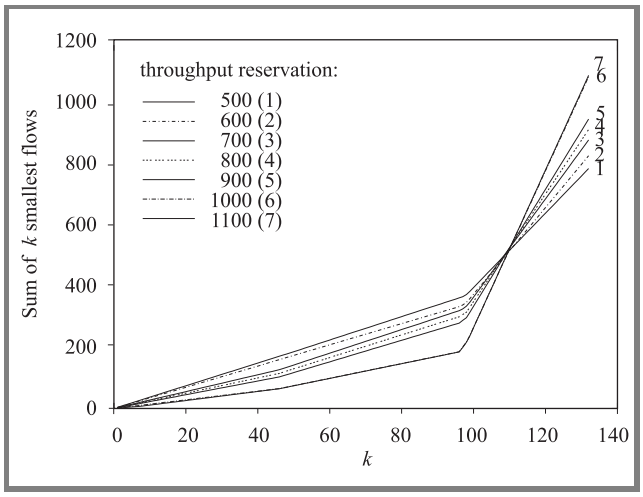

Fig. 3. Flows distributions for varying throughput reservation with $r = 0.03$.

As throughput reservation η_m^r increases, the cheaper flows receive more throughput at the expense of more expensive (longer) flows. For values of η_m^r above 1100, some flows were starved, and therefore these outcomes were not considered further. Under moderate throughput requirements, as r increases, the medium flows gain at the expense of the larger ones thus enforcing more equal distribution of flows (one may observe flattening of the curves). On the other hand, with higher throughput reservations the larger flows are protected by this requirement and increase of r causes that the medium flows gain at the expense of the smallest flows (one may observe convexification of the curves). For values of r higher than 0.04, the increase of the throughput reservation resulted in flow starvation.

Note from Fig. 4 that the boundary between the smallest flows for $\eta_m^r = 500$ and for $\eta_m^r = 1100$ is not in the same position. The reason for this is the upper constraint on link capacities. For $\eta_m^r = 500$, there are 8 flows that ought be in the middle group of flows but they cannot, since flows in the middle group receive so much throughput that the constraints on link capacity would be violated. Consequently, these flows are downgraded to the group of smallest flows and they receive the same amount of throughput as the smallest flows, due to the fairness rules.

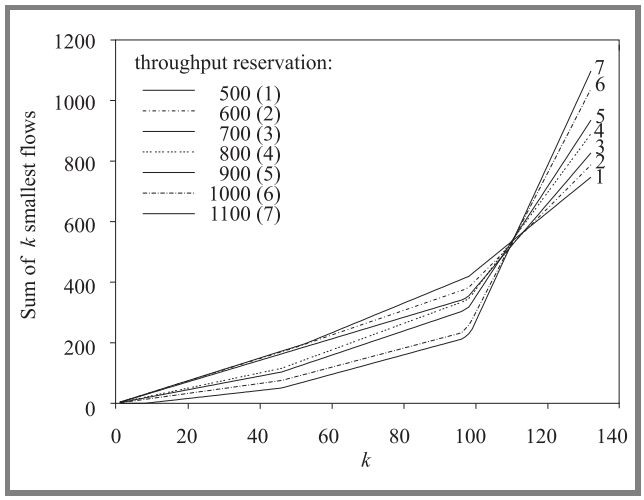

Fig. 4. Flows distributions for varying throughput reservation with $r = 0.04$.

In our experiments the throughput reservation was effectively used to find outcomes with the desired network throughput. Note that, especially for large throughput reservations, the optimization procedure automatically found outcomes that divided flows into four categories according to their path costs. This demonstrates that our methodology is cost-aware, and that it guarantees fairness to all flows with the same path cost (if link capacity constraints do not interfere). For the lowest throughput reservation of $\eta_m^r = 500$ and $r = 0.04$, the outcome was close to a perfectly fair distribution. Thus methodology described in this paper, can offer the user an opportunity to choose from a large gamut of different outcomes and control the trade-off between fairness and efficiency.

We have also tested an alternative scheme of the preference modeling within our reference point method implementation. Namely, we analyzed the initial scheme (see Section 4) based on the reference flow distribution given as a linear sequence $\phi_k = \phi_1(1+(k-1)r)$ with the (relative) slope coefficient r thus leading to the cumulated reference levels increasing quadratically $\bar{\theta}_k(\phi) = \phi_1 k(2+(k-1)r)/2$ is strictly implemented. The sequence was applied to construct all the reservation levels including η_1^r for the minimum flow and η_m^r for the network throughput. Although the value of η_m^r, due to the represented throughput criterion, had to be selected (varying) directly. Therefore, all the other reservation levels were formed according to the linearly increasing sequence of the ordered values with slope (step) r where the reservation level for the minimal flow ϕ_1 had allocated a value guaranteeing that $\eta_m^r = \phi_1 m(2+(m-1)r)/2$. Thus, the two parameters have been used to define the reference distribution: the reservation level η_m^r for the total throughput and the slope r for the linearly increasing sequence but (opposite to the scheme from Section 5) ϕ_1 has not been fixed.

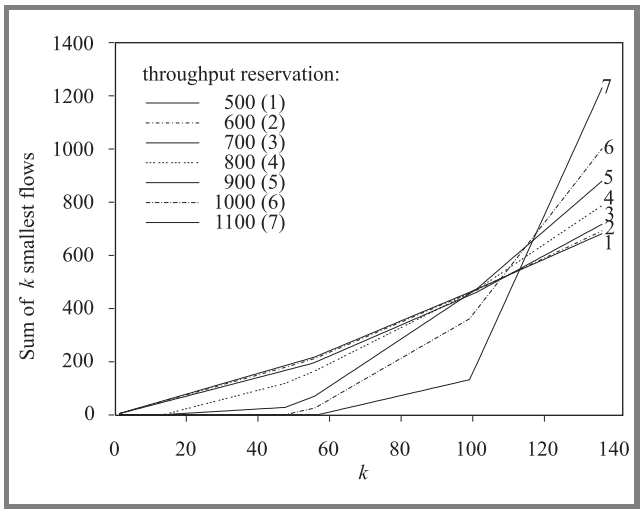

Fig. 5. Results for varying throughput reservation with $r = 0.02$ defining all other reservation levels.

We have applied this preference model to the first network dimensioning problem consisted of the single paths requirements, free link capacity and upper limits on capacity for certain links. The results of the experiment with $r = 0.02$ and varying η_m^r are shown in Fig. 5 with the corresponding absolute Lorenz curves. As η_m^r increases, the cheaper flows receive more throughput at the expense of more expensive (longer) flows. It turned out that except from relatively minor throughput requirements (values 500 to 700), increasing values of η_m^r introduced significant inequity among flows and numerous flows were starved. Similar solutions appeared for various values of r. Therefore, we have abandon such a two parameter control scheme and we have decided that the throughput criterion should be rather operated independently from the others. Such an approach to control the search for a compromise fair and efficient network dimensioning has been confirmed by the computational experiments as described in Section 5.

Overall, the experiments on the sample network topology demonstrated the versatility of the described methodology for equitable optimization. The use of reference levels, actually controlled by a small number of simple parameters, allowed us to search for compromise solutions best fitted to various possible preferences of a network designer. Using appropriate reference point based procedure, one should be able to find a satisfactory fair and efficient network dimensioning pattern in a few interactive steps.

6. Concluding remarks

Network dimensioning problems today must take into account the existence of elastic traffic. The usual approach is to maximize the amount of elastic traffic that uses the best-effort network service, since this increases the multiplexing gain. This approach is equivalent to maximizing network throughput for elastic traffic in a network dimensioning problem. However, this may lead to a starvation and unfair treatment of diverse network flows and, as a consequence, to customer dissatisfaction. While it is true that elastic traffic has no strict QoS requirements, it is also true that the utility of a customer that uses best-effort network services depends on the amount of available throughput.

These considerations lead to the problem of fair and efficient network dimensioning for elastic traffic. In our previous research and in this paper, we have shown that this problem leads to a tradeoff between fairness (where the goal is to decrease differences in throughput for different flows) and efficiency (increasing total network throughput). We have also shown that the problem of fair and efficient network dimensioning is a multiple criteria problem that has many possible solutions (Pareto-optimal solutions that are also optimal for the initial problem without fairness constraints). Previous work on the problem always found a single solution. This did not allow to control the basic tradeoff between fairness and efficiency.

In this paper, we have used the reference point methodology, a standard multiple criteria optimization method that allows for good controllability and the complete parameterization of nondominated solutions. While looking for fairly efficient network dimensioning, the reference point methodology can be applied to the cumulated ordered outcomes. Our initial experiments with such an approach to the problem of network dimensioning with elastic traffic have confirmed the theoretical advantages of the method. We were easily able to generate various (compromise) fair solutions, although the search for fairly efficient compromise solutions was controlled by only two parameters. One of these parameters was a reservation level for the network throughput. The second parameter allowed the network designer to control the difference in throughputs of cheaper and more expensive flows. Still, flows with the same cost were always treated fairly. Moreover, the obtained solutions divided flows into categories determined by flow cost. These characteristics demonstrate that the model is cost-aware and fulfills the axioms of equitable optimization. Also, the achieved total network throughputs in our solutions were higher than the throughput obtained by the max-min fairness method.

Acknowledgments

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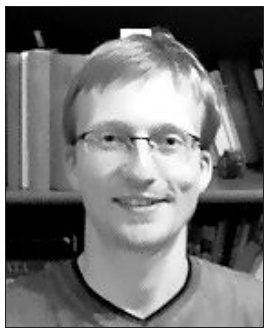

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Paper

A new algorithm for calculating the most reliable pair of disjoint paths in a network

Teresa Gomes, José Craveirinha, and Artur Violante

Abstract— In various types telecommunication networks, namely mobile ad hoc networks, WDM networks and MPLS networks, there is the necessity of calculating disjoint paths for given node to node connections in order to increase the reliability of the services supported by these networks. This leads to the problem of calculating a pair of disjoint paths (or a set of disjoint paths) which optimises some measure of performance in those networks. In this paper we present an algorithm, designated as OptDP, for obtaining the most reliable pair of disjoint paths based on the loopless version of MPS, a very efficient *k*-shortest path algorithm, and on Dijkstra algorithm. Since to the best of our knowledge there is no other proposal of an algorithm capable of solving exactly the same problem we perform a comparison with the application to this problem of the DPSP algorithm which calculates a set of disjoint paths with high reliability. Also a comparison with a simplified version (designated as NopDP) of the proposed algorithm, which stops after a maximal number *F* of candidate pairs of paths have been found, is presented. The comparison also includes the percentage of cases in which both algorithms were not capable of finding the optimal pair.

Keywords— reliability, OR in telecommunications, routing.

1. Introduction

In various types telecommunication networks, namely mobile ad hoc networks, wavelength division multiplexing (WDM) networks and multiprotocol label switching (MPLS) networks, there is the necessity of calculating disjoint paths for given node to node connections in order to increase the reliability (hence improving the quality of service – QoS) of the services supported by these networks. This leads to the problem of calculating a pair of disjoint paths (or a set of disjoint paths) which optimises some measure of performance in those networks.

Performance measures are generally defined having as basis the cost of the arcs. If the metric associated with each arc and the function to be optimised are additive measures, the calculation of a set of disjoint paths can be made using the well known algorithm proposed in [10]. If only a pair of paths is required, then the efficient polynomial time algorithm proposed by Suurballe and Tarjan [11] will solve the problem. If the most reliable pair of paths is desired, the algorithm in [11] cannot be used because the reliability of the union of the two paths is not additive in the path reliabilities.

The problem of finding the set of paths which maximises the two-terminal (ie., node-to-node) reliability metric has not received much attention according to Papadimitratos *et al.* [9]. These authors propose an algorithm (with polynomial worst case complexity) which calculates a set of disjoint paths (without length constraints) with high reliability, that was applied in the context of mobile ad hoc networks (MANET).

In [5] an algorithm is proposed for obtaining k disjoint paths between two different nodes, s and t, in a network with k different costs on every edge, such that the total cost of the paths is minimised (where the jth edge-cost is associated with the jth path).

The approach used in the present work has similarities with the enhancement of the two-step-approach [4] and with the iterative two-step-approach (ITSA) algorithm [8] for optimal diverse routing with shared protection in connection-oriented networks, where the arc costs of the protection path depend on the selected working path. Note that, in our case, the cost of the protection path does not depend on the selected working path, and the function to be optimised is not a linear combination of the costs of the working and protection paths, unlike the problem in [8].

In this paper we present an algorithm, designated as OptDP, for obtaining the most reliable pair of disjoint paths based on the loopless version of the k-shortest paths algorithm MPS [7] and on Dijkstra algorithm. The basic structure is analogous to the one of the algorithm RLDPC-BF proposed by the authors in [2] for calculating the most reliable pair of disjoint paths with a maximum number of arcs per path, based on KD (k-shortest paths with at most D arcs [3]) and Bellman-Ford algorithms.

Since to the best of our knowledge there is no other proposal of an algorithm capable of solving exactly the same problem we performed a comparison with the disjoint path selection protocol (DPSP) algorithm [9] which calculates a set of disjoint paths with high reliability. Note that this algorithm does not guarantee the determination of the optimal set of disjoint paths in terms of reliability. We compared the proposed algorithm with DPSP when this is used to obtain a pair of disjoint paths with high reliability DPSP(k = 2), in terms of central processing unit (CPU) time, for four sets of test networks, with low and high reliability in the edges, a number of nodes (n) varying from 50 to 500 and number of edges, $m = 3n$ and $m = 2n$. Also a comparison with a simplified version (designated as NopDP) of our algorithm which stops after a maximal number F of candidate

pairs of paths has been found, is presented (the used implementation considers $F = 5$). The comparison also includes the percentage of cases in which both algorithms were not capable of finding the optimal pair.

The major conclusion of the computational experiments with the test networks was that the simplified version of the proposed algorithm (NopDP) always performed better than DPSP(k = 2) in terms of CPU and enabled a significant larger relative number of optimal pairs of paths to be obtained. As for the exact algorithm (OptDP) it is less efficient than DPSP(k = 2) for networks with high reliability but becomes more efficient for networks with low reliability (this improvement is particularly significant in networks with $m = 3n$).

The paper is organised as follows. In Section 2 the problem will be formalised and in Section 3 the proposed algorithm is described. In Section 4 the DPSP algorithm is shortly reviewed. Experimental results from the three algorithms are shown and discussed in Section 5, followed by some final remarks in Section 6.

2. Problem formulation

Let $G = (N,L)$ be a directed graph where $N = \{v_1, v_2, \ldots, v_n\}$ is the node set and L the arc (or link) set, composed of ordered pairs of elements in N, where n represents the cardinality of set N. Let $l = (i, j)$ be an arc where j is the head of l and i its tail. A path from s to t ($s,t \in N$) in this graph will be specified by the sequence $p = \langle s,(s,v_1),v_1,\ldots,(v_w,t),t\rangle$, where all $l = (v,u) \in p$ belong to L. If all nodes in p are different it is called a loopless path. Although up till now only the term path was used, the loopless condition is implicitly assumed. The word "loopless" will continue to be omitted until explicit reference is needed.

Each link $l \in L$ has a probability $p_L(l)$ of being operational. Nodes are assumed not to fail. In a network where links fail (independently) and one seeks link disjoint paths from s to t, a *cost matrix* $[c_{ij}]$ of dimension $n \times n$ is defined such that the cost of an arc is the additional cost of introducing that arc in a path:

$$c_{ij} = \begin{cases} -\ln p_L(l) & \text{if } l = (i,j) \in L \\ +\infty & \text{if } l = (i,j) \notin L \end{cases} . \qquad (1)$$

The cost of a path $p = \langle s,(s,v_1),v_1,\ldots,(v_w,t),t\rangle$ is $\mathcal{C}(p) = \sum_{(v_i,v_j)\in p} c_{v_iv_j}$, and its reliability is:

$$\Pr(p) = e^{-\mathcal{C}(p)}, \qquad (2)$$

where $\Pr(p)$ represents the probability of path p being operational. Equation 2 establishes a relation between the cost of a path and its reliability. Using the cost matrix $[c_{ij}]$, the enumeration of the k-shortest paths is equivalent to enumerating, by decreasing order of their reliability, the k most reliable paths.

The most reliable pair of link disjoint paths (p_w, p_v) has a reliability given by:

$$\max_{p_w,p_v} \Pr(p_w \cup p_v) = \Pr(p_w) + (1 - \Pr(p_w))\Pr(p_v), \qquad (3)$$

where p_w and p_v are the working and protection paths, respectively. As can be seen from Eq. (3) $\Pr(p_w \cup p_v)$ cannot be written as a linear function of the costs of p_w and p_v. Two disjoint paths may have minimum $C(p_w) + C(p_v)$ but they may not be the paths with maximal $\Pr(p_w \cup p_v)$.

3. Description of OptDP

The sequential generation of paths p_i (selected by decreasing reliability order) can be made by using any k-shortest path ranking algorithm. In this work the loopless version of MPS algorithm was chosen [7], due to its efficiency [6].

For each i-shortest path p_i (where i represents the order of a selected path – p_i is a candidate working path) there may exist more than one link disjoint path (p_j, a candidate protection path for p_i). The path p_j which maximises $\Pr(p_i \cup p_j)$ (with p_i fixed) will be the one with highest reliability among all the feasible paths; therefore a subalgorithm is needed for efficiently obtaining the most reliable path disjoint with p_i. This algorithm can simply be the Dijkstra algorithm applied to graph G with the links in p_i (temporarily) removed, the algorithm execution being stopped as soon as the destination note t is selected as a minimum distance node.

The proposed algorithm, designated by OptDP, requires a condition to detect that the calculated disjoint path pair is optimal.

Suppose that for each path p_w, the most reliable link disjoint path p_v was obtained, such that at any given step of the algorithm the only recorded pair of paths is the one with the highest $\Pr(p_w \cup p_v)$. Considering that the next (most reliable) path, generated by the k-shortest path subalgorithm, to be selected in the main algorithm is p_i ($i > w$) such that:

$$\Pr(p_i) + (1 - \Pr(p_i))\Pr(p_i) \leq \Pr(p_w) + (1 - \Pr(p_w))\Pr(p_v), \qquad (4)$$

then (p_w, p_v) is the pair of paths with maximal reliability. The verification of this statement is straightforward. Let p_j be the most reliable path link disjoint with p_i, if $\Pr(p_j) \leq \Pr(p_i)$ then any other pair of paths obtained from this point onwards will always have reliability less than $\Pr(p_i) + (1 - \Pr(p_i))\Pr(p_i)$ therefore lower than $\Pr(p_w \cup p_v)$; if $\Pr(p_j) > \Pr(p_i)$ then p_j was previously generated and the reliability of the corresponding pair was not greater than the one of the current best pair, thence this case is irrelevant. Note that this optimal stopping condition (4) is the same as in the algorithm [2] proposed by the authors for calculating the most reliable pair of disjoint paths with length constraints.

Having established the optimal stopping rule of the algorithm (OptDP), its flowchart is presented in Fig. 1. The ex-

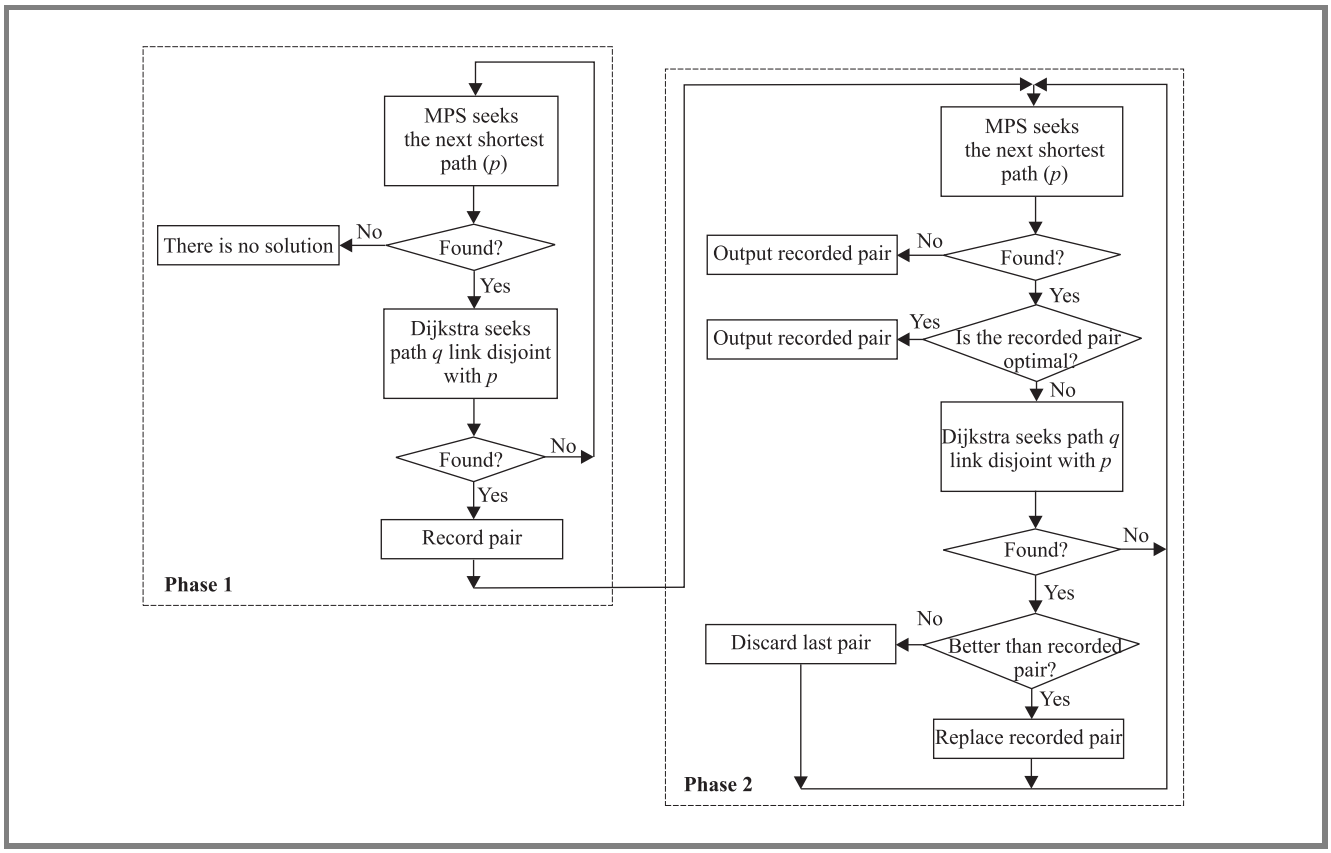

Fig. 1. Flowchart of OptDP.

perimental results will show that, although the optimal disjoint path is frequently among the first ones which are obtained by OptDP, sometimes it is difficult to verify the optimal stopping condition. So a variant of the algorithm was implemented, NopDP, which counts the number of generated path pairs and stops when the optimal stopping condition is satisfied or when the number of generated path pair reaches a pre-established value F, returning the current best pair of disjoint paths.

The main structure of the algorithm has two phases shown in Fig. 1: the obtainment of the first pair of link disjoint paths and the search and/or detection of the optimal pair of paths. In the first phase the algorithm may terminate without finding a solution: no single link disjoint pair of paths was identified (in a connected network this situation can only occur if the network is 1-line-connected).

Having completed phase 1 (we assume that at least a disjoint path pair is always found) and having recorded a pair of link disjoint paths, the second phase of the algorithm consists of improving this solution (whenever possible) until either the recorded pair of paths is detected to be optimal, or no more (working) paths can be found. This last condition implies that the best recorded pair is in fact the optimal one.

If the graph $G^u(N,l)$ which represents the structure of a telecommunications network is undirected, the proposed algorithm can still be used. Each (undirected) edge is replaced by two directed arcs in opposite directions with cost equal to the cost of the edge and the corresponding directed version, G, of G^u is used by OptDP. All the edges of a working path p_w have to be removed (temporarily) from the network graph before running the Dijkstra algorithm; in this case this is done by removing (temporarily) from G all arcs in p_w and also the corresponding arcs in opposite direction.

4. A brief overview of DPSP algorithm

The DPSP algorithm, by Papadimitratos *et al.* [9], iteratively builds a set of disjoint paths of high reliability, for undirected networks. The implementation of DPSP uses the directed graph corresponding to the undirected network under analysis.

Let us assume that at given step of the algorithm, k disjoint paths have already been obtained and are stored in set D_k ($D_k = \cup_{i=1}^{k}\{p_i\}$). The arcs which belong to the paths in set D_k are (temporarily) removed form the network graph, making their cost equal to ∞. The reverse arcs, that is the arcs in opposite direction corresponding to the arcs in D_k (recall that each edge in G^u is represented in G by two arcs in opposite directions) have its cost (temporarily) set to its symmetrical value. Using a shortest path algorithm which works with negative arc costs (in a graph without negative cycles) the more reliable path, p_c, is found in this modified graph and it is the candidate path which

Table 1
Test networks

n		50	100	150	200	250	300	350	400	450	500
$m=2n$	$d(G)$	5-7	7-8	8-9	8-10	8-10	9-10	9-11	9-11	10-11	10-12
	$\bar{d}(u,v)$	4.0	4.6	5.0	5.2	5.4	5.5	5.6	5.7	5.9	5.9
$m=3n$	$d(G)$	4-5	5-6	6-7	6-7	6-7	6-7	7-8	7-8	7-8	7-8
	$\bar{d}(u,v)$	3.4	3.9	4.1	4.3	4.4	4.5	4.6	4.7	4.7	4.8

Explanations: n – the number of nodes, m – the number of arcs, $d(G)$ – the network diameter, $\bar{d}(u,v)$ – the average node distance.

enables D_{k+1} to be obtained from D_k. If no interlacing exists, that is if the candidate path has no arcs with negative costs, which means p_c is disjoint with all the paths in D_k, then p_c is added to the set: $D_{k+1} = D_k \cup \{p_c\}$. If an interlacing exists, the algorithm will evaluate whether the removal of this interlacing and the corresponding change in the set of disjoint paths will lead to an increase in its reliability. Let A be the sub-set of paths in D_k which interlace with the candidate path p_c. Let I be the interlacing between A and p_c (the sub-set of arcs in paths in A the reverse arcs of which belong to p_c). Let B be the set of paths which would result from the removal of that interlacing (for details see [9]), then (by omission the algorithm does not remove the interlacing I):

1) $P_{op} = \Pr(p_c) \times \prod_{l \in I} p_L(l)$,
2) $m_1 = 1 - (1 - P_{op}) \prod_{p_i \in A} (1 - \Pr(p_i))$,
3) $m_2 = 1 - \prod_{p_j \in B} (1 - \Pr(p_j))$,
4) if $m_1 < m_2$ then remove the interlacing I.

According to [9] m_1 captures the reliability of the original path set and the candidate shortest path (p_c) and m_2 the the reliability of the original path set after removing the interlacing. If m_1 is greater than (or equal to) m_2 the interlacing is not removed. In this case, one or more arcs in I are removed from the graph (their costs is set to ∞) and their original costs are stored in a list. This will ensure that the next candidate path will be different from p_c. If $m_1 < m_2$ then the interlacing I is removed and the costs of the arcs in the interlacing recover their original values; the corresponding reverse arcs also recover their original costs. This procedure is repeated until no more candidate paths exist.

In the implemented version of DPSP the modified version of Dijkstra, as described in [1], was used for obtaining the shortest path between a pair of nodes in a network with negative costs. When the interlacing is not removed, all the arcs in the interlacing are removed from the graph.

The DPSP algorithm can be used for obtaining a pair of disjoint paths of high reliability, stopping when D_2 is obtained. This version of DPSP will be designated by DPSP(k = 2). The DPSP(k = 2) algorithm starts by obtaining the most reliable path which is stored in set D_1. If a candidate path p_c is obtained such that no interlacing exists between D_1 and p_c then $D_2 = D_1 \cup \{p_c\}$; if an interlacing I is obtained between D_1 and p_c, the metrics m_1 and m_2 are calculated and if the interlacing is removed D_2 is obtained, otherwise the algorithm proceeds by changing arc costs so that a new candidate path p_c will possibly be obtained.

5. Experimental results

Results are presented for undirected networks, with low connectivity, as indicated in Table 1. These types of features are common in wavelength division multiplexing (WDM) optical networks. For each number of nodes n, ten different networks were randomly generated[1] with the same number of arcs and nodes; the arc reliabilities were randomly generated in $[1-5\cdot10^{-4}, 1-10^{-6}]$ and $[0.8, 0.99]$. The first range of reliability values is adequate for WDM networks and the second for mobile ad hoc networks. Two different network densities were used: $m = 2n$ and $m = 3n$. In order to capture as faithfully as possible the algorithm dependences on the range of link reliabilities, the networks were obtained as follows. Firstly two sets of networks, for $m = 3n$ and $m = 2n$, and $p_L(l) \in [1-5\cdot10^{-4}, 1-10^{-6}]$, were obtained (as already mentioned, for each value of n, 10 networks were randomly generated). Secondly, using the same topological structure of the previous 2×100 networks, two new sets were obtained where the link costs were randomly generated in the range $[0.8, 0.99]$.

Due to the low network connectivity a great variation in CPU time used by OptDP was observed depending on the *s-t* pair. Therefore for each network a pair of disjoint paths was seeked for all $(n \times (n-1))$ node pairs[2] and the average CPU time obtained per pair of disjoint paths for each node pair in the set of all *s-t* pairs with t fixed (for all nodes t). This allows MPS (and therefore OptDP) to re-use the tree of shortest paths from all nodes to t and the ordered set of the network arcs.

[1] The used program for network generation was kindly borrowed from José Luis Santos.

[2] Due to the nature of MPS the cost of obtaining the optimal disjoint pair from s to t and from t to s is not identical.

Finding a pair of disjoint paths was easy, but detecting the optimality condition was sometimes difficult (in the sense that a large number of paths had to generated) but it was always successfully achieved by OptDP. So although OptDP has to generate a significant number of pairs of disjoint paths, in order to verify the optimality stopping condition, it was verified, in the test networks, that the optimal path was one of the first four paths (for networks with $m = 3n$) in 99% of the cases on average (97% was the lowest value obtained for all test networks). In Fig. 2 it

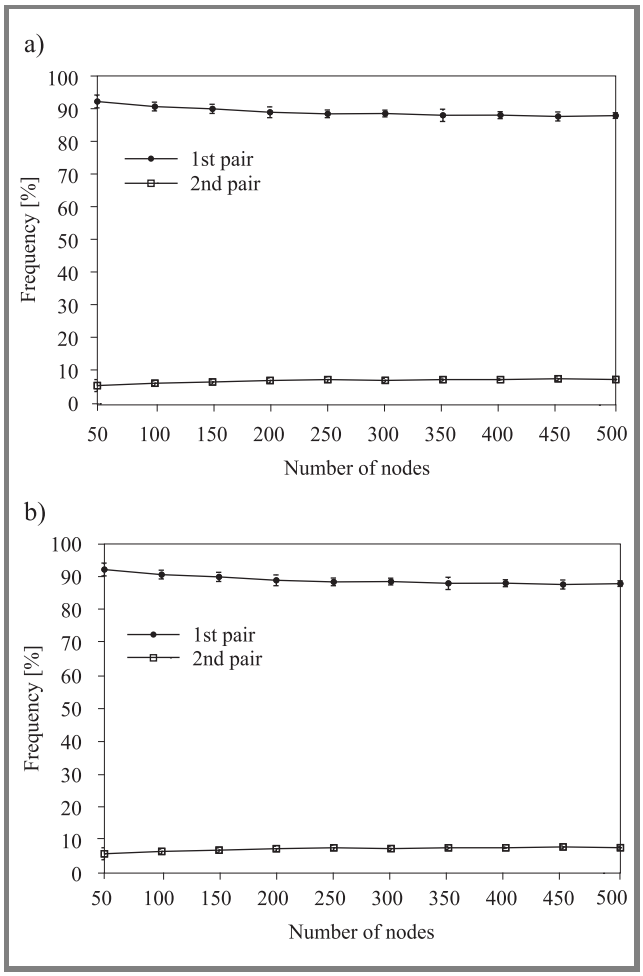

Fig. 2. Frequency of cases, where the optimal pair is either the first or second pair for $m = 3n$. (a) $p_L(l) \in [1-5\cdot10^{-4}, 1-10^{-6}]$; (b) $p_L(l) \in [0.8, 0.99]$.

can be seen that the first and second pairs are the optimal paths most of the times (results for networks with $m = 2n$ are similar and therefore are not presented). On average the third and fourth pairs are the optimal ones for at most 5% of the cases. For all tested cases the first pair, calculated by OptDP, is optimal with a frequency between 85% (for $m = 3n$ and high reliability networks) and 96% (for $m = 2n$ and high reliability networks) of the cases and the second pair is the optimal one with a frequency between 3% and 12% (both upper and lower bounds were obtained in networks with high reliability, with $m = 2n$).

Based on these results, which strongly suggest that the first four pairs represent a great percentage of the total number of optimal pairs, it was decided to implement a "shorter" version of the algorithm, NopDP. This algorithm is similar to OptDP but will sometimes return path pairs the optimality of which was not confirmed. Results will be presented for NopDP, when $F = 5$, which means the algorithm either stops because the 5th pair was obtained, or because it was not necessary to generate more than 4 path pairs before detecting that the optimality condition was true.

The CPU times per node pair are presented in Figs. 3 and 4 – the PC used was a Pentium IV at 2.8 GHz and 500 Mb of RAM. The average values (per network) obtained by OptDP presented some variation and therefore an error bar was added, centred in the average μ of the collected samples (one sample per network) which goes from

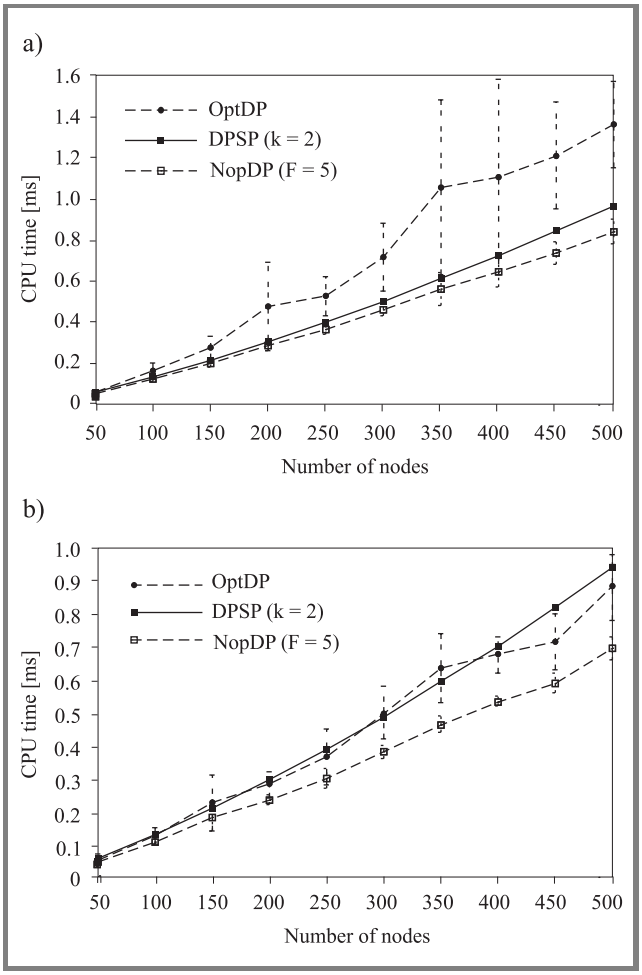

Fig. 3. CPU time per pair of nodes in the networks for $m = 3n$. (a) $p_L(l) \in [1-5\cdot10^{-4}, 1-10^{-6}]$; (b) $p_L(l) \in [0.8, 0.99]$.

$\max(0, \mu-\sigma)$ to $\mu+\sigma$, where σ is the standard deviation of the sample. The purpose of this bar was to show the variability of the results in the case of OptDP. The DPSP(k = 2) algorithm does not present significant variation, therefore no error bar was added in this case. An error bar was

also added to the results of NopDP(F = 5) to show that the variability in this case is rather small, when compared to OptDP.

In Figure 3 CPU times for networks with $m = 3n$ for low and high edge reliability, are presented. Figure 3a shows that OptDP is the less efficient algorithm for $p_L(l) \in [1-5\cdot10^{-4}, 1-10^{-6}]$. However, from Fig. 3b with $p_L(l) \in [1-5\cdot10^{-4}, 1-10^{-6}]$ it is not clear which algorithm is the less efficient, because the line for OptDP is interlaced with the line for DPSP(k = 2). On the other hand NopDP(F = 5) is consistently more efficient than OptDP and DPSP(k = 2).

In the case of networks with $m = 2n$ (see Fig. 4) OptDP continues to be the less efficient approach for $p_L(l) \in [1-5\cdot10^{-4}, 1-10^{-6}]$ and although it improves its relative performance for $p_L(l) \in [0.8, 0.99]$, it remains the less efficient approach (except for smaller networks: $n = 50, 100, 150$). On the other hand NopDP(F = 5) continues to be the best approach, as far has CPU per node pair is concerned.

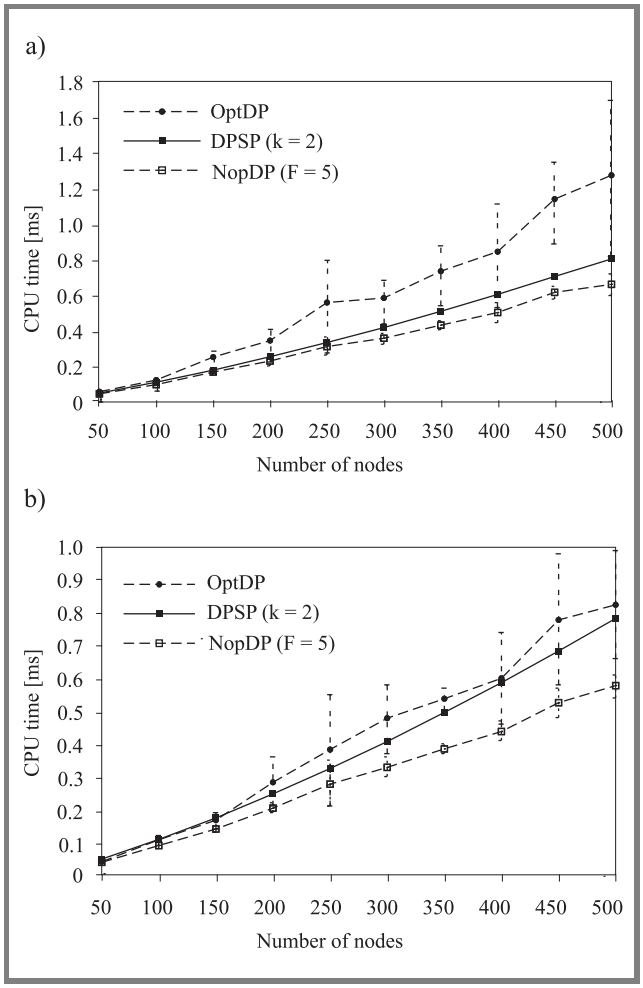

Fig. 4. CPU time per pair of nodes in the networks for $m = 2n$. (a) $p_L(l) \in [1-5\cdot10^{-4}, 1-10^{-6}]$; (b) $p_L(l) \in [0.8, 0.99]$.

It should be noted that when DPSP(k = 2) (or DPSP) is used, the algorithm has no way of knowing whether the optimal pair (set) of disjoint paths has been obtained. NopDP on the other hand knows that some of its solutions are indeed optimal (the remaining might be optimal, but NopDP did not run long enough to confirm their optimality or they may be sub-optimal). In Table 2, the average frequency of NopDP(F = 5) termination because an optimal solution was found, is presented. The minimal and maximal values obtained in all experiments were 68.3% and 98.9%.

Table 2

NopDP(F = 5) exits with the detection of the optimal condition

n	$m = 3n$		$m = 2n$	
	A	B	A	B
50	93.8	95.5	91.6	95.0
100	88.8	92.6	89.3	91.0
150	87.3	91.2	85.0	90.8
200	84.8	91.5	85.8	88.6
250	85.1	91.5	83.2	88.5
300	83.6	90.1	84.2	86.9
350	81.6	88.8	83.6	87.1
400	83.3	89.8	83.7	88.8
450	82.8	91.2	81.2	86.3
500	83.3	89.3	82.7	87.6

Explanations: A – high reliability, $p_L(l) \in [1-5\cdot10^{-4}, 1-10^{-6}]$; B – low reliability, $p_L(l) \in [0.8, 0.99]$.

Finally to confirm which of the algorithms, DPSP(k = 2) or NopDP(F = 5) did obtain the greater number of optimal solutions, an analysis was made (based on the optimal reliability value obtained by using OptDP) of the values returned by DPSP(k = 2) and NopDP(F = 5) (using 12 significant digits). Observing the results in Table 3 the first observation is that DPSP obtains sub-optimal solutions in 1.1%–2.9% of the cases while NopDP only fails in less than 1% of the cases for high reliability networks and in less than 0.5% of the cases for low reliability networks. The average values of the relative differences of the reliability of the obtained sub-optimal solution with respect to the optimal one are shown in Tables 3 and 4. These differences are in the same range for both algorithms, with a slight increase for NopDP(F = 5) in the case of less reliable networks.

Results for networks with $m = 2n$, regarding the frequency of sub-optimal solutions, are presented in Table 4. For DPSP(k = 2) this frequency is in the range 2.0%–3.2% while NopDP(F = 5) continues to present values under 1% for high reliability networks and under 0.5% for low reliability networks (with one exception: 0.51%). The relative errors of reliability (for sub-optimal node pairs) for $m = 2n$ are greater than for $m = 3n$, for both DPSP(k = 2) and NopDP(F = 5). The relation between this relative error is around 3 when comparing NopDP(F = 5) and DPSP(k = 2) and the relative frequency of sub-optimal solutions is around 9 for low reliability networks and around 4 for

Table 3

Frequency of non optimal pairs obtained with DPSP(k = 2) and NopDP(F = 5), and corresponding average reliability relative error (of non optimal pairs) for $m = 3n$

	$m = 3n$ $p_L(l) \in [1-5\cdot10^{-4}, 1-10^{-6}]$					$m = 3n$ $p_L(l) \in [0.8, 0.99]$			
	DPSP(k = 2)		NopDP(F = 5)			DPSP(k = 2)		NopDP(F = 5)	
n	[%]	ΔPr	[%]	ΔPr	n	[%]	ΔPr	[%]	ΔPr
50	1.91	$1.6\cdot10^{-8}$	0.31	$4.9\cdot10^{-8}$	50	1.13	$1.9\cdot10^{-3}$	0.05	$7.0\cdot10^{-3}$
100	2.34	$1.6\cdot10^{-8}$	0.35	$4.7\cdot10^{-8}$	100	1.84	$2.2\cdot10^{-3}$	0.18	$5.8\cdot10^{-3}$
150	2.52	$1.6\cdot10^{-8}$	0.42	$4.7\cdot10^{-8}$	150	2.09	$2.3\cdot10^{-3}$	0.24	$5.9\cdot10^{-3}$
200	2.74	$1.7\cdot10^{-8}$	0.68	$5.1\cdot10^{-8}$	200	2.10	$2.3\cdot10^{-3}$	0.27	$6.1\cdot10^{-3}$
250	2.78	$1.6\cdot10^{-8}$	0.61	$4.7\cdot10^{-8}$	250	2.12	$2.4\cdot10^{-3}$	0.25	$6.7\cdot10^{-3}$
300	2.75	$1.7\cdot10^{-8}$	0.82	$5.0\cdot10^{-8}$	300	2.17	$2.5\cdot10^{-3}$	0.32	$7.1\cdot10^{-3}$
350	2.86	$1.7\cdot10^{-8}$	0.94	$4.6\cdot10^{-8}$	350	2.24	$2.5\cdot10^{-3}$	0.41	$7.1\cdot10^{-3}$
400	2.73	$1.7\cdot10^{-8}$	0.88	$4.9\cdot10^{-8}$	400	2.23	$2.7\cdot10^{-3}$	0.35	$7.3\cdot10^{-3}$
450	2.93	$1.8\cdot10^{-8}$	0.87	$5.2\cdot10^{-8}$	450	2.03	$2.6\cdot10^{-3}$	0.29	$7.0\cdot10^{-3}$
500	2.69	$1.8\cdot10^{-8}$	0.85	$5.6\cdot10^{-8}$	500	2.21	$2.6\cdot10^{-3}$	0.39	$7.0\cdot10^{-3}$

Table 4

Frequency of non optimal pairs obtained with DPSP(k = 2) and NopDP(F = 5), and corresponding average reliability relative error (of non optimal pairs) for $m = 2n$

	$m = 2n$ $p_L(l) \in [1-5\cdot10^{-4}, 1-10^{-6}]$					$m = 2n$ $p_L(l) \in [0.8, 0.99]$			
	DPSP(k = 2)		NopDP(F = 5)			DPSP(k = 2)		NopDP(F = 5)	
n	[%]	ΔPr	[%]	ΔPr	n	[%]	ΔPr	[%]	ΔPr
50	2.38	$4.9\cdot10^{-8}$	0.29	$8.7\cdot10^{-8}$	50	1.99	$4.4\cdot10^{-3}$	0.10	$9.8\cdot10^{-3}$
100	2.48	$4.3\cdot10^{-8}$	0.40	$1.35\cdot10^{-7}$	100	2.43	$5.9\cdot10^{-3}$	0.15	$1.47\cdot10^{-2}$
150	2.75	$4.6\cdot10^{-8}$	0.62	$1.22\cdot10^{-7}$	150	2.50	$5.1\cdot10^{-3}$	0.23	$1.31\cdot10^{-2}$
200	2.78	$4.7\cdot10^{-8}$	0.60	$1.33\cdot10^{-7}$	200	2.70	$5.3\cdot10^{-3}$	0.37	$1.38\cdot10^{-2}$
250	3.00	$5.1\cdot10^{-8}$	0.77	$1.46\cdot10^{-7}$	250	2.82	$5.4\cdot10^{-3}$	0.39	$1.24\cdot10^{-2}$
300	2.77	$5.0\cdot10^{-8}$	0.73	$1.47\cdot10^{-7}$	300	2.90	$6.0\cdot10^{-3}$	0.46	$1.38\cdot10^{-2}$
350	2.95	$5.1\cdot10^{-8}$	0.76	$1.56\cdot10^{-7}$	350	2.87	$5.9\cdot10^{-3}$	0.46	$1.45\cdot10^{-2}$
400	2.83	$4.8\cdot10^{-8}$	0.80	$1.44\cdot10^{-7}$	400	2.72	$6.0\cdot10^{-3}$	0.38	$1.36\cdot10^{-2}$
450	3.17	$5.3\cdot10^{-8}$	0.93	$1.48\cdot10^{-7}$	450	2.88	$5.9\cdot10^{-3}$	0.51	$1.41\cdot10^{-2}$
500	2.90	$5.3\cdot10^{-8}$	0.85	$1.56\cdot10^{-7}$	500	2.83	$6.1\cdot10^{-3}$	0.43	$1.46\cdot10^{-2}$

high reliability networks when comparing DPSP(k = 2)) and NopDP(F = 5). Therefore the results for NopDP(F = 5) are significantly more favourable than for DPSP(k = 2).

6. Conclusions

A new algorithm, OptDP, for obtaining the most reliable pair of edge disjoint paths, and a "shorter" variant, NopDP, which does not always guarantee the generated path pair is optimal, have been proposed. Algorithm DPSP was also reviewed and a "truncated" version DPSP(k = 2) was used for obtaining a pair of disjoint paths with high reliability.

The performances of OptDP, NopDP(F = 5) and DPSP(k = 2) were evaluated through numerous experiments for randomly generated networks, with different connectivities. For each value of connectivity two sets of networks were generated, one with low reliability and the other with high reliability.

These experiments enabled the good performance of OptDP to be put in evidence for less reliable networks when compared with DPSP(k = 2). In particular NopDP(F = 5) was shown to be a good compromise between precision (number of optimal solutions obtained) and required CPU time.

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Paper

Extensions of the minimum labelling spanning tree problem

Raffaele Cerulli, Andreas Fink, Monica Gentili, and Stefan Voß

Abstract— In this paper we propose some extensions of the minimum labelling spanning tree problem. The main focus is on the minimum labelling Steiner tree problem: given a graph *G* with a color (label) assigned to each edge, and a subset *Q* of the nodes of *G* (basic vertices), we look for a connected subgraph of *G* with the minimum number of different colors covering all the basic vertices. The problem has several applications in telecommunication networks, electric networks, multimodal transportation networks, among others, where one aims to ensure connectivity by means of homogeneous connections. Numerical results for several metaheuristics to solve the problem are presented.

Keywords— network design, metaheuristics, spanning trees, labelling trees, Steiner tree problem.

1. Introduction

Many real-world problems can be modelled by means of graphs where a label or a weight is assigned to each edge and the aim is to optimize a certain function of these weights. In particular, one can think of problems where the objective is to find homogeneous subgraphs (respecting certain connectivity constraints) of the original graph. This is the case, e.g., for telecommunication networks (and, more generally, any type of communication networks) that are managed by different and competing companies. The aim of each company is to ensure the service to each terminal node of the network by minimizing the cost (i.e., by minimizing the use of connections managed by other companies).

This kind of problem can be modelled as follows. The telecommunication network is represented by a graph $G = (V,E)$ where with each edge $e \in E$ is assigned a set of colors L_e and each color denotes a different company that manages the edge. The aim of each company is to define a spanning tree of G that uses the minimum number of colors. When the graph represents a transportation network and the colors, assigned to each edge, represent different modes of transportation, then looking for a path that uses the minimum number of colors from a given source s to a given destination t means to look for a path connecting s and t using the minimum number of different modes of transportation.

We focus on the *minimum labelling Steiner tree problem* (MLSteiner): given a graph $G = (V,E)$, with a label (color) assigned to each edge and a subset $Q \subseteq V$ of nodes of G (basic vertices or nodes), we look for an acyclic connected subgraph of G spanning all basic nodes and using the minimum number of different colors. This problem is an extension of the *minimum labelling spanning tree problem* (MLST): given a graph G with a label (color) assigned to each edge we look for a spanning tree of G with the minimum number of different colors.

In this paper, first we review the earlier results existing in the literature to solve the MLST. Then we discuss how these approaches can be easily extended to efficiently solve the MLSteiner and present a comprehensive study of experimental results.

The sequel of the paper is organized as follows. Section 2 summarizes existing approaches for the MLST as well as some important references on the Steiner problem in graphs. In Section 3 we sketch some extensions of the MLST related to the MLSteiner which is the focus of this study. Section 4 presents our modifications of the solution approaches for the MLST to solve the MLSteiner. In Section 5 we present experimental results, and, finally, Section 6 gives some further research options.

2. Literature review

2.1. Earlier approaches to solve the MLST

The MLST was initially addressed by Broersma and Li [2]. They proved, on the one hand, that the MLST is $\mathcal{NP}$-hard by reduction from the minimum dominating set problem, and, on the other hand, that the "opposite" problem of looking for a spanning tree with the maximum number of colors is polynomially solvable. Independently, Chang and Leu [6] provided a different $\mathcal{NP}$-hardness proof of the problem by reduction from the set covering problem. They also developed two heuristics to determine feasible solutions of the problem and tested the performance of these heuristics by comparison with the results of an exact approach based on an A^* algorithm.

Krumke and Wirth [14] formulated an approximation algorithm (in the sequel referred to as maximum vertex covering algorithm – MVCA) with logarithmic performance guarantee and showed also that the problem cannot be approximated within a constant factor. Wan *et al*. [19] provided a better analysis of the greedy algorithm given in [14] by showing that its worst case performance ratio is at most $\ln(n-1)+1$ where n denotes the number of nodes of the given graph, i.e., $n = |V|$. Recently, Xiong *et al*. [23] obtained the better bound $1+\ln b$ where each color appears at most b times. Moreover, Xiong *et al*. [21] proposed a genetic algorithm to solve the MLST and provided some experimental results.

In [5] we presented several metaheuristic approaches to solve the MLST (namely, simulated annealing, reactive tabu search, the pilot method and variable neighborhood search) and compared them with the results provided by the MVCA heuristic presented in [14, 21]. Recently, a modification of our pilot method combined with the genetic algorithm of Xiong *et al*. [21] was shown to be effective by [22].

A variant of the problem has been studied by Brüggemann *et al*. [3], where the MLST with bounded color classes has been addressed. In this variant, each color of the graph is assumed to appear at most r times. This special case of the MLST is polynomially solvable for $r = 2$, and $\mathcal{NP}$-hard and APX-complete for $r \geq 3$. Local search algorithms for this variant, that are allowed to switch up to k of the colors used in a feasible solution have been studied, too. For $k = 2$, the authors showed that any local optimum yields an $\frac{(r+1)}{2}$-approximation of the global optimum, and this bound is best possible. For every $k \geq 3$, there exist instances for which some local optimum is a factor of $\frac{r}{2}$ away from the global optimum.

2.2. The Steiner tree problem

Consider an undirected connected graph $G = (V,E)$ with node set V, edge set E, and nonnegative weights associated with the edges. Given a set $Q \subseteq V$ of specified vertices (called terminals or basic vertices) *Steiner's problem* in graphs (SP) is to find a minimum cost subgraph of G such that there exists a path in the subgraph between every pair of basic vertices. In order to achieve this minimum cost subgraph additional vertices from the set $S := V \setminus Q$, called Steiner vertices, may be included. Since all edge weights are assumed to be nonnegative, there is an optimal solution which is a tree, called Steiner tree.

Correspondingly, *Steiner's problem in directed* graphs (SPD) is to find a minimum cost directed subgraph of a given graph that contains a directed path between a root node and every basic vertex. Applications of the SP and the SPD are frequently found in many problems related to network design and telecommunications. Beyond that, SP and SPD have equal importance also for the layout of connection structures in networks as, e.g., in topological network design, location science and VLSI (very large scale integrated circuits) design.

The SP is a well-studied problem and there is a wealth of excellent reference providing information on Steiner problems, such as [11]. Additional surveys on quite broad aspects of Steiner tree problems are provided by [10, 16, 20] as well as, most recently, [17].

3. Extension of the MLST

Steiner tree problems refer to important problem classes in graphs. The SP may be called one of the most important combinatorial optimization problems. Modifications and generalizations of Steiner tree problems will certainly arise and become a core focus of research and telecommunications applications including additional online optimization problems as well as stochastic optimization approaches. In that sense we have defined the MLSteiner as an extension of both, the MLST as well as the SP. But in this section we go beyond this.

Examples for possible generalizations may include, e.g., a weighted labelling Steiner tree problem with budget constraints. Here we are given a graph G with a label (color) assigned to each edge and we look for a spanning tree with respect to a given subset Q of the nodes of G with the minimum number of different colors. Furthermore, one may incorporate some weights on the edges and define a budget constraint on the sum of the weights of included edges while still minimizing the number of labels or more versatile capacitated Steiner tree problems.

Other ideas on generalizations of the MLST refer to certain ring network design problems with or without budget constraints (see, e.g., [8]) that may be formulated in terms of minimizing the number of labels once they are to be defined and considered. While these generalizations may prove to be important, subsequently we focus on the MLSteiner.

4. Different metaheuristic approaches to solve the MLSteiner

We aim for a Steiner tree which connects all required or basic nodes with a minimum number of colors/labels. Within the steps of the search process only those edges that are colored according to the currently activated colors may be used. For describing different metaheuristic approaches for the MLSteiner we heavily rely on our previous approaches for the MLST described in [5]. We have adapted our code for the MLST so that the algorithm checks whether the resulting subgraph (restricted to the edges with actually used or activated colors) connects all required nodes. If there are disconnections, large penalty values are added to the objective function so that the search process is directed towards feasibility.

Before going into detail let us introduce some notation. Given an undirected graph $G = (V,Q,E)$ with V being the set of nodes, $Q \subseteq V$ the subset of basic nodes and E denoting the set of edges, let c_e be the color (label) associated with edge $e \in E$ and $L = \{c_1, c_2, \ldots, c_l\}$ be the set of all colors. We denote by $C(F) = \bigcup_{e \in F} c_e$ the set of colors assigned with edges in $F \subseteq E$. Any subgraph T of G can be represented by the set of its colors $C(T)$. Given a set of colors C, we define by $V(C)$ the subset of nodes of G covered by the edge set defined by C, i.e., $V(C) = \{i \in V : e \in E$ is incident to i and $c_e \in C\}$. A set of colors C is *feasible* for the MLSteiner if and only if the corresponding set of edges defines a connected subgraph $G_C = (V', E')$ that spans all the basic nodes of G, i.e., $V' \cap Q = Q$. (Moreover, we note in passing that we assume $|L_e| = 1$ throughout the remainder of this paper. That is, exactly one color is assigned to each edge.).

4.1. Greedy

The algorithm starts with an empty set of edges. Then, it iteratively selects one color among the unused ones and inserts all edges of that color in the graph until all the basic nodes are connected. At each iteration it tests all the unused colors and chooses a color in that way that the decrease in the number of Steiner connected components is as large as possible, where we define a Steiner connected component as a connected component $H = (V', E')$ of the graph that contains at least one basic node, e.g., $V' \cap Q \neq \emptyset$. The proposed algorithm is illustrated below.

Algorithm: The greedy heuristic

Let $C = \emptyset$ be the set of used colors.

Repeat
  let H be the subgraph of G restricted to edges with colors from C;
  let H' be the subgraph of H restricted to the Steiner connected components of H;
  for all $c_i \in L \setminus C$ **do**
    determine the number of Steiner connected components when inserting all edges with color c_i in H;
  end for
  choose color c_i with the smallest resulting number of Steiner connected components and do: $C = C \cup \{c_i\}$;
until H' is connected.

The greedy strategy we adopt differs from the MVCA heuristic since it carries out operations on the Steiner connected components of subgraph H, while MVCA considers all the connected components of such a graph.

The running time of the proposed greedy strategy is $O(l^2 n)$, where l is the total number of different colors in G. Indeed, the *repeat* loop will take $O(l)$ steps and we have $O(ln)$ to carry out the *for*-loop.

Since the MLST is a special case of the MLSteiner, then, by applying the same reasoning introduced in [19], we can derive the following approximation result.

Theorem 1: Given any MLSteiner instance with n nodes and q basic nodes ($q < n$, $n > 1$), the greedy algorithm provides an $(\ln(q-1)+1)$-approximation.

4.2. Variable neighborhood search

Variable neighborhood search (VNS) goes back to Mladenović and Hansen [15]. The underlying idea of VNS is to generalize the classical local search based approaches by considering a multi-neighborhood structure, i.e., a set of pre-selected neighborhood structures $\mathcal{N} = \{N_1, N_2, \ldots, N_s\}$ such that $N_j(C)$, $j = 1, 2, \ldots, s$ is the set of solutions in the jth neighborhood of C. The basic VNS algorithm, applied to solve the MLSteiner, is described below.

Algorithm: The basic VNS algorithm

Step 1. Consider an initial feasible solution $C \subseteq L$ and set $k \leftarrow 1$.

Step 2. Generate at random a solution $C' \in N_k(C)$.

Step 3. Apply a local search algorithm, starting from the initial solution C', to obtain a local optimum C''.

Step 4. If $|C''| < |C|$ then: $C \leftarrow C''$ and set $k \leftarrow 1$ otherwise $k \leftarrow k+1$.

Step 5. If $k \leq k_{\max}$ then go to Step 1, else Stop.

We implemented VNS by using three different neighborhood structures, in order to check whether one neighborhood is better than the other. In particular, given a feasible color set C, we consider the following neighborhood structures:

- ***k – switch neighborhood*** $N_k^1(C)$
  A set $C' \in N_k^1(C)$ if and only if we can get the color set C' from the color set C by removing up to k colors from C and adding up to k new colors. That is, $N_k^1(C) = \{C' \subseteq L : |C' \backslash C| \leq k \text{ and } |C \backslash C'| \leq k\}$.

- ***k – covering neighborhood*** $N_k^2(C)$
  A set $C' \in N_k^2(C)$ if and only if the common colors between C and C' cover at least k basic nodes. That is, $N_k^2(C) = \{C' \subseteq L : |V(C' \cap C) \cap Q| \geq k$ and $|C'| \leq |C|\}$.

- ***k – mixed neighborhood*** $N_k^3(C)$
  A set $C' \in N_k^3(C)$ if and only if C' contains exactly $|C| - k$ colors in common with C and all the remaining different colors cover a greater number of basic vertices. That is, $N_k^3(C) = \{C' \subseteq L : |C \backslash C'| = k$, $|C' \backslash C| \leq k$ and $|V(C \backslash C') \cap Q| \leq |V(C' \backslash C) \cap Q|\}$.

For each of the neighborhood structures described above, the procedure starts from an initial feasible solution C provided by the greedy algorithm described in Subsection 4.1. At each generic iteration the VNS:

– selects at random a feasible solution C' in the neighborhood $N_k^i(C)$;

– applies a local exchange strategy that, for a maximum number $h_{\max}$ of iterations, tries to decrease the size of C' to obtain a possible better solution C'' by removing π labels and adding up to π new labels, where $\pi = 2, 3, \ldots, |C'|$;

– defines the new neighborhood to be explored in the next iteration.

In our implementation of VNS, we let parameter $k_{\max}$ vary during the execution, that is $k_{\max} = \min\{|C|, \frac{l}{4}\}$, where $|C|$ is the "size" of the current feasible solution whose neighborhood is being explored.
In the sequel we refer to the implementations of VNS using $N_k^1()$, $N_k^2()$ and $N_k^3()$ as VNS1, VNS2 and VNS3, respectively.

4.3. Simulated annealing

Simulated annealing (SA) extends basic local search by allowing moves to worse solutions [13]. The basic concept of SA is the following: starting from an initial solution (in our implementation from an empty set of activated colors as in the greedy heuristic), successively, a candidate move is randomly selected. This move is accepted if it leads to a solution with a better objective function value than the current solution, otherwise the move is accepted with a probability that depends on the deterioration Δ of the objective function value. The acceptance probability is computed according to the Boltzmann function as $e^{-\Delta/T}$, using a temperature T as control parameter.
Following [12], the value of T is initially high, which allows many worse moves to be accepted, and is gradually reduced through multiplication by a parameter *cooling factor* according to a geometric cooling schedule. Given a parameter *size factor*, *size factor* $\times\, l$ candidate moves are tested (note that l denotes the neighborhood size) before the temperature is reduced. The starting temperature is determined as follows: given a parameter *initial acceptance fraction* and based on an abbreviated trial run, the starting temperature is set so that the fraction of accepted moves is approximately *initial acceptance fraction*. A further parameter, *frozen acceptance fraction* is used to decide whether the annealing process is *frozen* and should be terminated. Every time a temperature is completed with less than *frozen acceptance fraction* of the candidate moves accepted, a counter is increased by one, while this counter is re-set to 0 each time a new best solution has been obtained. The whole procedure is terminated when this counter reaches a parameter *frozen limit*. For our implementation we follow the parameter setting of [12], which was reported to be robust for various problems. That is, we use $\alpha = 0.95$, *initial acceptance fraction* = 0.4, *frozen acceptance fraction* = 0.02, *size factor* = 16 and *frozen limit* = 5.

4.4. Reactive tabu search

The basic paradigm of *tabu search* (TS) is to use information (in the sense of an adaptive memory) about the search history to guide local search approaches to overcome local optimality (see [9] for a survey on tabu search). In general, this is done by a dynamic transformation of the local neighborhood. Based on some sort of memory certain moves may be forbidden, they are defined tabu (and appropriate move attributes such as a certain index indicating a specific color put into a list, called tabu list). As for SA, the search may imply acceptance of deteriorating moves when no improving moves exist or all improving moves of the current neighborhood are set tabu. At each iteration a best admissible neighbor may be selected. A neighbor, respectively a corresponding move, is called admissible, if it is not tabu.
Reactive TS (RTS) aims at the automatic adaptation of the tabu list length [1]. The idea is to increase the tabu list length when the tabu memory indicates that the search is revisiting formerly traversed solutions. A possible specification is the following. Starting with a tabu list length s of 1, it is increased to $\min\{\max\{s+2, s\times 1.2\}, b_u\}$ every time a solution has been repeated, taking into account an appropriate upper bound b_u (to guarantee at least one admissible move). If there is no repetition for some iterations, we decrease it to $\max\{\min\{s-2, s/1.2\}, 1\}$. To accomplish the detection of a repetition of a solution, one may apply a trajectory based memory using hash codes.
For RTS, it is appropriate to include means for diversifying moves whenever the tabu memory indicates that one is trapped in a certain basin of attraction. As a trigger mechanism one may use, e.g., the combination of at least three solutions each having been traversed three times. A very simple escape strategy is to perform randomly a number of moves (depending on the average of the number of iterations between solution repetitions). For our implementation of RTS we consider as initial solution (as for the SA and the greedy heuristic) an empty set of activated colors. As termination criterion we consider a given time limit.

4.5. Pilot method

Using a greedy construction heuristic such as the MVCA as a building block or application process, the pilot method is a metaheuristic with the primary idea of performing repetition exploiting the application process as a look ahead mechanism [7, 18]. In each iteration (of the pilot method) one tentatively determines for every possible local choice (i.e., move to a neighbor of the current solution, called master solution) a look ahead or pilot solution, recording the best results in order to extend at the end of the iteration the master solution with the corresponding move. This strategy may be applied by successively performing, e.g., a construction heuristic for all possible local choices (i.e., starting a new solution from each incomplete solution that can result from the inclusion of any not yet included element into the current incomplete solution).
We apply the pilot method in connection with a greedy local search strategy operating on a solution space that includes incomplete (infeasible) solutions and a neighborhood that considers the addition of colors (see MVCA). We take into account infeasibilities by adding appropriate penalty values. The pilot method successively chooses the best local move (regarding the additional activation of one color) by evaluating such neighbors with a steepest descent until a local optimum, and with that a feasible solution, is obtained. (Note that as for the MVCA, at the end it may be beneficial to greedily drop colors while retaining feasibility.)

5. Experimental results

In this section we report some of our computational results. We considered different groups of instances in order to evaluate how the performance of the algorithms is influenced by both

– the number and distribution of the basic nodes;

– the distribution of the labels on the edges.

In particular, we defined different scenarios based on different parameter settings: n – number of nodes of the graph; l – total number of colors assigned to the graph; m – total number of edges of the graph computed by $m = \frac{d(n-1)n}{2}$, where d is a measure of density of the graph, and, q – the number of basic nodes of the graph. Parameter settings are: $n = 50, 100$, $l = 0.25n, 0.5n, n, 1.25n$, $d = 0.2, 0.5, 0.8$ and $q = 0.2n, 0.4n$, for a total of 48 different scenarios. For each scenario we generated ten different instances. All the generated data are available upon request from the authors.

Our results are reported in Tables 1–4. In each table the first three columns show the parameters characterizing the different scenarios (n, l, d, while the values of q determine the different tables). The remaining columns give the results of the greedy heuristic and of our metaheuristics: variable neighborhood search (VNS1, VNS2 and VNS3), simulated annealing, reactive tabu search and the pilot method, respectively. For the results reported on the greedy heuristic we note that we have implemented the idea to try at the end to greedily drop colors while retaining feasibility. In two of all 480 cases this reduced the objective value by 1.

In general we can say that the pilot method behaves best with respect to solution quality. The SA is usually outperformed by all other metaheuristics and RTS overall behaves a bit better than the VNS for VNS2 and VNS3. The RTS and VNS1 are somewhat incomparable as there does not seem to be a clear picture which method behaves best with respect to solution quality. Among the VNS implementations the first version seems to provide better results than the other two implementations.

Closer inspection of the results reveals a few probably unusual behaviours. First of all, we encountered a considerable role of how ties are broken. Assuming that a tie is broken unfavorably and one encounters an increase or decrease of the objective function by 1, the percentage deviation is affected considerably as most problem instances tend to have very small objective function values (considering the pilot method, only in 6 of the cases with $q = 0.2n$

Table 1
Computational results for $n = 50$ and $q = 0.2n$

n	l	d	Greedy	VNS1	VNS2	VNS3	SA	RTS	Pilot
50	12.5	0.8	1.0	1.0	1.0	1.0	1.0	1.0	1.0
50	12.5	0.5	1.1	1.1	1.1	1.1	1.1	1.1	1.1
50	12.5	0.2	2.1	2.0	2.0	2.1	2.0	2.0	2.0
50	25	0.8	1.2	1.2	1.2	1.2	1.2	1.2	1.2
50	25	0.5	1.9	1.9	1.9	1.9	1.9	1.9	1.9
50	25	0.2	2.9	2.9	2.9	2.9	2.9	2.9	2.9
50	50	0.8	2.0	2.0	2.0	2.0	2.0	2.0	2.0
50	50	0.5	2.8	2.8	2.6	2.7	2.7	2.8	2.6
50	50	0.2	4.0	3.9	3.9	4.0	4.0	4.0	3.9
50	62.5	0.8	2.3	2.0	2.0	2.0	2.2	2.2	2.0
50	62.5	0.5	3.1	2.8	2.9	3.0	3.0	3.0	2.8
50	62.5	0.2	4.4	4.3	4.4	4.5	4.4	4.4	4.3
Sum			28.8	27.9	27.9	28.4	28.4	28.5	27.7

Table 2
Computational results for $n = 100$ and $q = 0.2n$

n	l	d	Greedy	VNS1	VNS2	VNS3	SA	RTS	Pilot
100	25	0.8	1.0	1.0	1.0	1.0	1.0	1.0	1.0
100	25	0.5	1.5	1.5	1.5	1.5	1.5	1.5	1.5
100	25	0.2	2.2	2.1	2.1	2.1	2.1	2.1	2.1
100	50	0.8	1.9	1.9	1.9	1.9	1.9	1.9	1.9
100	50	0.5	2.0	2.0	2.0	2.0	2.0	2.0	2.0
100	50	0.2	3.3	3.2	3.2	3.3	3.5	3.3	3.2
100	100	0.8	2.4	2.1	2.2	2.2	2.5	2.4	2.0
100	100	0.5	3.0	3.0	3.0	3.1	3.4	3.0	3.0
100	100	0.2	4.9	4.6	5.1	5.0	5.2	4.8	4.6
100	125	0.8	2.8	2.8	2.8	2.8	3.0	2.8	2.8
100	125	0.5	3.5	3.4	3.5	3.6	4.0	3.5	3.4
100	125	0.2	5.7	5.3	5.9	5.7	6.2	5.6	5.4
Sum			34.2	32.9	34.2	34.2	36.3	33.9	32.9

Table 3
Computational results for $n = 50$ and $q = 0.4n$

n	l	d	Greedy	VNS1	VNS2	VNS3	SA	RTS	Pilot
50	12.5	0.8	1.0	1.0	1.0	1.0	1.0	1.0	1.0
50	12.5	0.5	1.4	1.4	1.4	1.4	1.4	1.4	1.4
50	12.5	0.2	2.6	2.6	2.6	2.6	2.6	2.6	2.6
50	25	0.8	2.0	2.0	2.0	2.0	2.0	2.0	2.0
50	25	0.5	2.0	2.0	2.0	2.0	2.0	2.0	2.0
50	25	0.2	4.3	3.9	3.9	3.9	3.9	4.0	3.9
50	50	0.8	2.4	2.2	2.3	2.2	2.5	2.4	2.1
50	50	0.5	3.2	3.0	3.0	3.0	3.1	3.0	3.0
50	50	0.2	5.9	5.5	5.9	5.8	5.8	5.8	5.3
50	62.5	0.8	2.7	2.7	2.7	2.8	2.9	2.7	2.6
50	62.5	0.5	3.6	3.3	3.2	3.5	3.7	3.6	3.2
50	62.5	0.2	6.2	6.6	6.7	6.5	6.4	6.2	6.0
Sum			37.3	36.2	36.7	36.7	37.3	36.7	35.1

Table 4
Computational results for $n = 100$ and $q = 0.4n$

n	l	d	Greedy	VNS1	VNS2	VNS3	SA	RTS	Pilot
100	25	0.8	1.0	1.0	1.0	1.0	1.0	1.0	1.0
100	25	0.5	1.9	1.9	1.9	1.9	1.9	1.9	1.9
100	25	0.2	3.1	3.0	3.0	3.0	3.0	3.0	3.0
100	50	0.8	2.0	2.0	2.0	2.0	2.0	2.0	2.0
100	50	0.5	2.4	2.2	2.2	2.2	2.4	2.3	2.2
100	50	0.2	4.8	4.4	4.5	4.8	4.4	4.6	4.3
100	100	0.8	3.0	3.0	3.0	3.0	3.0	3.0	3.0
100	100	0.5	3.9	3.6	3.6	3.6	3.8	3.9	3.6
100	100	0.2	6.6	6.9	7.6	7.0	7.4	6.6	6.5
100	125	0.8	3.0	3.0	3.0	3.0	3.4	3.0	3.0
100	125	0.5	4.1	4.1	4.2	4.3	4.9	4.1	4.0
100	125	0.2	7.6	8.1	9.2	8.2	8.1	7.6	7.0
Sum			43.4	43.2	45.2	44.0	45.3	43.0	41.5

and 32 of the cases with $q = 0.4n$ the objective turned out to be larger than 5, i.e., 6, 7, or 8). In this sense, a random neighbor selection within the SA and the VNS implementations may already lead to an unfavorable objective function value that is difficult to be overcome which explains the few cases where the SA results are even worse than those of the greedy approach.

For RTS the approach first mimics the behaviour of a steepest descent like the greedy heuristic. Based on the way infeasibilities are penalized, the method usually stays within the feasible region so that the method may be caught within some basin of attraction related to the first local optimum found. That is, the RTS does not really work as expected, since in most cases (about 95%) the best results have been obtained within the first second of the computation. After that the method did not find improvements quite often even if they would have been possible.

The computations for our methods have been made on a Pentium IV 1.8 GHz. The termination criteria for the different methods follow the descriptions given above. The RTS is terminated after a time limit of 10 seconds for instances with $n = 50$ and after 40 seconds for $n = 100$. In general the computational times are moderately increasing for decreasing values of d, they are also increasing for increasing values of q and l, and they are considerably increasing for an increasing number of nodes. While RTS has a given time limit, computational times for the other methods tend to be below those numbers for larger values of d for all methods while they become slightly larger than those for RTS in case of the VNS implementations for $d = 0.2$. Computational times for the VNS implementations mainly depend on graph density: the more sparse the graph the larger the times. The computational times for the pilot method mainly depend on the number of nodes and the value of l. If l increases then the times for the pilot method may easily become considerably larger than those of SA and RTS but also larger than those of the VNS. Detailed computational times are reported in Table 5 for the largest instances to get a feeling about the general behaviour of our methods.

Table 5
Computational times [s] for $n = 100$ and $q = 0.4n$

n	l	d	VNS1	VNS2	VNS3	SA	RTS	Pilot
100	25	0.8	0.4	0.2	4.6	11.1	40.0	1.0
100	25	0.5	0.6	0.6	2.8	9.7	40.0	1.2
100	25	0.2	19.2	23.4	46.0	6.6	40.0	1.4
100	50	0.8	1.1	1.2	7.0	22.1	40.1	6.5
100	50	0.5	12.0	5.2	28.2	17.4	40.0	6.1
100	50	0.2	49.9	81.6	77.0	12.3	40.0	9.1
100	100	0.8	15.3	10.9	57.8	43.5	40.1	43.3
100	100	0.5	44.8	101.9	50.3	33.7	40.0	43.0
100	100	0.2	72.0	128.2	96.2	21.7	40.0	63.5
100	125	0.8	29.0	66.0	36.8	54.0	40.1	73.1
100	125	0.5	50.5	75.5	53.4	40.5	40.1	76.7
100	125	0.2	94.3	174.8	128.0	27.3	40.0	115.1

We should note that a detailed analysis of the results reveals that the pilot method usually does not need as much time as shown to find the indicated solutions, since in almost all cases the best result has been obtained in the first few seconds of the computations. This gives a strong hint that a small evaluation depth (see [18]) may be used to reduce the computation times without discarding solution quality.

6. Conclusions and further research

In this paper we have considered a generalization of the minimum labeling spanning tree problem to the case where not necessarily all but only a subset of required nodes need to be spanned. Common metaheuristics have successfully been applied to this generalization and the results are in line with our expectation gained from experimentation with the original labeling spanning tree problem. The most visible result is that the pilot method outperforms the other approaches with respect to solution quality while the computation times of the pilot method can be considerably larger than those of reactive tabu search or simulated annealing especially for larger problem instances. The computation times of our implementations for the pilot method and the variable neighborhood search are somewhat comparable with some exceptions for smaller densities of the given graphs where the pilot method may be faster. This motivates one direction of our further research consisting in the combination of the two metaheuristics that seem to behave better, the pilot method and the VNS1 [4]. Moreover, the results have been obtained for one generalization of the labeling spanning tree problem and future research refers also to extending those ideas to other generalizations such as the one also proposed in this paper considering additional budget constraints. Moreover, allowing for more than one color assigned to each edge poses an interesting case motivated by some applications.

Another step in our research refers to developing various mathematical programming formulations for the MLST as well as the MLSteiner to obtain optimal solutions to better judge on the quality of our heuristic solutions at least for small and moderately sized problem instances.

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Paper

Distributed, asynchronous algorithms for network control with contracted flow rates – a review

Andrzej Karbowski

Abstract— The paper reviews current algorithms for distributed, asynchronous control of networks when the customer is guaranteed to get some predetermined (e.g., as a part of a service level agreement – SLA) values of flow. Two cases are considered – both with single and multiple commodity. It is assumed, that the flow cost functions are convex with special attention devoted to linear and strictly convex cases.

Keywords— computer networks, asynchronous algorithms, distributed optimization, routing, quality of service, service level agreement.

1. Introduction

In the network optimization (e.g., in the Internet traffic control) one of the most important problems is elaboration of such policies, which guarantee that the service provider will deliver to the customer the contracted amount of the commodity flow (bandwidth in data networks). This amount is rigid and usually specified in service level agreement (SLA).
One may distinguish two situations:

- When the operator is obliged to deliver to all network nodes (customers) the contracted amount of flow of the same commodity, no matter from where; such situation is typical for power and water networks; in the Internet subnetworks with contracted access rates and many connection points to other operators' subnetworks as well as some peer-to-peer (P2P) computational grids may be modelled in this way.
- When the operator is responsible for the delivery of different commodities at the contracted level. Such problems are characteristic for transportation networks, and for data networks with higher quality of service (QoS) demands, such as networks with virtual connections (that is with isochronous traffic), for example: voice over Internet Protocol (VoIP), video-conferences, video on demand, etc.

Because networks are bigger and bigger, it is more and more difficult to control them effectively. Hence, in the recent years a special attention is paid to decentralized optimization and control algorithms. Especially, their asynchronous versions catch great attention of scientists dealing with management and control of networks. It is due to the flexibility and scalability of asynchronous algorithms. In such algorithms, the computing nodes may use outdated information and communicate with each other at different, random times, but this does not destroy the convergence. Moreover, the situation in networks, for example traffic in data networks, is changing dynamically. Asynchronous algorithms, using always the latest information, which is measured in the neighbourhood of the computing nodes, lead the network towards the current global optimum. Nowadays, for such a big and nonhomogenous network as Internet, it is even difficult to imagine an effective optimization mechanism that would not be distributed and asynchronous!

In the article such algorithms for single and multicommodity networks with contracted flow rates will be presented. Several different approaches will be considered, such as: standard primal-dual, ε-relaxation, an approach based on minimum first derivative length principle, an approach based on multiple adaptive traffic engineering method. It is assumed, that the flow cost functions are convex with special attention devoted to linear and strictly convex cases.

This paper is complementary to the paper "Distributed asynchronous algorithms in the Internet – new routing and traffic control methods" [7] presented at *DSTIS 2004 Conference* and closes the series of review papers devoted to distributed asynchronous network algorithms presented by the author at DSTIS conferences.

2. The optimization of flows in single commodity networks with linear and nonlinear cost functions

2.1. General convex functions

We consider a directed graph consisting of n nodes (routers). Let us denote by N the set of all these nodes, by A the set of all arcs (that is, the set of all links in the network) and by N_i the set of neighbours of the ith node (that is, the set of all nodes from the set N, to which arcs starting from i go). Let us assume, that every arc $(i,j) \in A$ is characterized by a continuous, convex function $a_{ij}(f_{ij})$ of the cost of the realization of the flow f_{ij} from the node i to j and box flow constraints:

$$f_{ij} \in F_{ij} = [b_{ij}, c_{ij}]. \tag{1}$$

With every node $i \in N$ we connect a given supply $s_i > 0$ or demand $s_i < 0$. Our goal is the calculation of such

a distribution of flows between nodes, that all demands are satisfied and the total cost of these flows is minimal, that is

$$\min_{f} \sum_{(i,j)\in A} a_{ij}(f_{ij}), \tag{2}$$

$$\sum_{\{j|(j,i)\in A\}} f_{ji} + s_i = \sum_{\{j|(i,j)\in A\}} f_{ij}, \ \forall i \in N, \tag{3}$$

$$b_{ij} \le f_{ij} \le c_{ij}, \ \forall (i,j) \in A, \tag{4}$$

where f is a vector of all flows. Equations (3) result from the balance of flows in the nodes (so-called 1st Kirchhoff rule).

We assume, that supplies and demands s_i are balanced over the network, that is:

$$\sum_{i\in N} s_i = 0 \tag{5}$$

and, of course, that the set of admissible solutions (i.e., the set of possible combinations of coordinates of the vector of flows f) is not empty.

In the Internet such problem may be important for wide area operators, who have several connection points with other operators.

To solve this problem we formulate a Lagrange function [1], but taking into account only balance constraints (3). If we denote the multiplier corresponding to the ith node equation as p_i, it will be:

$$L(f,p) = \sum_{(i,j)\in A} a_{ij}(f_{ij}) + \sum_{i\in N} p_i \left(\sum_{\{j|(j,i)\in A\}} f_{ji} + s_i - \sum_{\{j|(i,j)\in A\}} f_{ij} \right). \tag{6}$$

Making some simple transformations we can present the Lagrange function in the following way:

$$L(f,p) = \sum_{(i,j)\in A} \Big[a_{ij}(f_{ij}) - (p_i - p_j) f_{ij} \Big] + \sum_{i\in N} p_i s_i. \tag{7}$$

Hence, the dual function in problem (2)–(4) will have the form:

$$\begin{aligned} L_D(p) &= \min_{b\le f\le c} \left\{ \sum_{(i,j)\in A} \Big[a_{ij}(f_{ij}) - (p_i - p_j) f_{ij} \Big] + \sum_{i\in N} p_i s_i \right\} \\ &= \sum_{(i,j)\in A} \left\{ \min_{b_{ij}\le f_{ij}\le c_{ij}} \Big[a_{ij}(f_{ij}) - (p_i - p_j) f_{ij} \Big] \right\} + \sum_{i\in N} p_i s_i \\ &= \sum_{(i,j)\in A} L_{ij}(p_i - p_j) + \sum_{i\in N} p_i s_i, \end{aligned} \tag{8}$$

where L_{ij} is a component of the dual function corresponding to the arc (i,j), that is:

$$L_{ij}(p_i - p_j) = \min_{b_{ij}\le f_{ij}\le c_{ij}} \Big[a_{ij}(f_{ij}) - (p_i - p_j) f_{ij} \Big]. \tag{9}$$

According to the duality theory [1, 6], the solution of the problem (2)–(4) may be obtained by the solution of the dual problem:

$$\max_{p\in\mathbb{R}^n} L_D(p). \tag{10}$$

To find the optimal solution of the problem (10) one may use gradient of the dual function L_D, with coordinates:

$$\begin{aligned} \frac{\partial L_D}{\partial p_i} &= -\sum_{\{j|(j,i)\in A\}} L'_{ji}(p_j - p_i) + \sum_{\{j|(i,j)\in A\}} L'_{ij}(p_i - p_j) + s_i \\ &= \sum_{\{j|(j,i)\in A\}} f_{ji} - \sum_{\{j|(i,j)\in A\}} f_{ij} + s_i. \end{aligned} \tag{11}$$

The equivalent statement of optimality conditions stemming from duality theory is, that a flow vector $\hat{f}$ is optimal if and only if it is primal feasible, that is $\hat{f}_{ij} \in F_{ij} \ \forall \ i,j$ and there exists a price vector $\hat{p}$ satisfying together with $\hat{f}$ the following conditions (called complementary slackness conditions – CS) [3]:

$$a^-_{ij}(\hat{f}_{ij}) \le \hat{p}_i - \hat{p}_j \le a^+_{ij}(\hat{f}_{ij}). \tag{12}$$

In this expression leftmost and rightmost are, respectively, the left and the right derivatives of the arc cost function. Usually we deal with smooth cost functions and we have:

$$a^-_{ij}(\hat{f}_{ij}) = a^+_{ij}(\hat{f}_{ij}) = a'_{ij}(\hat{f}_{ij}). \tag{13}$$

In practical numerical calculations, while looking for a good approximation of the optimal solution, a relaxed version of the CS conditions proved to be very useful. For a given scalar $\varepsilon > 0$ inequalities (12) are replaced by the following:

$$a^-_{ij}(f_{ij}) - \varepsilon \le p_i - p_j \le a^+_{ij}(f_{ij}) + \varepsilon. \tag{14}$$

These conditions are called ε-complementary slackness conditions, ε-CS for short. The optimization approach which applies ε-CS conditions is called ε-relaxation method [3]. It consists in adjusting flows ("flow push") and increasing prices ("price rise") at appropriate nodes in such a way, that ε-CS conditions are maintained. This algorithm may be implemented in a distributed, asynchronous version [2], where each node i is a processor that updates its own price and its arcs flows, and exchanges information with its forward

$$F_i = \{j|(i,j)\in A\} \tag{15}$$

and backward

$$B_i = \{j|(j,i)\in A\} \tag{16}$$

adjacent nodes.

The information available at node i for any time t is as follows:

- $p_i(t)$: the price of node i;
- $p_j(i,t)$: the price of node $j \in F_i \cup B_i$ communicated by j to i at some earlier time;
- $f_{ij}(i,t)$: the estimate of the flow on the arc $(i,j), j \in F_i$, available at node i at time t;
- $f_{ji}(i,t)$: the estimate of the flow of arc $(j,i), j \in B_i$ available at node i at time t.

At each time t, each node i may be in one of the following four phases:

1. *Idle phase.* Node i does nothing.

2. *Computational phase.* Node i computes the surplus $g_i(t)$:

$$g_i(t) = \sum_{j \in B_i} f_{ji}(i,t) - \sum_{j \in F_i} f_{ij}(i,t) + s_i \,. \tag{17}$$

If $g_i(t) < 0$, node i does further nothing. Otherwise the following values:

$$p_i(t),\ \ f_{ij}(i,t),\ j \in F_i,\ \ f_{ji}(i,t),\ j \in B_i \tag{18}$$

are updated. The updating is performed due to the following procedure:

Step 1: (Calculation of the push list and the flow margin)
Given a flow-price vector satisfying the ε-CS conditions, *the push list* L_i of node $i, \forall i \in N$, is defined as follows:

$$\begin{aligned} L_i \ = \ & \{(i,j) | \varepsilon/2 < p_i(t) - p_j(i,t) \\ & - a_{ij}^{+}(f_{ij}(i,t)) \le \varepsilon\} \\ & \cup \{(j,i) | -\varepsilon \le p_j(i,t) - p_i(t) \\ & - a_{ji}^{-}(f_{ji}(i,t)) < -\varepsilon/2\} \,. \end{aligned} \tag{19}$$

For each arc (i,j) or (j,i) in the push list L_i, the supremum of σ for which

$$p_i(t) - p_j(i,t) \ge a_{ij}^{+}(f_{ij}(i,t) + \sigma) \tag{20}$$

or, respectively,

$$p_j(i,t) - p_i(t) \le a_{ji}^{-}(f_{ij}(i,t) - \sigma) \,,$$

is called the *flow margin*.

Step 2: (Scan of the push list)
If $L_i = \emptyset$ go to Step 4.

Step 3: (δ-Flow push)
Choose an arc from the push list L_i and let

$$\delta = \min(g_i(t), \text{ flow margin of the chosen arc}). \tag{21}$$

Increase f_{ij} by δ if (i,j) is the arc, or decrease f_{ji} by δ if (j,i) is the arc. If as a result the surplus becomes zero, go to the next iteration; otherwise, go to Step 2.

Step 4: (Price rise)
Increase the price p_i by the maximum amount that maintains ε-CS conditions. Go to the next iteration.

3. *Output phase.* The values of $p_i(t), f_{ij}(i,t), f_{ji}(i,t)$, computed during the computational phase, are communicated to the adjacent nodes $j \in F_i \cup B_i$.

4. *Input phase.* Node i receives from one or more adjacent nodes $j \in F_i \cup B_i$ a message containing the price $p_j(t')$ and the arc flow $f_{ij}(j,t')$ (when $j \in F_i$) or $f_{ji}(j,t')$ (when $j \in B_i$), computed by node $j, j \in F_i \cup B_i$, at some earlier time $t' < t$.
On the basis of this information, the node i updates $p_j(i,t)$ and $f_{ij}(i,t)$ if $j \in F_i$, ($f_{ji}(i,t)$, if $j \in B_i$).
If $p_j(t') \ge p_j(i,t)$, then $p_j(i,t) = p_j(t')$.
In addition, if $j \in F_i$, the value of $f_{ij}(i,t)$ is replaced by $f_{ij}(j,t')$ if

$$p_i(t) < p_j(t') + a_{ij}^{+}(f_{ij}(j,t')) + \varepsilon \ \text{ and } \ f_{ij}(j,t') < f_{ij}(i,t). \tag{22}$$

In the case of $j \in B_i$, the value of $f_{ji}(i,t)$ is replaced by $f_{ji}(j,t')$ if

$$p_j(t') \ge p_i(t) + a_{ji}^{-}(f_{ji}(j,t')) - \varepsilon \ \text{ and } \ f_{ji}(j,t') > f_{ji}(i,t). \tag{23}$$

The algorithm terminates if there is a time t_k such that, for all $t \ge t_k$:

$$g_i(t) = 0 \ \ \forall i \in N \,,$$

$$f_{ij}(i,t) = f_{ij}(j,t) \ \ \forall (i,j) \in A \,,$$

$$p_j(t) = p_j(i,t) \ \ \forall j \in F_i \cup B_i \,.$$

It may be shown, that the algorithm converges if the initial prices and flows satisfy ε-CS conditions, the nodes never stop executing iterations and communication and assuring that the old information is eventually purged from the system [2].

2.2. Linear cost functions

In this case the optimization problem has the following form:

$$\min_{f} \sum_{(i,j) \in A} a_{ij}(f_{ij}) = \alpha_{ij} f_{ij} \,, \tag{24}$$

$$\sum_{\{j | (j,i) \in A\}} f_{ji} + s_i = \sum_{\{j | (i,j) \in A\}} f_{ij}, \ \ \forall i \in N \,, \tag{25}$$

$$b_{ij} \le f_{ij} \le c_{ij}, \ \forall (i,j) \in A \,. \tag{26}$$

We may apply the algorithm presented in Subsection 2.1 in a simplified version [4][1], taking as:

- arc cost derivatives:

$$a_{ij}^{'}(f_{ij}) = \alpha_{ij}, \tag{27}$$

[1] Chronologically the asynchronous version of the ε-relaxation algorithm for linear minimum cost flow problems was presented much earlier in [4]; the version for convex problems from [2] was its extension.

- the ε-CS conditions:

$$f_{ij} < c_{ij} \Rightarrow p_i - p_j \leq \alpha_{ij} + \varepsilon, \ \forall (i,j) \in A, \quad (28)$$

$$b_{ij} < f_{ij} \Rightarrow p_i - p_j \geq \alpha_{ij} - \varepsilon, \ \forall (i,j) \in A, \quad (29)$$

- the push list:

$$L_i = \{(i,j) | \, p_i(t) = p_j(i,t) + \alpha_{ij} + \varepsilon \ \text{ and } \ f_{ij}(i,t) < c_{ij}\}$$
$$\cup \{(j,i) | \, p_i(t) = p_j(i,t) - \alpha_{ji} + \varepsilon \ \text{ and } \ b_{ji} < f_{ji}(i,t)\}, \quad (30)$$

- the flow margin:

$$\sigma = \begin{cases} c_{ij} - f_{ij}(i,t) & j \in F_i \\ f_{ji}(i,t) - b_{ji} & j \in B_i \end{cases}, \quad (31)$$

- the replacement conditions for flow estimates ((22) and (23)):
  - in the case of $j \in F_i$, the value of $f_{ij}(i,t)$ is replaced by $f_{ij}(j,t')$ if

$$p_i(t) < p_j(t') + \alpha_{ij} \ \text{ and } \ f_{ij}(j,t') < f_{ij}(i,t), \quad (32)$$

  - in the case of $j \in B_i$, the value of $f_{ji}(i,t)$ is replaced by $f_{ji}(j,t')$ if

$$p_j(t') \geq p_i(t) + \alpha_{ji} \ \text{ and } \ f_{ji}(j,t') > f_{ji}(i,t). \quad (33)$$

Application to the shortest path problem. For a single connection it is also possible to formulate a shortest path problem to find a shortest path from node s to node d as the following linear minimum flow cost problem [3]:

$$\min_f \sum_{(i,j) \in A} \alpha_{ij} f_{ij}, \quad (34)$$

$$\sum_{\{j|(j,i) \in A\}} f_{ji} - \sum_{\{j|(i,j) \in A\}} f_{ij} = \begin{cases} -1 & \text{if } i = s \\ 1 & \text{if } i = d \\ 0 & \text{otherwise} \end{cases} \quad \forall i \in N, \quad (35)$$

$$0 \leq f_{ij}, \ (i,j) \in A. \quad (36)$$

It may be solved in a distributed, asynchronous way through the ε-relaxation algorithm. The optimal path will be made of those arcs (i,j) for which $\hat{f}_{ij} = 1$ (for the remaining $\hat{f}_{ij} = 0$).

2.3. Strictly convex arc cost functions

In this section we assume, that the functions $a_{ij}(f_{ij}), (i,j) \in A$ in problem (2)–(5) are strictly convex. This problem, both the formulation and the basic features were taken from the book [4].

First, let us notice from Eqs. (7) and (5), that the optimal Lagrange multipliers are not unique, because one may add to all of them the same constant and the function value will not change. Hence, it is worthwhile to fix one of the coordinates of the vector p and take for example:

$$p_1 = r, \quad (37)$$

where r is an arbitrary nonzero real constant.

To solve the dual optimization problem (10) it is necessary to solve the family of scalar optimization problems (9), separately for every arc. They are very easy to solve, often even analytically.

According to the duality theory [1, 6] the function L_D and all functions L_{ij} are concave. It can be proved [4], that the algorithm of the Lagrange multiplier iteration of the form:

$$p_i := \begin{cases} r & i = 1 \\ \arg\max\limits_{\xi} L_D(p_1, p_2, \ldots, p_{i-1}, \xi, p_{i+1}, \ldots, p_n) & i = 2,3,\ldots,n \end{cases} \quad (38)$$

is also an order preserving mapping and it is convergent in a totally asynchronous version.

In fact, it is not necessary to perform optimization of L_D with the ith coordinate. To explain it let us define for every $i \in N \setminus \{1\}$ a *point-to-set* mapping R_i, which assigns to every Lagrange multipliers vector p a set of all prices which maximize the dual function L_D with respect to the ith price p_i, that is:

$$R_i(p) = \arg\max_{\xi} L_D(p_1, p_2, \ldots, p_{i-1}, \xi, p_{i+1}, \ldots, p_n). \quad (39)$$

It can be proved, that if the problem (2)–(4) and (5) is feasible, that is:

$$\sum_{\{j|(i,j) \in A\}} b_{ij} - \sum_{\{j|(j,i) \in A\}} c_{ji} \leq s_i \leq \sum_{\{j|(i,j) \in A\}} c_{ij} - \sum_{\{j|(j,i) \in A\}} b_{ji} \quad (40)$$

then the set $R_i(p)$ is either a singleton or a closed interval. Due to the concavity and differentiability of L_D every point $\xi \in \mathbb{R}$ belonging to $R_i(p)$ is a root of the scalar equation:

$$\frac{\partial L_D(p_1, p_2, \ldots, p_{i-1}, \xi, p_{i+1}, \ldots, p_n)}{\partial p_i} = 0 \quad (41)$$

that is, due to Eq. (11):

$$R_i(p) = \left\{ \xi : \sum_{\{j|(j,i) \in A\}} L'_{ji}(p_j - \xi) = \sum_{\{j|(i,j) \in A\}} L'_{ij}(\xi - p_j) + s_i \right\}. \quad (42)$$

Let us denote now as $\underline{R}_i(p)$ and $\overline{R}_i(p)$, respectively, the left and the right end of this interval. It turns out, that both these functions preserve the order. In the consequence,

the algorithm of the Lagrange multiplier iteration of the form:

$$p_i := \begin{cases} r & i = 1 \\ \gamma \underline{R}_i(p) + (1-\gamma)\overline{R}_i(p) & i = 2,3,\ldots,n \end{cases} \quad (43)$$

is equivalent to (38) for every $\gamma \in [0,1]$ and of course also convergent in a totally asynchronous version [4].
If it is not difficult to calculate the intervals $R_i(p), i = 1,\ldots,n$, one may propose another algorithm, which is convergent under the partial asynchronism assumptions [4, 9]. In this algorithm the ith coordinate is changed due to the iteration:

$$p_i := \gamma \cdot p_i + (1-\gamma) \arg \min_{\xi \in R_i(p)} |\xi - p_i| \;\; i = 1,2,3,\ldots,n \quad (44)$$

with $0 < \gamma < 1$. The computational experiments [9] showed, that this algorithm is considerably faster than the algorithm (43).

3. The optimization of flows in multicommodity networks with contracted transmission rates for virtual connections

Now we will consider a more complicated situation, where from the network we expect not only the transport of the total volume of traffic, from all sources to all destination nodes, but also the guarantees on the flow between given pairs of nodes. So we will deal with networks which actually provide virtual connections, in other words with virtual-circuit data networks.
We define a set W of origin-destination pairs and assume, that for every connection $w = (s,d)$, $s,d \in N, s \neq d$ the total flow r_w may be split to several paths P_w. We also assume, that the sets P_w for different w are disjoint. We will denote by A_p the set of all arcs (links) belonging to (i.e., forming) the path p.
Let x_p be the flow through a particular path $p \in P_w$. According to our assumptions, the flow f_{ij} through an arc (i,j) equals:

$$f_{ij} = \sum_{p \in P_{ij}} x_p \,, \quad (45)$$

where $P_{ij} = \{p : (i,j) \in A_p\}$ is the set of all paths traversing arc (i,j). Denoting, as before, by $a_{ij}(f_{ij})$ the cost of assuring the flow f_{ij} in the arc (i,j), we may formulate the optimization problem as:

$$\min_x \left[z(x) = \sum_{(i,j)\in A} a_{ij}(f_{ij}) = \sum_{(i,j)\in A} a_{ij}\left(\sum_{p \in P_{ij}} x_p \right) \right], \quad (46)$$

$$\sum_{p \in P_w} x_p = r_w, \quad \forall w \in W \,, \quad (47)$$

$$x_p \geq 0, \quad \forall p \in P_w, \quad \forall w \in W \,. \quad (48)$$

It turns out [4, 10], that the optimal distribution of path flows may be obtained by distributed partially asynchronous iterations of path flows x_p, grouped with respect to the realized connections w. The so-called "minimum first-derivative length" (MFDL) principle is applied. It says, that we should allocate more traffic to this path from the set P_w, for which the partial derivative of the cost function $\sum_{(i,j)\in A_p} \frac{\partial a_{ij}}{\partial x_p}$ is minimal. Applying the Taylor expansion series, it can be easily proved, that it guarantees for small amounts of shifted flow the decrease of the total cost. The assessment of MFDL path may be performed locally for every connection, that is in a distributed way, and asynchronously.
Since the information on flows in different arcs $(i,j) \in A_p$ for $p \in P_w$ comes from different times (e.g., the data concerning closer nodes is more recent) the wth processor, which calculates path flows of the wth connection, actually uses an estimate $\tilde{f}_{ij}^w(t)$ of these flows in some time window before the time of calculations t:

$$\tilde{f}_{ij}^w(t) = \sum_{\tau=t-B}^{t} \eta_{ij}^w(t,\tau) f_{ij}(\tau) \,, \quad (49)$$

where $f_{ij}(\tau)$ is the actual flow at time τ in the arc (i,j), B is the length of the time window, and $\eta_{ij}^w(t,\tau)$ are (usually unknown) nonnegative coefficients such that:

$$\sum_{\tau=t-B}^{t} \eta_{ij}^w(t,\tau) = 1 \,. \quad (50)$$

Let us denote the estimate of the derivative of the cost of the flow along the path $p \in P_w$ calculated at time t by $\lambda_p(t)$, that is:

$$\lambda_p(t) = \sum_{(i,j)\in A_p} a'_{ij}(\tilde{f}_{ij}^w(t)) \quad (51)$$

and the index of the MFDL path by p_m, that is:

$$\lambda_{p_m}(t) = \min_{p \in P_w} \lambda_p(t) \,. \quad (52)$$

In the general model it is assumed, that flows are not changed immediately and two phases are distinguished: the calculation of desired flows $\bar{x}_p$ and their realization x_p. According to this model, the new (actual) routing $x_p(t+1)$, $p \in P_w$ is determined as a convex combination of the desired routing $\bar{x}_p(t)$ and the current one $x_p(t)$:

$$x_p(t+1) = \beta_p(t)\bar{x}_p(t) + (1-\beta_p(t))x_p(t), \;\; p \in P_w \,, \quad (53)$$

where $0 < \beta < \beta_p(t) \leq 1$ are generally unknown coefficients reflecting a smooth (with geometric rate) movement from the current to the desired routing. Of course whichever they are, the transmission rate constraints (47) have to be satisfied.

The desired flows $\bar{x}_p$ for all paths in the connection w are calculated differently for the MFDL path and for the remaining ones. For paths $p \neq p_m$ the following formula is used:

$$\bar{x}_p(t) = \max\left\{0, x_p(t) - \frac{\gamma}{H_p(t)}\left(\lambda_p(t) - \lambda_{p_m}(t)\right)\right\}, \quad (54)$$

where $\gamma > 0$ is a stepsize and $H_p(t)$ is an estimate of the second derivative length of path p

$$H_p(t) = \sum_{(i,j)\in A_p} a''_{ij}(\tilde{f}^w_{ij}).$$

Afterwards, for the MFDL path the desired flow is calculated from the expression:

$$\bar{x}_{p_m}(t) = r_w - \sum_{p\in P_w, p\neq p_m} \bar{x}_p(t). \quad (55)$$

It may be proved, that there exists some $\gamma_0(B)$ such that for $0 < \gamma < \gamma_0(B)$ the described algorithm implemented asynchronously converges, delivering the minimum total cost of transmission $z(x)$. Luo and Tseng [8] showed, that when the cost function a_{ij} (e.g., the expected delay) on each link is a strictly convex function on the link flow, the sequence generated by this algorithm converges in the space of path flows at a linear rate.

It is possible to apply instead of (54) and (55) another scaled gradient algorithm:

$$\bar{x}_w(t+1) = \left[\bar{x}_w(t) - \gamma M_w^{-1}\lambda_w(t)\right]^+_{M_w(t)}, \quad (56)$$

where $\bar{x}_w$, λ_w are vectors formed of components $\bar{x}_p$, λ_p for $p \in P_w$, $M_w(t)$ is a symmetric positive definite matrix (usually it is an estimate of the Hessian matrix $\frac{\partial^2 z}{\partial x_w^2}$, and the algorithm (56) is an approximation of the projected Newton method), $[.]^+_{M_w(t)}$ denotes the projection on the simplex

$$\left\{x_w \middle| \sum_{p\in P_w} x_p = r_w \text{ and } x_p \geq 0, \forall p \in P_w\right\} \quad (57)$$

with respect to the norm $||x_w||_{M_w(t)} = \left(x'_w M_w(t) x_w\right)^{\frac{1}{2}}$. However, since this algorithm takes into account the current value of the desired flows $\bar{x}_w(t)$ instead of the current value of the actual flows $x_w(t)$ one may expect that it will be slower in the adaptation to sudden changes in the problem data r_w. Surprisingly, the replacement in Eq. (56) $\bar{x}_w(t)$ with $x_w(t)$ destroys the descent property and the convergence of the algorithm [4].

Recently Elwalid *et al.* [5] noticed, that the above scheme may be successfully adapted to Internet traffic engineering in multiprotocol label switching (MPLS) networks. They introduced two changes:

- They do not distinguish between the actual $x_p(t)$ and the desired $\bar{x}_p(t)$ source rates, that is a new rate vector is calculated from the formula:

$$x_w(t+1) = \left[x_w(t) - \gamma\lambda_w(t)\right]^+, \quad (58)$$

  where $[.]^+$ denotes the projection on the feasible space Eq. (57) with respect to the Euclidean norm. The justification is, that if one is only dealing with IP datagrams it is reasonable to assume that each ingress node can shift its traffic among the label switched paths available to it immediately after each update.

- They relax the assumption that at time t each source has available the current first derivative lengths Eq. (51) and uses it in place of the gradient in the update algorithm. Instead, they assume, that at time t, the source may only have outdated first derivative lengths. Moreover, the source uses a weighted average over several past lengths in the update algorithm. That is, the price used in algorithm (58) is calculated in the following way:

$$\lambda_p(t) = \sum_{\tau=t-B}^{t} \sum_{(i,j)\in A_p} \rho^w_{ij}(t,\tau)\, a'_{ij}(\tilde{f}^w_{ij}(\tau)), \quad (59)$$

  where $\tilde{f}_{ij}(\tau)$ is an estimate of flow in the arc (i,j) calculated at time τ Eq. (49), B is the length of the time window, and $\rho^w_{ij}(t,\tau)$ are (usually unknown) nonnegative coefficients such that:

$$\sum_{\tau=t-B}^{t} \rho^w_{ij}(t,\tau) = 1. \quad (60)$$

  This is because, in the distributed and decentralized implementation of the algorithm, the source can only estimate the first derivative lengths through noisy measurement.

Despite these differences, stability of this algorithm (called MATE – from multipath adaptive traffic engineering) in [5] has been established using the same techniques as in [4, 10].

4. Conclusions

All presented distributed, asynchronous optimization methods for data networks management may be interesting to network operators. For mass client market traffic balance routing (see Section 2) may be sufficient. For more demanding users: state services, governmental institutions, big companies, banks, etc., the model with guaranteed connection rates (see Section 3) should be applied. While in the first case prices, i.e., Lagrange multipliers, are only some internal indicators guiding the network towards the optimum and the balance of resources and demands, without the monetary consequences, in the second case they may be more useful. Namely, it is possible to use them directly to calculate online the cost of high-quality connections or to draw up a new price list for future SLAs.

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Paper

FR/ASimJava: a federated approach to parallel and distributed network simulation in practice

Andrzej Sikora and Ewa Niewiadomska-Szynkiewicz

Abstract— The paper addresses issues associated with the application of federations of parallel/distributed simulators to large scale networks simulation. We discuss two principal paradigms for constructing simulations today. Particular attention is paid to an approach for federating parallel/distributed simulators. We describe the design and performance of frame relay network simulator (FR/ASimJava) implemented based on a Java-based library for distributed simulation – ASimJava. Six practical examples – six networks operating under frame relay – are presented to illustrate the operation of the given software tool. The focus is on the efficiency of presented network simulator.

Keywords— parallel simulation, computer networks simulation, frame relay, federated simulators.

1. Introduction

Network simulation is an important tool for researches that allows to analyze the behavior and performance of the considered network and verify new ideas. A variety of software environments simulating packets transmission through the network are available today. There are a number of possible sets of criteria that could be used for network simulators comparison, e.g., the model size, the execution time, memory requirements, scalability, programming interface, etc. Different tools are optimized for different purposes. The comparative study of some popular simulators are reported in many papers, e.g., the results of the performance study involving: JavaSim [7], ns-2 [19], SSFNet-Java [18], and SSFNet-C++ [17] are described in [13], the comparison of ns-2, JavaSim and OPNET [15] is concluded in [9].

We are involved in large heterogenous networks simulating in near real time. The main difficulty in packet level simulation is the enormous computational power, i.e., speed and memory requirements needed to execute all events involved by packets transmission through the network. Another problem is scalability, i.e., how a given simulator scales for large topologies and high speed links. Parallel and distributed simulation has already proved to be very useful when performing the analysis of different real complex systems [20]. It allow us to reduce the computation time of the simulation programme, to execute large programs that cannot be put on a single processor and to better reflect the structure of physical system. Last years a new paradigm for constructing parallel and distributed simulations was developed. It is based on the idea of federating disparate simulators, utilizing runtime infrastructure to interconnect them. In this paper we investigate issues concerning federations of parallel simulators. A novel approach to scalability and efficiency of parallel/distributed frame relay (FR) network simulation is described and discussed.

2. An approach for federating parallel/distributed simulators

Parallel/distributed discrete-event simulation can be described in terms of logical processes (LPs) and communicate with each other through message-passing. LPs simulate the real life physical processes (FPs). Each logical process starts processing as a result of event occurrence (from the event list or having received a new message). It performs some calculations and generates one or more messages to other processes.

The calculation tasks executed in parallel require explicit schemes for synchronization. Two simulation techniques are considered [9]: synchronous and asynchronous. Synchronous simulation is implemented by maintaining a global clock (global virtual time – GVT). Events with the smallest time-stamp are removed from the event lists of all LPs for parallel execution. Parallelism of this technique is limited because only events with time-stamps equal to that of the global clock can be executed during an event cycle. Asynchronous simulation is much more effective due to its potentially high performance on a parallel platform. In asynchronous simulation each logical process maintains its own local clock (local virtual time – LVT). Local times of different processes may advance asynchronously. Events arriving at the local input message queue of a logical process are executed according to the local clock and the local schedule scheme.

Synchronization mechanisms fall into two categories: conservative and optimistic. They differ in their approach to time management. Conservative schemes avoid the possibility of causality error occurring. These protocols determine safe events that can be executed. Optimistic schemes allow occurrence of causality errors. They detect such error and provide mechanisms for its removal. The calculations are rolled back to a consistent state by sending out antimessages. It is obvious that in order to allow rollback all results of previous calculations have to be recorded. Now, there

are two basic directions to take when developing parallel and distributed simulators [2]:

– development of a problem dedicated (specialized) simulators, specific to the environment for which they were created;

– development of general purpose simulators designed as federations of disparate simulators, utilizing runtime infrastructure (RTI) software to interconnect them.

In the case of the first paradigm the simulation engine, interface, libraries and tools to create new high performance simulators are defined. It is difficult, in general, for the user to modify and apply such software to new environments. A second paradigm results in coarse-grained set of simulators creating federation. It is assumed that his entire simulators are viewed as black boxes. They are designated as federates. The runtime infrastructures used for federates interconnecting are typically designed for coarse granularity concurrency. This approach is utilized in high level architecture (HLA) [5] standard for distributed discrete-event simulation. The main advantage is high possibility of simulation models reuse. However, we pay for this universal applicability. This approach imposes certain restrictions concerning the structure of the federation members. In addition federates have to obey some rules of the federation that are included in.

The federated, distributed simulation consists of a collection of autonomous simulators that are interconnected using RTI software. The RTI implements relevant services required by the federated simulation environment. The most important services are: time synchronization among federated simulators, secure and efficient communication and scalable platform architecture.

Network analysis and modeling concentrate on studying network components (from devices to requirements and performance levels) and their inputs and outputs. We are interested to evaluate simulated system operation under various real-life conditions. Two important characteristics of networks: interconnectivity and levels of hierarchy (from network core to user access level) [10] have to be concerned. Based on these characteristics we can define a set of network submodels. We assume that the simulator of each submodel implements only a part of the network being simulated. The advantage of such application is that there is no need to provide the shared memory access to describe the whole simulated network system. Physical connectivity between federated simulators executes a set of logical connections between submodels. An example of this approach is shown in Fig. 1.

Using the paradigm of federated simulators, an implementation of a federation of asynchronous parallel software modules simulating frame relay network was developed and examined.

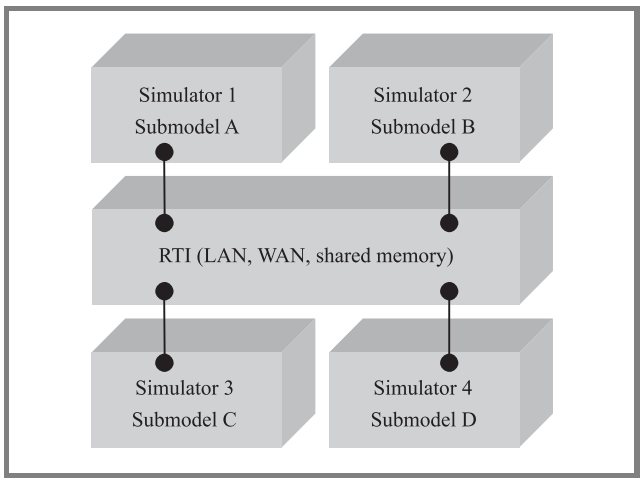

Fig. 1. An architecture of federated simulator.

3. Parallel simulation of frame relay networks

3.1. ASimJava library

Frame relay/asynchronous simulation Java (FR/ASimJava) network simulator is a parallel/distributed fast simulator of frame relay networks. It was implemented based on ASimJava, a Java-based library for large-scale systems simulation. Although ASimJava was described in [14], we provide a brief summary here, to make the paper self-contained. The ASimJava library admits to do parallel and distributed discrete-event simulations that can be described in terms of logical processes and communicate with each other through message-passing. LPs simulate the real life physical processes. Each logical process starts processing as a result of event occurrence (from the event list or having received a new message). It performs some calculations and generates one or more messages to other processes. The calculation tasks executed in parallel require explicit schemes for synchronization. The synchronous and asynchronous [20] variants of simulators are available. In the case of asynchronous approaches four synchronization protocols are provided: conservative protocol with null messages (CMB) [11], window conservative protocol [12], time warp (TW) [8], moving time window protocol (MTW) [16].

The simulator built upon ASimJava classes has hierarchical structure. The simulated system is partitioned into several subsystems (subtasks), with respect to functionality and data requirements. Each subsystem is implemented as LPs. Each LP can be divided into smaller LPs. Hence, the logical processes are nested (Fig. 2). Calculation processes belonging to the same level of hierarchy are synchronized. The module-oriented architecture of ASimJava library allows developers to add new components. One of these modules is bidirectional interface to XML configuration and state save file that uses ASimL language – XML schema[1] specification for building XML file with description of pa-

[1]See http://www.w3.org standard

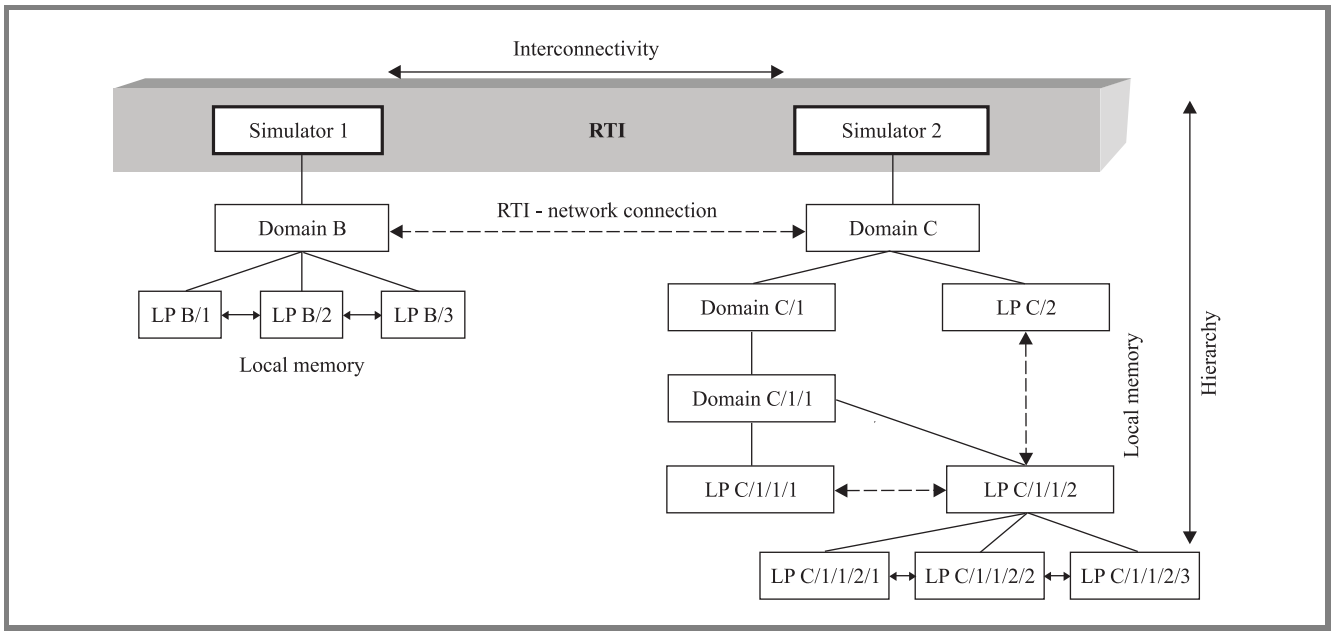

Fig. 2. A federation of network simulators consisting of two members: Simulator 1 and Simulator 2.

rameterized system model. Simulator configuration can be fully loading (run, re-run) from XML file, it may contain any number of user-defined parameters.
Two types of simulators can be distinguished:

1) the simulator consisting only of classes provided in ASimJava; the structure of the simulated system together with all model parameters is created using ASimJava graphical interface or may be read from an XML file;

2) new simulator – the user's task is to implement the subsystems' simulators responsible for adequate physical systems simulation; he can create his application applying adequate classes from the ASimJava libraries and including his own code – numerical part of the application.

As one of the ASimJava's principle goals was portability and usage in heterogeneous computing environments. Two versions of ASimJava are implemented: parallel and distributed. It is possible to join both of them in one simulator. The JXTA technology platform provided by Sun Microsystems was used to interprocess communication in the case of distributed version of the library. JXTA is a set of open, generalized peer-to-peer (P2P) protocols that allow any connected device on the network to communicate and collaborate as peers (see Fig. 3). The JXTA protocols are independent of any programming language, and multiple implementations exist for different environments. This technology enables developers to build and deploy interoperable P2P services and applications. The JXTA protocols standardize the manner in which peers:

– discover each other peer,

– self-organize into peer groups,

– advertise and discover network services,

– securely communicate with each other,

– monitor each other peer remotely.

The ASimJava software framework is suitable to solve many small and large scale problems, based on simulation. The package is flexible and can be easily extended by software modules, which are specific to a chosen application.

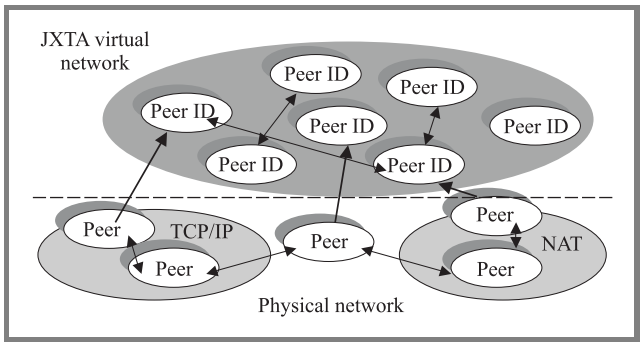

Fig. 3. JXTA logical network mapping. Explanations: ID – identification number, NAT – network address translation, TCP/IP – transmission control protocol/Internet Protocol.

3.2. Description of FR/ASimJava simulator

Frame relay is a high-performance wide area network (WAN) protocol that operates at the physical and data link layers of the open system interconnection (OSI) reference model. This is a standard protocol for local area network (LAN) internetworking which provides a fast and efficient method of transmitting information from a user

device to LAN bridges and routers. Frame relay is an example of a packet-switched technology. Variable-length packets are used for more efficient and flexible data transfers. These packets are switched between various segments in the network until the destination is reached. Internationally, frame relay was standardized by the International Telecommunication Union – Telecommunications Standards Section (ITU-T) [4]. In the United States, frame relay is an American National Standards Institute (ANSI) standard [6].
Frame relay traffic is described based on several characteristic parameters. The detailed information about FR parameters one can find on frame relay forum webside [3].

The FR/ASimJava simulator implements the following parameters describing traffic characteristics in frame relay networks:

- committed information rate (*CIR*);
- excess information rate (*EIR*);
- committed information size (*Bc*):
  $Bc = CIR \times Tc$,
  where Tc denotes assumed time interval;
- excess information size (*Be*):
  $Be = EIR \times Tc$;
- a physical line speed of the interface connecting to the frame relay network (*access rate*):
  $\sum_i^I CIR_{ij} \leq access\ rate_j$,
  where $i \in I$ denotes client and j connection.

The following features of FR protocol are taken into considerations in our implementation:

- permanent virtual circuit (PVC);
- switched virtual circuit (SVC);
- data terminal equipment (DTE) – an edge, access routers:
  - classifying traffic – any number of class that can be used in quality of service (QoS) mechanism,
  - measuring and marking excess traffic (if greater than *CIR*) with bit discard eligibility DE = 1,
  - shaping input traffic (leaky bucket, token bucket),
  - dropping "bursty" traffic (if greater than $CIR + EIR$),
  - backward explicit congestion notification (BECN) and forward explicit congestion notification (FECN) reaction,
  - SVC negotiation,
  - stochastic traffic generators for all virtual circuits;
- Data circuit-terminating equipment (DCE) – switches (see Fig. 4):
  - quality of service: input and output buffers management: first in, first out (FIFO), priority queuing (PQ), class-based weighted fair queuing (CBWFQ), PQ-CBWFQ discipline,
  - BECN and FECN signaling,
  - SVC negotiation,
  - switching and routing.

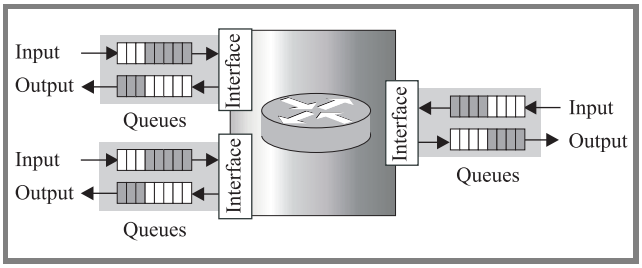

Fig. 4. Exampled architecture of switch used in FR/ASimJava simulator.

All parameters of simulation model, network topology, characteristics of data flows (traffic) and frame relay mechanism are saved in XML configuration file. This file can be simply modified and reused in many simulations.

4. Case study results

In the presented case study we evaluate the complexity of frame relay network simulation. In this paper the results of experiments performed for four network configurations, examples E1–E4 describing different model size, and two variants of implementation – S (sequential) and D (distributed) are discussed. The detailed descriptions, i.e., network models and traffic characteristics are given in Table 1. During the tests, we measured the simulation time (the execution time of each experiment). We assumed in all tests the same simulated time – 30 seconds of physical network operation. The objective of presented case study was to compare the efficiency of parallel, federated simulators with the sequential realization. To compare the performance of packet-level simulators we used two characteristics, i.e., simulation time (execution time) in miliseconds and average simulator speed simulated packets transmissions per second (PTS) [4] defined in Eq. (1):

$$PTS \approx \left(\frac{N_F \cdot P_F \cdot H_F}{T} \right), \tag{1}$$

where T denotes the execution time, N_F – the number of flows (edge router to edge router), P_F – the number of packets sent per flow, H_F – the average hops per flow (queuing, transmitting over link, etc.). The presented definition ignores lost packets, protocol generated packets.

Table 1
Results of experiments: four exampled frame relay networks (variant S)

Example	Network model	LPs number	Packets number	Simulation time [ms]	*PTS*
E1	1 switch 2 interfaces 2 edge routers 2 links 1.544 Mbit/s	7	$(\sim 13500 \cdot 2) = 27000$	4800	22.5
E2	1 switch 6 interfaces 6 edge routers 6 links 1.544 Mbit/s	19	$(\sim 13500 \cdot 6) = 81000$	12900	25.1
E3	2 switches 14 interfaces 12 edge routers 12 links 1.544 Mbit/s 1 link 44.736 Mbit/s	42	$(\sim 13500 \cdot 12) = 162000$	24300	46.6
E4	3 switches 22 interfaces 18 edge routers 18 links 1.544 Mbit/s 2 links 44.736 Mbit/s	65	$(\sim 13500 \cdot 18) = 243000$	34100	71.2

Table 2
Computer systems used during experiments

Computer systems	AMD Athlon-M 1.2 GHz, 512 RAM	AMD Sempron 1.67 GHz, 512 RAM	AMD Sempron 1.67 GHz, 512 RAM
C1	X		
C2	X	X	
C3	X	X	X

The results of simulation experiments performed on single machine (a computer system C1 described in Table 2) are presented in Table 1. It can be observed that the execution time of experiment performed for exampled network E4 exceeds the real time operation of the physical network (the simulation time is greater than simulated, virtual time).

The second series of experiments was performed in the network of computers. Two hardware platforms were considered: C2 – the network of two machines, C3 – the network of three machines (see Table 2).

Two exampled networks E3 and E4 were taken into considerations. The simulator of the whole network was composed of two federated simulators in the case of example E3 and three federated simulators in the case of E4 (see Fig. 5). The calculations of each member of federation were performed by separate computer. The window conservative scheme described in [12, 14] was applied to federated simulators synchronization.

The submodels configurations, execution time of each experiment and simulators speeds are given in Table 3.

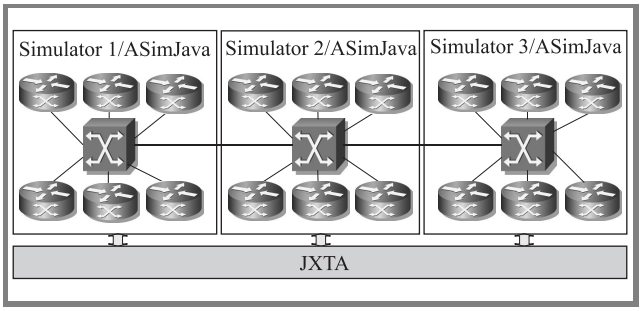

Fig. 5. Federated simulator of exampled network E4.

As expected, we can observe that federated, distributed simulation can seriously speed up simulations of network operation w.r.t. sequential implementation. The calculation speed-up depends on the size of considered network model and assumed degree of parallelism. It should be indicated that in the case of distributed implementation the reserve of efficiency to meet real time requirements is quite large (see E4 in Table 3).

Table 3
Results of experiments: two exampled frame relay networks (variant D)

Example	Network model	LPs number	Packets number	Simulation time [ms]	*PTS*
E3	1 switch 7 interfaces 6 edge routers 6 links 1.544 Mbit/s 1 link 44.736 Mbit/s	21	$(\sim 13500 \cdot 12) = 162000$	19300	58.7
	1 switch 6 interfaces 6 edge routers 1 link 1.544 Mbit/s	21			
E4	1 switch 7 interfaces 6 edge routers 6 links 1.544 Mbit/s 1 link 44.736 Mbit/s	21	$(\sim 13500 \cdot 18) = 243000$	19300	**125.2**
	1 switch 6 interfaces 6 edge routers 6 links 1.544 Mbit/s 1 link 1.544 Mbit/s	21			
	1 switch 8 interfaces 6 edge routers 6 links 1.544 Mbit/s	23			

5. Conclusions

In this paper we described the federated approach to parallel and distributed simulation of frame relay networks. We demonstrated that this approach is suitable to perform fast simulations of large-scale networks. Our experiences with federated, distributed network simulations confirm the ability of the federated simulation approach to achieve large and detailed simulation models. JXTA peer-to-peer technology and ASimJava library allow us to use Internet and other computer networks as a secure simulation platform.

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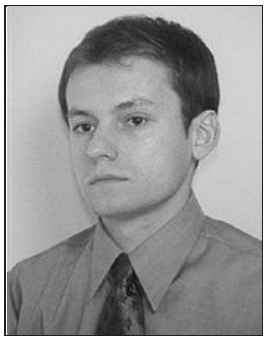

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Paper

Coherence of radial implicative fuzzy systems with nominal consequents

David Coufal

Abstract— In the paper we are interested in the question of coherence of radial implicative fuzzy systems with nominal consequents (radial I-FSs with NCs). Implicative fuzzy systems are fuzzy systems employing residuated fuzzy implications for representation of IF-THEN structure of their rules. Radial fuzzy systems are fuzzy systems exhibiting the radial property in antecedents of their rules. The property simplifies computational model of radial systems and makes the investigation of their properties more tractable. A fuzzy system has nominal consequents if its output is defined on a finite unordered set of possible actions which are generally quantitatively incomparable. The question of coherence is the question of under which conditions we are assured that regardless the input to the system is, an output of the system exists, i.e., the output is non-empty. In other words, a fuzzy system is coherent if it has no contradictory rules in its rule base. In the paper we state sufficient conditions for a radial I-FS with NCs to be coherent.

Keywords— implicative fuzzy system, radial fuzzy system, nominal output space, coherence.

1. Introduction

In the theory of fuzzy systems there are generally recognized two approaches to the representation of IF-THEN rules and their groups – rule bases [3, 7, 8]. They are the *conjunctive* and the *implicative* approach. In the conjunctive approach, IF-THEN structure of a rule is represented by a fuzzy conjunction and individual rules are combined by a fuzzy disjunction. In the implicative approach, IF-THEN structure of a rule is represented by a fuzzy implication and individual rules are combined by a fuzzy conjunction.

Radial fuzzy systems are fuzzy systems which have membership functions of fuzzy sets in their rules represented by radial functions and exhibit the radial property. The radial property is the shape preservation property related to antecedents (IF parts) of IF-THEN rules. The presence of this property simplifies the computational model of radial fuzzy systems and enables an effective study of their properties.

Fuzzy systems with nominal consequents (THEN parts) are those systems with finite, generally unordered, output spaces. Such a space forms the universe of discourse on which fuzzy sets in consequents are specified. Such an universe can be treated as a set of possible actions which are quantitatively incomparable.

The question of coherence of a fuzzy system is an important question related mainly to the theory of implicative fuzzy systems [4, 9]. The request for coherence of an implicative system can be seen as the request for the non-presence of contradictory rules in its rule base, for if the rules are contradictory there exists an input making the output of the system to be empty. As a typical example of contradictory rules consider the situation if (simultaneously) one rule indicates *go left* action and the other *go right* action.

In the literature, the question of coherence was discussed mainly for fuzzy systems with ordinal consequents [2, 4], i.e., for the systems having consequents' fuzzy sets specified on ordered universes of discourse, typically on real line $\mathcal{R}$.

In this paper we are interested in the study of coherence of radial fuzzy systems with nominal consequents (radial I-FSs with NCs). In the next section we introduce the computational model of these systems in an explicit way. Section 3 is the main section of the paper and contains two theorems stating sufficient conditions for coherence of a radial I-FS with NCs. The paper concludes by Section 4.

2. Radial I-FSs with NCs

We consider the standard architecture of a fuzzy system. That is, the system consists of four building blocks – *singleton* fuzzifier, implicative rule base, compositional rule of inference (CRI) inference engine, and a defuzzification block. The flow of a signal is as usual, i.e., from the fuzzifer to the defuzzifier through the inference engine [7, 8].

2.1. Computational model of I-FSs with NCs

Under the implicative approach, a rule base consisting of m, $m \in \mathcal{N} = \{1,2,\dots\}$, rules has the following mathematical representation:

$$RB(\boldsymbol{x},y) = \bigwedge_{j=1}^{m} A_j(\boldsymbol{x}) \to B_j(y), \tag{1}$$

where A_j is the representation of the antecedent in the jth rule, $j = 1,\dots,m$, B_j is the consequent fuzzy set, $\to$ is a *residuated* fuzzy implication and $\bigwedge$ a fuzzy conjunction. Typically $\bigwedge = \star$, where $\star$ is the t-norm which is used to form antecedents of rules and for specification of $\to$, see below.

Antecedents A_js are generally specified on n-dimensional, $n \in \mathcal{N}$, input space $X = \mathcal{R}^n$ and represented in the standard way as

$$A_j(\boldsymbol{x}) = A_{j1}(x_1) \star \cdots \star A_{jn}(x_n), \tag{2}$$

where $\boldsymbol{x} \in \mathcal{R}^n$, $\boldsymbol{x} = (x_1,\dots,x_n)$, A_{ji}, $i = 1,\dots,n$, are one-dimensional fuzzy sets defined on respective one-

dimensional parts of input space (in fact these are real lines $\mathcal{R}$), and $\star$ is a t-norm representing a fuzzy conjunction (and linguistic connective). As mentioned, this $\star$ is usually also used for representing $\bigwedge$ in Eq. (1).

As we are interested in fuzzy systems with nominal consequents we consider output space Y to correspond to an *unordered finite* set (nominal space) of $l \in \mathcal{N}$ generally mutually incomparable actions y_k, $k = 1,\ldots,l$, i.e., $Y = \{y_1,\ldots,y_l\}$. Consequents' fuzzy sets B_js are then considered to be specified on Y.

Let the input to a fuzzy system be $\boldsymbol{x}^* \in \mathcal{R}^n$. As we consider the singleton fuzzifier to be employed in the system the general CRI formula for computing output fuzzy set B' from the fuzzy system is simplified into the form

$$B'(y) = RB(\boldsymbol{x}^*, y), \tag{3}$$

where RB is the representation of rule base. Employing the implicative rule base Eq. (1) the above reads as

$$B'(y_k) = \bigwedge_{j=1}^{m} A_j(\boldsymbol{x}^*) \to B_j(y_k). \tag{4}$$

Introducing m individual output fuzzy set B'_j, each related to the single rule j and defined by

$$B'_j(y_k) = A_j(\boldsymbol{x}^*) \to B'_j(y_k), \tag{5}$$

the overall output is specified as

$$B'(y_k) = B'_1(y_k) \star \cdots \star B'_m(y_k). \tag{6}$$

To proceed let us recall the concept of residuated implication and its properties. Residuated fuzzy implications are generalizations of Boolean implications. A residuated implication $\to_\star$ is derived on the basis of its associated t-norm $\star$ according to formula

$$a \to_\star b = \sup\{c \in [0,1] \,|\, a \star c \leq b\}. \tag{7}$$

Examples of residuated fuzzy implications are the *Gödel implication* derived from the minimum t-norm: $a \to_{\mathrm{M}} b = 1$ iff $a \leq b$ and $a \to_{\mathrm{M}} b = b$ iff $a > b$; and the *Goguen implication* derived from the product t-norm: $a \to_{\mathrm{P}} b = 1$ iff $a \leq b$ and $a \to_{\mathrm{P}} b = b/a$ iff $a > b$. For details about residuated implications see [6, 7].

An important property, valid for any residuated implication $\to$ (in the sequel we will not explictly indicate the associated t-norm $\star$), is

$$a \to b = 1 \quad \text{iff} \quad a \leq b. \tag{8}$$

On the basis of this property the computational model of an I-FS with NCs is stated as follows:

Let the consequents fuzzy sets B_js be normal, i.e., for each rule j there exists a k such that $B_j(y_k) = 1$. Then the core (or kernel) of B'_j set is specified as $core(B'_j) = \{y_k \,|\, B'_j(y_k) = 1\}$. Due to the normality of B_j, $core(B'_j) \neq \emptyset$. Going back to how B'_j sets are defined, formula (5), and employing the property (8) we can see that depending on value of $A_j(\boldsymbol{x}^*)$ another y_k(s) can occur in $core(B'_j)$. More specifically, for a given input $\boldsymbol{x}^*$, an $y_k \in Y$ is in $core(B'_j)$ if and only if $A_j(\boldsymbol{x}^*) \leq B_j(y_k)$. Let us denote $core(B'_j)$ for a given input $\boldsymbol{x}^*$ by $I_j(\boldsymbol{x}^*)$. The following specification formula for $I_j(\boldsymbol{x}^*)$ can be adopted:

$$I_j(\boldsymbol{x}^*) = \{\, y_k \in Y \,|\, A_j(\boldsymbol{x}^*) \leq B_j(y_k) \,\}. \tag{9}$$

With respect to the overall output B' of an I-FS with NCs, which is given by formula (6), let us assume that for a given input $\boldsymbol{x}^*$ the corresponding output fuzzy set B' is normal and let us denote its core by $I(\boldsymbol{x}^*)$. From the properties of t-norms ($x_1 \star \ldots \star x_n = 1$ iff $x_i = 1$ for all i) we obtain $I_j(\boldsymbol{x}^*)$ to be determined as the intersection of particular cores $I_j(\boldsymbol{x}^*)$, i.e.,

$$I(\boldsymbol{x}^*) = \bigcap_{j=1}^{m} I_j(\boldsymbol{x}^*). \tag{10}$$

If $I(\boldsymbol{x}^*) \neq \emptyset$, then $y_k \in I(\boldsymbol{x}^*)$ are those actions from Y which are fully consistent with the input $\boldsymbol{x}^*$ under the given implicative rule base. That is, they make the evaluation of all rules in the rule base to be (simultaneously) 1, so they are natural candidates for the output of the I-FS for the given input $\boldsymbol{x}^* \in \mathcal{R}^n$.

Let as assume fuzzy set B' to be normal for any input $\boldsymbol{x}^*$ and therefore $core(B') = I(\boldsymbol{x}^*) \neq \emptyset$ for any $\boldsymbol{x}^*$. If we take as the output of I-FS with NCs an element from $I(\boldsymbol{x}^*)$ we can consider this process as defuzzification of B' set. The answer to the question of which concrete action from $I(\boldsymbol{x}^*)$ is taken (what deffuzification method is used) depends on concrete application. Here we will consider as output for given input $\boldsymbol{x}^* \in \mathcal{R}^n$ the whole set $I(\boldsymbol{x}^*)$. Formally written, the computational model of an I-FS with NCs has the form

$$\mathrm{I-FSNC}(\boldsymbol{x}^*) = I(\boldsymbol{x}^*) = \bigcap_{j=1}^{m} I_j(\boldsymbol{x}^*), \tag{11}$$

$$\mathrm{I-FSNC}(\boldsymbol{x}^*) = \bigcap_{j=1}^{m} \{\, y_k \,|\, A_j(\boldsymbol{x}^*) \leq B_j(y_k) \,\}. \tag{12}$$

Let us show that for the computation of an I-FS (with NCs) only firing rules are important. Indeed, let $\boldsymbol{x}^*$ be an input into the system, then two cases are possible with respect to the jth rule: either the rule does not fire, i.e., $A_j(\boldsymbol{x}^*) = 0$ or it fires, i.e., $A_j(\boldsymbol{x}^*) > 0$.

With respect to the first case of $A_j(\boldsymbol{x}^*) = 0$, we get immediately $I_j(\boldsymbol{x}^*) = Y$ on the basis of property (8) of residuated implication ($a = 0$). Forming the final output $I(\boldsymbol{x}^*)$ by intersection (11), we see that if we exclude $I_j(\boldsymbol{x}^*)$ from the intersection, then the result remains the same. Thus, if a rule in an I-FS does not fire then it can be excluded from the computation, under the assumption that at least one another rule fires. If none of rules fires, i.e., if $A_j(\boldsymbol{x}^*) = 0$ for all j, then $I(\boldsymbol{x}^*) = Y$.

With respect to general formula (11) there are two questions important. The first is related to how to specify $I_j(\boldsymbol{x}^*)$ sets

in an explicit way. The second is related to the assumption of non-emptiness of $I(\boldsymbol{x}^*)$ for any $\boldsymbol{x}^* \in \mathcal{R}^n$. The first question can be answered more explicitly in connection with the class of so called radial fuzzy systems. The second relates to the concept of coherence and for radial systems is treated in Section 3. Let us now introduce the class of radial fuzzy systems.

2.2. Radial I-FSs with NCs

The concept of a radial implicative fuzzy system with nominal consequents is defined as follows:

Definition 1: An implicative fuzzy system with nominal consequents is *radial* if:

(1) There exists a continuous function $act : [0,+\infty) \to [0,1]$, $act(0) = 1$ as follows: (a) either there exists $z_0 \in (0,+\infty)$ such that *act* is strictly decreasing on $[0,z_0]$ and $act(z) = 0$ for $z \in [z_0,+\infty)$ or (b) *act* is strictly decreasing on $[0,+\infty)$ and $\lim_{z\to+\infty} act(z) = 0$.

(2) Fuzzy sets in antecedent and consequent parts of the jth rule are specified as

$$A_{ji}(x_i) = act\left(\left|\frac{x_i - a_{ji}}{b_{ji}}\right|\right), \tag{13}$$

$$B_j(y_k) = \mu_{kj}, \tag{14}$$

where for each B_j there exists at least one action y_k such that $\mu_{kj} = 1$, i.e., fuzzy sets B_js are normal; $n,m,l \in \mathcal{N}$; $i,j,k = 1,\ldots,n,m,l$, respectively; $\boldsymbol{x} \in \mathcal{R}^n$, $\boldsymbol{x} = (x_1,\ldots,x_n)$; $y_k \in Y = \{y_1,\ldots,y_l\}$; $\boldsymbol{a}_j \in \mathcal{R}^n$, $\boldsymbol{a}_j = (a_{j1},\ldots,a_{jn})$; $\boldsymbol{b}_j \in \mathcal{R}^n_+$, $\boldsymbol{b}_j = (b_{j1},\ldots,b_{jn})$, (i.e., $b_{ji} > 0$); $\mu_{kj} \in [0,1]$.

(3) For each $\boldsymbol{x} \in \mathcal{R}^n$ the *radial property* holds, i.e.,

$$A_j(\boldsymbol{x}) = A_{j1}(x_1) \star \cdots \star A_{jn}(x_n) = act(||\boldsymbol{x} - \boldsymbol{a}_j||_{\boldsymbol{b}_j}), \tag{15}$$

where $||\cdot||_{\boldsymbol{b}_j}$ is a scaled version of some ℓ_p norm in $\mathcal{R}^n$. This norm is common to all rules of the fuzzy system.

Let us comment on the definition to clarify the concept. An I-FS with NCs is radial if it satisfies three requirements. Before we dicuss these requirements let us recall the concepts of radial function and ℓ_p norm, both defined in $\mathcal{R}^n$ space.

Radial functions are generally defined by formula $f(\boldsymbol{x}) = \Phi(||\boldsymbol{x} - \boldsymbol{a}||)$, where Φ is a function from $[0,+\infty)$ to $\mathcal{R}$, $||\cdot||$ is a norm in $\mathcal{R}^n$ and $\boldsymbol{a} \in \mathcal{R}^n$ is a central point of the function. Concerning radial fuzzy systems, the class of so-called ℓ_p norms in $\mathcal{R}^n$ is important [5]. The definition formula of ℓ_p norms depends on parameter $p \in [1,+\infty]$ and reads as follows:

$$\begin{aligned} &||\boldsymbol{u}||_p = (|u_1|^p + \cdots + |u_n|^p)^{1/p} \quad \text{for} \quad p \in [1,+\infty), \\ &||\boldsymbol{u}||_\infty = \lim_{p\to+\infty} ||\boldsymbol{u}||_p = \max_i\{|u_i|\}. \end{aligned} \tag{16}$$

Scaled ℓ_p norms, denoted by $||\cdot||_{p_{\boldsymbol{b}}}$, are derived from corresponding ℓ_p norms by incorporating a vector $\boldsymbol{b} \in \mathcal{R}^n_+$ of scaling parameters, $\boldsymbol{b} = (b_1,\ldots,b_n)$, $b_i > 0$, into the above formulas. That is,

$$\begin{aligned} &||\boldsymbol{u}||_{p_{\boldsymbol{b}}} = (|u_1/b_1|^p + \ldots + |u_n/b_n|^p)^{1/p};\ p \in [1,+\infty), \\ &||\boldsymbol{u}||_{\infty_{\boldsymbol{b}}} = \lim_{p\to+\infty} ||\boldsymbol{u}||_{p_{\boldsymbol{b}}} = \max_i\left\{|u_i/b_i|\right\}. \end{aligned} \tag{17}$$

Clearly, original unscaled ℓ_p norms are obtained from scaled ones by choosing $\boldsymbol{b} = \boldsymbol{1} = (1,\ldots,1)$. The most prominent examples of scaled ℓ_p norms are scaled octaedric ($p = 1$), Euclidean ($p = 2$) and cubic ($p = +\infty$) norms.

Now we can discuss the definition of a radial fuzzy system. The first two requirements are related to the specification of membership functions of fuzzy sets employed in IF-THEN rules. Especially, they relate to the shapes of one-dimensional fuzzy sets which form antecedents of rules.

The requirement (1) specifies the "shape" of one-dimensional fuzzy sets by specification of an *act* function. This function is considered to be generally non-increasing and can have two variants. The first variant corresponds to a strictly decreasing function, the other has strictly decreasing part and after reaching zero it is constant.

The requirement (2) is in fact the prescription which makes one-dimensional fuzzy sets A_{ji} to be radial. In one-dimensional space the norm correspond to the absolute value, central point corresponds to $a_{ji} \in \mathcal{R}$ and also (width) scaling parameter $b_{ji} \in \mathcal{R}_+$, ($\mathcal{R}_+ = (0,\infty)$, i.e., $b_{ji} > 0$) is employed. The shape is determined by *act* function.

Consequents' fuzzy sets B_js are required to be normal. In formula (14) the simplification of notation is adopted in form of $B_j(y_k) = \mu_{kj}$ (the indices are switched). Particular μ_{kj} can be seen as the membership degree of action y_k to the consequent of the jth rule.

The requirement (3) is in fact the radial property. The property requires a radial shape preservation of one-dimensional fuzzy sets in antecedents after their combination by a t-norm according to formula (2). Mathematically, the property is specified by equality (15). We can see that the property requires antecedents to be represented by radial functions (now in n-dimensional space $\mathcal{R}^n$) which have the same shape *act* as one-dimensional fuzzy sets A_{ji}. Moreover, central point $\boldsymbol{a}_j \in \mathcal{R}^n$ is required to be composed from one-dimensional central points a_{ji}, i.e., $\boldsymbol{a}_j = (a_{j1},\ldots,a_{jn})$. Similarly, scaling parameter $\boldsymbol{b}_j \in \mathcal{R}^n_+$ is required to be composed from one-dimensional scaling parameters b_{ji}, i.e., $\boldsymbol{b}_j = (b_{j1},\ldots,b_{jn})$.

The radial property is not trivial. If the specification of one-dimensional fuzzy sets A_{ji} is given together with a certain t-norm, then the specification of A_j is determined; and this specification need not have the form of a multidimensional radial function. For example, if triangular fuzzy sets are combined by the product t-norm then it can be shown [2] that the resulting representation of A_j does not exhibit the radial property in the sense of formula (15).

The question of which shapes (*act* functions) and t-norms can be combined so the radial property hold is partially

answered in [1, 2]. As an example of radial I-FSs let us present here Mamdani and Gaussian radial I-FSs [2].

In the Mamdani radial I-FS, the used t-norm is the minimum t-norm, corresponding residuated implication is the Goguen implication and *act* function has form $act(z) = \max\{0, 1-z\}$. This *act* function is of (1)(a) type of Definition 1. The resulting one-dimensional fuzzy sets are triangular and ℓ_p norm in antecedents is the cubic norm.

In the Gaussian radial I-FS, the used t-norm is the product t-norm, corresponding residuated implication is the Gödel implication and *act* function has form $act(z) = \exp(-z^2)$. This *act* function is of (1)(b) type of Definition 1. The resulting one-dimensional fuzzy sets are Gaussian curves and ℓ_p norm in antecedents is the Euclidean norm. In the case of this system the radial property can be easily verified on the basis of well know behavior of Gaussian curves with respect to product (the product of one-dimensional Gaussian curves is a multidimensional Gaussian curve).

2.3. Computational model of radial I-FSs with NCs

In the previous section we have presented the notion of a radial I-FS with nominal consequents. In this section we will discuss its computational model in a more explicit way.

As we have mentioned in Subsection 2.1, with respect to the general computational model of an I-FS with NCs there are important two questions. The first relates to the specification of particular outputs $I_j(\boldsymbol{x}^*)$ and the second to the coherence of the system. In the case of a radial I-FS with NCs we have the following straightforward answer to the first question: if the system is radial, then

$$I_j(\boldsymbol{x}^*) = \{\, y_k \mid act(||\boldsymbol{x}^* - \boldsymbol{a}_j||)_{\boldsymbol{b}_j} \leq \mu_{kj} \,\}. \tag{18}$$

Thus, on the basis of the radial property, a $y_k \in I_j(\boldsymbol{x}^*)$ if and only if a transformed (by *act* function) scaled norm of distance of input $\boldsymbol{x}^*$ from central point $\boldsymbol{a}_j$ is lower or equal to the membership degree of y_k to the consequent fuzzy set B_j.

In fact, the computational gain from the presence of radial property is not so significant as for the radial I-FSs with ordinal consequents [2]. However, the radial property allows us to explicitly express for which inputs $\boldsymbol{x}^* \in \mathcal{R}^n$ the action y_k is not included in $I_j(\boldsymbol{x}^*)$. Based on formula (18), we know that $y_k \notin I_j(\boldsymbol{x}^*)$ iff $act(||\boldsymbol{x}^* - \boldsymbol{a}_j||)_{\boldsymbol{b}_j} > \mu_{kj}$. Now, the following chain of equivalent inequalities can be introduced: let for an input $\boldsymbol{x}^*$ the action $y_k \notin I_j(\boldsymbol{x}^*)$, then

$$act(||\boldsymbol{x}^* - \boldsymbol{a}_j||_{\boldsymbol{b}_j}) > \mu_{kj}, \tag{19}$$

$$act_+(||\boldsymbol{x}^* - \boldsymbol{a}_j||_{\boldsymbol{b}_j}) > \mu_{kj}, \tag{20}$$

$$||\boldsymbol{x}^* - \boldsymbol{a}_j||_{\boldsymbol{b}_j} < act_+^{-1}(\mu_{kj}), \tag{21}$$

$$||\boldsymbol{x}^* - \boldsymbol{a}_j||_{\boldsymbol{b}_j} < r_{kj}, \tag{22}$$

where $r_{kj} = act_+^{-1}(\mu_{kj})$. The reverse holds too, i.e., if (22) holds then $y_k \notin I_j(\boldsymbol{x}^*)$.

In the above chain of inequalities, if *act* is of type (1)(a) of Definition 1, then act_+ is the restriction of *act* function on interval $[0, z_0]$. This restriction is a strictly decreasing function and therefore $act_+^{-1} : [0,1] \to [0, z_0]$ is well defined. If *act* function is of type (1)(b), then $act_+ = act$ for $z \in [0, +\infty)$ and $act_+^{-1} = act^{-1}$ on domain $(0,1]$. We set by definition $act_+^{-1}(0) = +\infty$. Thus, also in this case $act_+^{-1} : [0,1] \to [0, +\infty]$ is well defined. Based on the definition of act_+^{-1} function we see that values r_{kj} are well defined too, and $r_{kj} \in [0, +\infty]$.

As we will see in the next section, the possibility of introduction of inequality (22), which would not be possible without presence of the radial property, helps significantly in testing the coherence of radial I-FSs with NCs.

3. Coherence of radial I-FSs with NCs

The question of coherence of an implicative fuzzy system is the question of non-presence of contradictory rules in the rule base of the system. Incoherence is indicated by empty output of the system for certain input(s). The emptiness is caused by non-existence of common points in outputs of individual rules in the rule base (the intersection of particular outputs is empty). In order to avoid this situation we are looking for at least sufficient conditions on parameters of the system which assure that the case of empty intersection cannot occur for any possible input. Thus, we can say that the system is coherent if and only if for any possible input it has a non-empty output. In this section we will investigate the coherence of radial I-FSs with NCs. In order to obtain sufficient conditions we start from the computational model of this class of systems.

Based on chain of inequalities (19)–(22) we can state for every action y_k and rule j so-called *region of incoherence* RIC_{kj} as the set of those inputs[1] $\boldsymbol{x} \in \mathcal{R}^n$ for which $y_k \notin I_j(\boldsymbol{x})$. We have

$$RIC_{kj} = \{\boldsymbol{x} \in \mathcal{R}^n \mid ||\boldsymbol{x} - \boldsymbol{a}_j||_{\boldsymbol{b}_j} < r_{kj} \,\}. \tag{23}$$

In other words, RIC_{kj} is the set of those inputs which exclude action y_k from output $I_j(\boldsymbol{x})$, or, it is the set of those inputs for which action y_k is not included in output $I_j(\boldsymbol{x})$. Clearly, if $\boldsymbol{x} \notin RIC_{kj}$, then $y_k \in I_j(\boldsymbol{x})$. Based on the above formula, the regions of incoherence RIC_{kj}s, $k = 1, \ldots, l$, $j = 1, \ldots, m$ can be seen as deformed (due to the scaling in norm) hyperballs in $\mathcal{R}^n$ space.

For every action y_k, $k = 1, \ldots, l$, let us introduce its region of incoherence RIC_k as the union of RIC_{kj} over all rules j, i.e.,

$$RIC_k = \bigcup_{j=1}^{m} RIC_{kj}. \tag{24}$$

RIC_k can be interpreted as follows: if an input $\boldsymbol{x}$ is in RIC_k, then there exists a rule j such that $y_k \notin I_j(\boldsymbol{x})$ and therefore y_k is not included in the overall output $I(\boldsymbol{x})$ of the system. The reverse holds too, i.e., if $y_k \notin I(\boldsymbol{x})$ then $\boldsymbol{x} \in RIC_k$.

[1] From now on we will denote the input by $\boldsymbol{x}$ instead of former $\boldsymbol{x}^*$.

The next step in specification of conditions of coherence is straightforward.

Lemma 1: A radial I-FS with NCs is coherent if and only if

$$RIC = \bigcap_{k=1}^{l} RIC_k = \bigcap_{k=1}^{l} \bigcup_{j=1}^{m} RIC_{kj} = \emptyset. \tag{25}$$

Proof: Inspecting the intersection of RIC_k over possible actions two cases can occur. If this intersection is empty, then there does not exist any input which would excluded simultaneously all actions from the output of the system, i.e., the system is coherent. On the other hand, if the intersection is non-empty then points (inputs) in this intersection are witnesses of incoherence, as they exclude simultaneously all actions from the overall output. □

In the sequel we will investigate the intersection of unions of deformed hyperballs presented by formula (25). Actually, the question is how to test that the intersection of unions of hyperballs is empty (or non-empty). To solve this question, let us explicitly remark that if $\mu_{kj} = 1$, then $r_{kj} = 0$ (because $act_+^{-1}(1) = 0$) and formula (23) yields $RIC_{kj} = \emptyset$. On the other hand, if $\mu_{kj} = 0$ and *act* is of (1)(b) type of Definition 1, then $r_{kj} = +\infty$ and $RIC_{kj} = \mathcal{R}^n$. Both cases are important as we will see below.

Intersection (25) is non-empty (the system is incoherent) if and only if there exists a permutation with repetition $\pi = (j_1, \ldots, j_l)$, $\pi(1) = j_1, \pi(2) = j_2, \ldots$ of rule indices $\{1, \ldots, m\}$ such that the intersection

$$I_\pi = RIC_{1,j_1} \cap RIC_{2,j_2} \cap \cdots \cap RIC_{l,j_l} \tag{26}$$

is non-empty. The length of the permutation is l, i.e., it equals to the number of actions. If we show that for any permutation π with repetition of length l from rule indices $\{1, \ldots, m\}$ the intersection (26) is empty, then the corresponding radial I-FS with NCs is coherent.

To better understand the above, it is worth to consider the scheme presented in Table 1. If the system is incoherent, then there exists an input $\boldsymbol{x}$ such that simultaneously $\boldsymbol{x} \in RIC_k$ for all k. Since RIC_k is given by union of RIC_{kj}, this can be interpreted as follows: for this $\boldsymbol{x}$ and for every row k in Table 1 there exists a column j such that $\boldsymbol{x} \in RIC_{kj}$. We code the indices of these columns as permutation $\pi(k)$. Clearly, if for every such permutation π the intersection $\bigcap_k RIC_k$ is empty then the system is coherent. This kind of testing of coherence will be elaborated in the sequel.

Table 1
Incoherence regions

	$j=1$	$j=2$	$\ldots$	$j=m$	$\bigcup_j RIC_{kj}$
$k=1$	RIC_{11}	RIC_{12}	$\ldots$	RIC_{1m}	RIC_1
$k=2$	RIC_{21}	RIC_{22}	$\ldots$	RIC_{2m}	RIC_2
			$\vdots$		
$k=l$	RIC_{l1}	RIC_{l2}	$\ldots$	RIC_{lm}	RIC_l
					$\bigcap_k RIC_k$

There are two problems related to the testing scheme proposed. First, how to test the emptiness of intersection (26) for a given permutation π. Second, how to cope with the curse of dimensionality as the number of all permutations is m^l for given number of rules m and actions l.

Let I_π of (26) be non-empty for given permutation π, $\pi(k) = j_k$, $k = 1, \ldots, l$, i.e., $\pi = (j_1, \ldots, j_l)$. Then for $\boldsymbol{x} \in I_\pi$ we have $\boldsymbol{x} \in RIC_{k,\pi(k)}$ for all k, which yields the following k inequalities:

$$||\boldsymbol{x} - \boldsymbol{a}_{\pi(k)}||_{\boldsymbol{b}_{\pi(k)}} < r_{k,\pi(k)} \quad \text{for} \quad k = 1, \ldots, l. \tag{27}$$

To proceed let us remark that for any scaled ℓ_p norm in $\mathcal{R}^n$ and any vector $\boldsymbol{b} \in \mathcal{R}^n_+$, $\boldsymbol{u} \in \mathcal{R}^n$ the following inequality holds:

$$(1/\max_i\{b_i\}) \cdot ||\boldsymbol{u}|| \le ||\boldsymbol{u}||_{\boldsymbol{b}}, \tag{28}$$

and therefore if $||\boldsymbol{u}||_{\boldsymbol{b}_j} < r_{kj}$ then $||\boldsymbol{u}|| < (\max_i\{b_{ji}\}) \cdot r_{kj}$. In the sequel we set $sr_{kj} = \max_i\{b_{ji}\} \cdot r_{kj}$ for all k, j. On the basis of this notation and inequality (28), inequalities (27) imply

$$||\boldsymbol{x} - \boldsymbol{a}_{\pi(k)}|| < sr_{k,\pi(k)} \quad \text{for} \quad k = 1, \ldots, l. \tag{29}$$

Summing the above inequalities we obtain

$$\sum_k ||\boldsymbol{x} - \boldsymbol{a}_{\pi(k)}|| < \sum_k sr_{k,\pi(k)}. \tag{30}$$

Now, reversing the implication and employing the properties of norms in $\mathcal{R}^n$ we get the following theorem.

Theorem 1: Let $\Pi_{m,l}$ be the set of all permutations with repetition of length l from the set of rule indices $\{1, \ldots, m\}$. Let $\overline{\boldsymbol{a}}_\pi$ be the average vector formed from vectors $\boldsymbol{a}_{\pi(1)}, \ldots, \boldsymbol{a}_{\pi(l)}$. If for every $\pi \in \Pi_{m,l}$

$$\frac{1}{2} \sum_k ||\overline{\boldsymbol{a}}_\pi - \boldsymbol{a}_{\pi(k)}|| \ge \sum_k sr_{k,\pi(k)}, \tag{31}$$

then the system is coherent.

Proof: The theorem is a direct corrolarly of inequality (30). By the triangle inequality, for any $\boldsymbol{x}, \boldsymbol{a}_{\pi(k)} \in \mathcal{R}^n$ we have

$$\sum_k ||\boldsymbol{x} - \boldsymbol{a}_{\pi(k)}|| \ge ||\sum_k (\boldsymbol{x} - \boldsymbol{a}_{\pi(k)})|| = ||l\boldsymbol{x} - \sum_k \boldsymbol{a}_{\pi(k)}|| = l \cdot ||\boldsymbol{x} - \overline{\boldsymbol{a}}_\pi||. \tag{32}$$

We have also the following l inequalities valid ($\pi(1) = j_1$, $\pi(2) = j_2, \ldots$):

$$||\boldsymbol{x} - \boldsymbol{a}_{\pi(1)}|| + ||\overline{\boldsymbol{a}}_\pi - \boldsymbol{x}|| \ge ||\overline{\boldsymbol{a}}_\pi - \boldsymbol{a}_{\pi(1)}||, \tag{33}$$

$$||\boldsymbol{x} - \boldsymbol{a}_{\pi(2)}|| + ||\overline{\boldsymbol{a}}_\pi - \boldsymbol{x}|| \ge ||\overline{\boldsymbol{a}}_\pi - \boldsymbol{a}_{\pi(2)}||, \tag{34}$$

$$\vdots$$

$$||\boldsymbol{x} - \boldsymbol{a}_{\pi(l)}|| + ||\overline{\boldsymbol{a}}_\pi - \boldsymbol{x}|| \ge ||\overline{\boldsymbol{a}}_\pi - \boldsymbol{a}_{\pi(l)}||. \tag{35}$$

Summing these inequalities we get

$$\sum_k ||\boldsymbol{x}-\boldsymbol{a}_{\pi(k)}|| + l\cdot||\overline{\boldsymbol{a}}_\pi - \boldsymbol{x}|| \geq \sum_k ||\overline{\boldsymbol{a}}_\pi - \boldsymbol{a}_{\pi(k)}||. \quad (36)$$

Since $l\cdot||\overline{\boldsymbol{a}}_\pi - \boldsymbol{x}|| = l\cdot||\boldsymbol{x}-\overline{\boldsymbol{a}}_\pi||$ and inequality (32) holds the above gives

$$\sum_k ||\boldsymbol{x}-\boldsymbol{a}_{\pi(k)}|| + \sum_k ||\boldsymbol{x}-\boldsymbol{a}_{\pi(k)}|| \geq \sum_k ||\overline{\boldsymbol{a}}_\pi - \boldsymbol{a}_{\pi(k)}||, \quad (37)$$

$$\sum_k ||\boldsymbol{x}-\boldsymbol{a}_{\pi(k)}|| \geq \frac{1}{2}\sum_k ||\overline{\boldsymbol{a}}_\pi - \boldsymbol{a}_{\pi(k)}|| \quad (38)$$

for every $\boldsymbol{x}\in\mathcal{R}^n$. Therefore the minimum of the left side is bounded from below by the constant which is given by the right side. If this constant is greater or equal to the sum of $sr_{k,\pi(k)}$ then there cannot exists an $\boldsymbol{x}$ for which inequality (30) holds and intersection I_π for this π is empty. If inequality (31) holds for every $\pi\in\Pi_{m,l}$ then the system is coherent. □

Theorem 1 states the sufficient condition for checking the emptiness of intersection I_π for given π. The problem is that in order to test the coherence of a radial I-FS with NCs we have to perform generally m^l tests for all permutations π from $\Pi_{m,l}$ set. This number can be slightly lowered on the basis of the following lemma.

Lemma 2: If a radial I-FS with NCs is coherent, then for each rule j there must exist an action y_k such that $\mu_{kj}=1$.

Proof: Due to the properties of *act* function we have $r_{kj}=0$ iff $\mu_{kj}=1$. If for some rule j and all actions y_k would be $\mu_{kj}<1$, then also $r_{kj}>0$ for all k and $\boldsymbol{a}_j\in RIC_{kj}$ for all k. Considering permutation $\pi=(j,\ldots,j)$ we would get $\boldsymbol{a}_j\in I_\pi$ and the system would be incoherent. Let us note that if $r_{kj}=0$, i.e., if $\mu_{kj}=1$, then $RIC_{kj}=\emptyset$ and $\boldsymbol{a}_j\notin RIC_{kj}$. □

The direct corollarly of the above lemma is the fact that if the necessary condition is satisfied, which is our case, see Definition 1, then the number of permutations which have to be tested can be lowered to $(m-1)^l$ and only proper permutations have to be generated for testing. A permutation is proper if there exists at least two k_1, k_2 such that $\pi(k_1)\neq\pi(k_2)$.

We proceed by introducing Table 2 which is similar to Table 1 and contains in each cell the value of sr_{kj}.

As $r_{kj}\in[0,+\infty]$ and $\max_i\{b_{ji}\}>0$, we have $sr_{kj}\in[0,+\infty]$. $sr_{kj}=0$ iff $r_{kj}=0$, which corresponds to $\mu_{kj}=1$ and consequently to $RIC_{kj}=\emptyset$. So if there is zero in the kth row and the jth column of Table 2, then $I_\pi=\emptyset$ for permutations having $\pi(k)=j$ and these permutations need not be tested.

On the other hand, $sr_{kj}=+\infty$ iff $r_{kj}=+\infty$. This corresponds to the situation of $\mu_{kj}=0$ and *act* being of (1)(b) type. In this case the region of incoherence is given by whole space $\mathcal{R}^n$. Actually, for any input $\boldsymbol{x}$ to the system we have $A_j(\boldsymbol{x})>0$ and therefore y_k can never occur in the output of the system.

Concerning the last row of Table 2, we denote by $k^*(j)$ the index k for which maximum of sr_{kj} is reached in the jth column of the table. Thus $k^*(j)=\text{argmax}_k\{sr_{kj}\}$ and $rs_{\max,j}=rs_{k^*(j),j}$.

Table 2
Incoherence limit points

	$j=1$	$j=2$	...	$j=m$
$k=1$	sr_{11}	sr_{12}	...	sr_{1m}
$k=2$	sr_{21}	sr_{22}	...	sr_{2m}
			⋮	
$k=l$	sr_{l1}	sr_{l2}	...	sr_{lm}
max	$sr_{\max,1}$	$sr_{\max,2}$	...	$sr_{\max,m}$

Now, let the so-called *symmetric regions of incoherence* $SRIC_{kj}$ for given k,j be specified according to the following formula:

$$SRIC_{kj}=\{\boldsymbol{x}\in\mathcal{R}^n \mid ||\boldsymbol{x}-\boldsymbol{a}_j||<sr_{kj}\}. \quad (39)$$

Recalling the discussion presented when sr values were introduced (formula (28)) we can see that if $\boldsymbol{x}\in RIC_{kj}$, then $\boldsymbol{x}\in SRIC_{kj}$, i.e., $RIC_{kj}\subseteq SRIC_{kj}$ for all k,j and also $RIC_{kj}\subseteq SRIC_{k^*(j),j}$ for constant j and $k=1,\ldots,l$. The specification of $SRIC_{k^*(j),j}$ enables us to state the following theorem.

Theorem 2: Let a radial I-FS with NCs consists of m rules. Let for any pair of different rules $j_1,j_2\in\{1,\ldots,m\}$, $j_1\neq j_2$, the following holds:

$$||\boldsymbol{a}_{j1}-\boldsymbol{a}_{j2}||\geq sr_{\max,j_1}+sr_{\max,j_2}. \quad (40)$$

Then the system is coherent.

Proof: To start let us show that for the special case of $l=2$, the factor 0.5 can be ommited in formula (31). Let $SRIC_i=\{\boldsymbol{x}\in\mathcal{R}^n\,|\,||\boldsymbol{x}-\boldsymbol{a}_i||<sr_i\}$ for some $\boldsymbol{a}_i\in\mathcal{R}^n$, $sr_i\geq 0$, $i=\{1,2\}$. Let $SRIC_1\cap SRIC_2\neq\emptyset$, then there exists an $\boldsymbol{x}$ such that $||\boldsymbol{x}-\boldsymbol{a}_1||<sr_1$, $||\boldsymbol{x}-\boldsymbol{a}_2||<sr_2$ and also $||\boldsymbol{x}-\boldsymbol{a}_1||+||\boldsymbol{x}-\boldsymbol{a}_2||<sr_1+sr_2$. Since by the triangle inequality we have $||\boldsymbol{x}-\boldsymbol{a}_1||+||\boldsymbol{x}-\boldsymbol{a}_2||\geq||\boldsymbol{a}_1-\boldsymbol{a}_2||$, the minimum of the left side is reached for both $\boldsymbol{x}=\boldsymbol{a}_1$, $\boldsymbol{x}=\boldsymbol{a}_2$ and has the value $||\boldsymbol{a}_1-\boldsymbol{a}_2||$. So we can conclude that if the intersection of two hyperballs $SCRI_i$, $i\in\{1,2\}$ is non-empty then $||\boldsymbol{a}_1-\boldsymbol{a}_2||<sr_1+sr_2$.

Now, assume that under the validity of the above theorem the system is incoherent. Then there must exist a proper permutation π such that I_π is non-empty. Let $j_1=\pi(k_1)\neq\pi(k_2)=j_2$ for some k_1, k_2, then $RIC_{k_1,j_1}\cap RIC_{k_2,j_2}\neq\emptyset$. As $RIC_{k_i,j_i}\subseteq SRIC_{k^*(j_i),j_i}$ for $i=\{1,2\}$ then we have also $SRIC_{k^*(j_1),j_1}\cap SRIC_{k^*(j_2),j_2}\neq\emptyset$ which implies $||\boldsymbol{a}_{j1}-\boldsymbol{a}_{j2}||<sr_{\max,j_1}+sr_{\max,j_2}$. A contradiction. □

The above theorem reduces the number of inequalities that have to be tested in order to check the coherence of the system to $m(m-1)/2$, as inequalities (40) are symmetric with respect to j_1, j_2. However, the reduction of the number of tests is for the price of lowering the specificity of the tests. That is, the testing according to Theorem 2 will state more systems possibly incoherent than when the testing according to Theorem 1 is adopted. The reason for this fact is that in the case of Theorem 1 we check the emptiness of intersection of l hyperballs. In the case of Theorem 2, regardless the number l of actions is, we always test the intersection of only two hyperballs.

4. Conclusions

In the paper we have introduced the concept of the radial implicative fuzzy system with nominal consequents (radial I-FS with NCs). We have presented its computational model and investigated the notion of coherence for this class of fuzzy systems. We have presented two theorems stating two sufficient conditions (in fact set of conditions/inequalities) which assure the coherence of a radial I-FS with NCs.

The first sufficient condition, stated by Theorem 1, is based on the radial property which helps to investigate the coherence, however, it suffers from the curse of dimensionality because the number of tests to verify the coherence is generally $(m-1)^l$, where m is the number of rules and l is the number of actions.

The second sufficient condition, stated by Theorem 2, needs only $m(m-1)/2$ tests for the verification of coherence. However, the specificity of the second sufficient condition is lower than of the first condition. That is why, we recommend to use the tests according to the first condition anywhere where this is computationally tractable (low values of m and mainly l).

Because the lack of specificity during the tests according to the second sufficient condition, the next direction in our research is to elaborate an efficient tree-like algorithm for testing the coherence based on the first sufficient condition. The basic idea of this algorithm is not to test permutations which are clear to yield empty intersections because they contain empty sub-intersections.

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Paper

Multiple criteria and multiple periods performance analysis: the comparison of telecommunications sectors in the Maghreb countries

Gerard Colson, Karim Sabri, and Mapapa Mbangala

Abstract— In four Maghreb countries (Algeria, Morocco, Mauritania and Tunisia), a considerable improvement of the situation of the telecommunication operators has been noticed during the nineties. The evolution in these countries was very different depending on their economic policies, their effort of reorganization of their telecommunication sector and their technological change. Theses differences will be exhibited and analysed by comparing the operators' performances over a decade (1992–2001). A first approach is based on the Malmquist DEA TFP index for measuring the total factors productivity change, decomposed into technical efficiency change and technological changes. Second, using the Promethee II method and the software ARGOS, a multiple criteria analysis is performed, taking into account a larger scope of analysis. A main issue is that the general performance ranking of sets of operators by country is almost the same according to the two methods, although the variables of inputs and output used for the index of Malmquist are different and narrower in the considered scope than the chosen families of criteria used in method Promethee and software ARGOS. Both methods of analysis provide however complementary useful detailed information, especially in discriminating the technological and management progresses for Malmquist and the two dimensions of performance for Promethee: that are the service to the community and the enterprises performances, often in conflict.

Keywords—case study, multiple criteria decision aid, Promethee, Malmquist DEA TFP index, African telecommunications reforms, dynamic performance analysis.

1. Introduction

Since the beginning of the eighties, the telecommunications sector knew two great evolutions: a fast evolution of technology on one part, and several policies implying regulatory reforms, the liberalization and the privatization on the other part. These evolutions differ according to countries and to their development levels. If the developed countries knew very early the waves of liberalization, privatization, and globalization of their economies, it is only at the end of this 20th century that the African public companies knew these phenomena, since they remained for a long time the kept hunting of the authorities. Some arguments were advanced to explain this tendency and why these companies kept a level of financial profitability and/or productivity which is generally regarded as very low, if not poor [12, 14]. The economic opening by the liberalization for the services market and the deregulation of the communication infrastructures sharpened the appetite of the principal telecommunications operators of the zone OECD [9, 15] and privatization has become common across Africa [8]. And, even if there are still very strong disparities between the various countries of the zones North Africa and Middle East, a true explosion shocked the telecommunications sector during these last years. The number of private fixed lines knew or will know a clear increase (up to 67% between 1999 and 2007 according to IDATE). First operators on Internet made their appearance. The national markets were opened to the competition and the services of mobile telephony have been developed. Let us concentrate ourselves on Maghrebian telephony.

Karim Sabri was interested in the regulatory reforms in five countries: Algeria, Libya, Mauritania, Morocco and Tunisia [17]. Like other developing countries in Africa and elsewhere, the Maghreb countries have modified seriously their lawful frameworks for attracting foreign private investors and they have recently opened their telecom networks to the competition and the privatization: the state monopoly of the telecom is finished. All of them have reorganized, at the end of the nineties, their set of legal rules for facilitating the needed foreign investment and settled different control authorities, Libya still being penalized by its past behaviour, reprobated by the international community. In the comparison of the four other countries, appeared several differences in terms of reforms and of key macroeconomic and sector's figures, these differences may be considerable in terms of macro economy and demography: the comparison of telecom performances must then lay upon ratios and productivities, rather independent of the sizes differences.

We shall focus this paper on two quantitative methods in view of comparing performances of the telecom Maghrebian sectors. The Sections 2 and 3 will present successively the Malmquist index and the multiple criteria ranking by the method Promethee II included in the software ARGOS. These latter two analysis highlight the evolution of the productivities and the service and enterprise performances of four countries sectors among the five quoted in the previous

paper: Algeria, Morocco, Mauritania, Tunisia, since it was impossible to obtain the corresponding figures for Libya.

2. A comparison of productivities of the telecom sectors based on the index of Malmquist

2.1. Methodology of the Malmquist index analysis

The index of Malmquist is often used to evaluate the performance of public utilities networks, using physical data which are more available than financial data [2–4]. The Malmquist quantity index is composed of ratios of distance functions. It measures the total factor productivity change (TFP) between two data points (K and K' in Fig. 1), each one representing a same firm (or a sector in our paper) by its 2 coordinates (X,Y), X being an input (a set of inputs) and Y an output (a set of outputs) of a production of this firm, calculated at two different times t and $t+1$. Between these two times, changes may have occurred in the used technology of production or (and) in the firm's management. The index is obtained by computing the ratio of the distances for each of these two data points relatively to a common technology (at a same period). This technology is obtained as the efficient frontier of production determined by the set of firms to be compared at a given time by means of non parametric programming techniques, well known in data envelopment analysis (DEA). One can read a complete description of this method in [2].

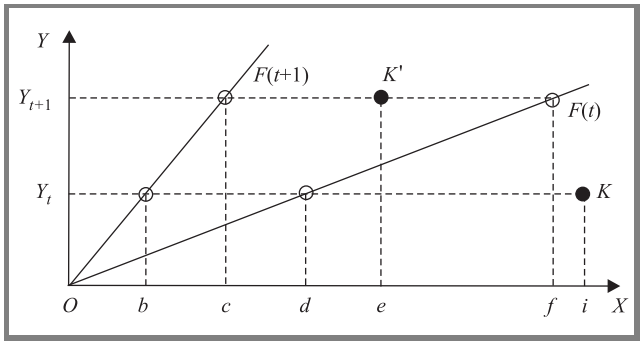

Fig. 1. The input based Malmquist index.

As presentation of this method, Fig. 1 [11] illustrates the input based Malmquist productivity index using constant returns to scale technology involving a single input and a single output. The technology at t is represented by $F(t)$ and at $t+1$ by $F(t+1)$, assuming that the firm produces at the points K and K' in periods t and $t+1$, respectively. In each period, the firm is operating below the technology for that period, meaning that there is a technical inefficiency in both periods. We can therefore compare and measure the firm's progress in term of productivity from period t to period $t+1$, and show that the improvement in productivity is due to the combination of the two factors: on one hand, the positive shift of the frontier is considered as the result of technical progress, while on the other hand, an improvement in the technical efficiency could allow the firm in K' to be closer to or even to reach the frontier of the period $t+1$. Since this frontier is determined by a set of country operators here, reaching it means only a relative efficiency.

The input based productivity index (M_{it}) for a firm i, in terms of the above distances along the x-axis (input axis) is given by

$$M_{it}(Y_{t+1},X_{t+1},Y_t,X_t) = \frac{Oe/Oc}{Oi/Od}\left[\frac{Oc\ Ob}{Of\ Od}\right]^{\frac{1}{2}}, \qquad (1)$$

where the first ratio $\frac{Oe/Oc}{Oi/Od}$ measures the technical efficiency and the last ratio measures the technical change by means of a geometric mean of such changes.

Technical efficiency is synonymous with production efficiency. From a production point of view, a company may be considered technically efficient when, for a given set of production factors, it succeeds in maximizing its output, or put in another view, it minimizes the total resources deployed (production factors) to attain a given production level, The associated gains in technical efficiency and productivity are mainly the result of improvements in the firm's managerial practices. Technical progress is also a source of productivity enhancement that may come from:

- new investments in equipment,
- innovation in the sector (staff training, availability of highly qualified managers, new production techniques as the introduction of cellular into each telecom network, etc.).

Our practical study relies on the physical data of four Maghrebian telecommunications sectors (Algeria, Morocco, Mauritania and Tunisia) over a ten years period (1992 to 2001). This method will allow us to know the origins of the positive or negative evolutions of the total productivity of the operators with a splitting into and a change of efficiency and a technological change. The first component often reflects the improvement of management within each network and the last one comes from the innovation (new investments).

2.2. Presentation of the data and choice of the variables

Table 1 gathers all the data available for the analysis, concerning the telecommunications operators in the four Maghreb countries during the period 1992–2001.

As output, we chose the outgoing total traffic in minutes for meaning operators production. We could have chosen for example the sales turnover that reflects the sold production of the operators. But several problems remaining about the availability of reliable data and diversity of the countable standards in each country encouraged us to avoid this kind of output.

For inputs, we adopted two inputs, very often used in DEA, that are the factors of work represented by the full time

Table 1

Inputs and output of the Malmquist DEA TFP index analysis

Sector of the country	Years	Outgoing total traffic [minutes]	Personnel (full time)	Principal lines
Algeria	1992	118 014 368	19 208	962 247
	1993	78 289 000	22 712	1 068 094
	1994	79 000 000	18 492	1 122 409
	1995	84 332 632	18 423	1 176 316
	1996	93 040 368	18 554	1 278 142
	1997	157 712 352	18 817	1 400 343
	1998	121 282 248	18 230	1 477 000
	1999	143 415 168	17 809	1 600 000
	2000	151 837 328	17 900	1 761 327
	2001	209 191 000	17 900	1 880 000
Morocco	1992	102 577 360	11 484	654 000
	1993	125 073 168	12 632	827 000
	1994	130 011 616	13 396	1 007 000
	1995	129 986 000	14 626	1 128 000
	1996	129 343 496	14 772	1 208 000
	1997	149 570 000	14 208	1 300 528
	1998	181 000 000	14 150	1 393 355
	1999	219 500 000	14 068	1 471 000
	2000	245 000 000	14 511	1 425 000
	2001	270 000 000	16 200	1 191 335
Mauritania	1992	4 357 334	400	6 750
	1993	4 277 511	410	7 499
	1994	4 503 822	456	8 426
	1995	4 127 943	451	9 249
	1996	4 889 159	443	10 204
	1997	5 475 163	456	13 045
	1998	6 300 266	454	15 030
	1999	8 078 267	480	16 525
	2000	9 029 041	720	18 969
	2001	9 800 000	600	25 199
Tunisia	1992	68 767 000	7 500	374 848
	1993	69 392 000	6 314	421 362
	1994	80 000 000	6 432	474 253
	1995	87 529 704	5 800	521 742
	1996	94 052 984	5 975	584 938
	1997	97 903 000	6 221	654 242
	1998	115 000 000	6 421	752 180
	1999	140 000 000	6 567	850 381
	2000	164 000 000	7 011	955 131
	2001	174 000 000	7 400	1 056 209

Sources: Algeria – Ministry for the Post and Telecommunication (MPT); Morocco – National Office of the Post and Telecommunications (NOPT); Mauritania – Office of the Post and Telecommunications (OPT); Tunisia – Tunisia Telecom.

Table 2
The mean productivity changes of the Maghrebian telecom of 1992 to 2001

Years	Malmquist index summary of annual means, converted in growths rates		
	Efficiency change [%]	Technological change [%]	Total factors productivity change [%]
1993/1992	–7.2	–2.2	–9.3
1994	–0.6	+5.6	+5.0
1995	–6.4	+4.9	–0.7
1996	–1.3	+5.7	+4.3
1997	+17.7	–1.6	+15.8
1998	–6.9	+10.0	+2.5
1999	+0.9	+17.5	+18.6
2000	+3.7	–1.9	+1.8
2001/2000	+12.3	–12.3	–1.5
Mean	**+1.2**	**+2.5**	**+3.7**
Note: all Malmquist index averages are geometric means.			

Table 3
The productivity changes of the Maghrebian telecom of 1992 to 2001 by countries

Sector of the country	Malmquist index summary of sector means converted in growths rates		
	Efficiency change [%]	Technological change [%]	Total factors productivity change [%]
Algeria	+0.5	+4.4	+4.9
Morocco	+2.2	+5.2	+7.6
Mauritania	0.0	–6.0	–6.0
Tunisia	+1.9	+7.1	+9.1
Mean	**+1.2**	**+2.5**	**+3.7**
Note: all Malmquist index averages are geometric means.			

personnel and of physical capital represented by the number of principal lines [18]. Let us recall that a principal telephone line is defined as a line of telephone connecting the equipment of the subscriber to the commutated public network, and giving him a particular interface with the telephone communication network.

2.3. Presentation and analysis of the results

From Table 2 we conclude that the total growth annual rate is 3.7% over the period 1992–2001. The decomposition of this rate shows that this growth comes primarily from the column "Technological change" which reflects the innovation in the telecommunications sector, maybe by the introduction of new technologies. This change can also be due to the entry of mobile telephony. Technological progress takes part at a rate of 2.5% in the growth rate. The remaining 1.2% of growth comes from the column "Efficiency change" that determines the evolution of the management of the sector. It is difficult to interpret the evolutions year per year since meaningful tendencies cannot be detected. We have converted the indices in growth rates[1].

Table 3 indicates for each year, which are the networks that contributed more than others to the improvement of productivity. Thus we can confirm that Tunisia comes at the head of the ranking while contributing at a rate of 9.1% to the total growth rate, followed by Morocco that presents 7.6%, then Algeria in third position with a rate of 4.9% and in last position Mauritania comes in showing the only negative rate of –6%. We can notice for the first three operators that the rates of the "Technological change" column are higher than those of the "Efficiency change" column, which confirms the idea that the annual total growth rate results primarily from the technical progress that reflects the innovation in the telecommunications sector and the introduction of new technologies, and that to the detriment of the management change. Nevertheless for Mauritania, the negative total productivity change (–6%) is due completely to the technological effect.

[1]Let us recall that indexes are f.i. for the last line 1.012 and 1.025 producing by multiplication: 1.037, thus 3.7% of growth decomposed into 1.2% and 2.5%. Thus the last column can be sometimes obtained by an addition as an approximation.

According to these average productivities growths of Table 3, we can rank the countries telecom sectors as follows:

1. Tunisia (+9.1%),
2. Morocco (+7.6%),
3. Algeria (+4.9%),
4. Mauritania (–6%).

3. Multi-criterion analysis by Promethee of the telecommunications performances of the Maghrebian operators

3.1. Data and ratios presentation

The data concerning the telecommunications operators in the four Maghrebian countries during the period 1992–2001 are gathered in Table 4 hereafter while on Table 5, we computed ratios being free of the rates of money changes and inflation.

3.2. Recalling the Promethee II method

Multiple criteria methods are well known in the literature [6, 16, 19]. One of the best known method is the second release of Promethee II by Brans *et al.* [1]. The Promethee II method is an outranking multiple criteria device that provides a preorder of items by making pair wise comparisons of these items (telecom sectors in our case), first for each criterion, and then for all criteria. The final ranking is obtained according to the decreasing order of the preference flows of the items. Among the six kinds proposed by the method, we used only one kind of criterion: the pseudo-criterion with a linear preference between the two thresholds (Fig. 2).

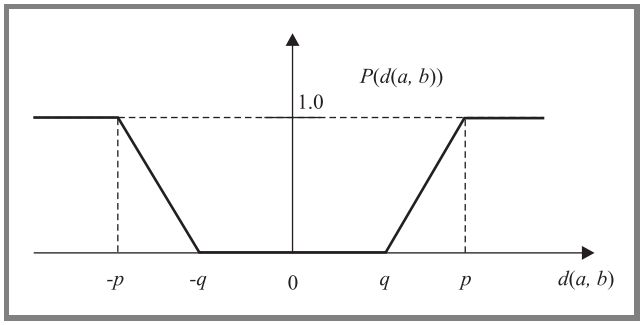

Fig. 2. The linear pseudo-criterion used in Promethee: $P(d) = 0$ if $|d| \leq q$, there is indifference; $P(d) = 1$ if $|d| > p$, there is a strong preference; $P(d) = (|d| - q)/(p - q)$ else , there is a weak preference.

Let a and b stand for two items and let $d(a,b)$ be the difference of their evaluations on a criterion c. We assume that a positive $d(a,b)$ corresponds to a preference for a over b. The preference function $P(d(a,b))$ is assumed to take the value 1 as soon as the preference is strong (= clearly stated), i.e., when $|d| > p$, the preference threshold, and is assumed to take the value 0 when an indifference between a and b is decided, since their evaluation difference does not reach the threshold q. Between these two decisions, a weak preference is expressed and P linearly increases with d.

Thus, this criterion states that a is surely preferred to b when $P(d(a,b)) = 1$. For the sake of simplicity, let us write $P_c(a,b) \equiv P_c(d(a,b))$: the preference function for the criterion c.

The method defines then a multi-criteria preference index as the weighted average of the preference functions P_c for all criteria. In our application, we considered that the three criteria of each point of view had the same weights[2]. The index $Phi(a,b)$ is computed by the next equation:

$$Phi(a,b) = \big(P_1(a,b) + P_2(a,b) + P_3(a,b)\big)/3\,. \qquad (2)$$

This index is called the (multi-criteria) preference flow of a over b. We are more confident that a is preferred to b according to all criteria of the considered family, when the flow value is closer to 1. Of course, a is surely preferred to b, when the unanimity of criteria is in favour of a, and $Phi(a,b) = 1$ then. At this stage, Promethee proposes to build a graph on the set K of considered items: its nodes are all of the compared items: $a,b,c,\ldots$ of K; the arcs joining two items are valued by $Phi(a,b)$ and $Phi(b,a)$ for a pair (a,b). Then, the method computes two flows for an item a:

$$Phi^+(a) = \sum_{b \in K} Phi(a,b)\,: \text{ the leaving flow;}$$
$$Phi^-(a) = \sum_{b \in K} Phi(b,a)\,: \text{ the entering flow.} \qquad (3)$$

One may interpret the leaving flow as a multi-criteria force of preference of a on the other items in K, and the entering flow as a multi-criteria preference weakness of a.

In Promethee II, a balance of flows is completed, delivering a net preference flow for each item a on all of the others items and for all criteria of the family:

$$Phi(a) = Phi^+(a) - Phi^-(a)\,: \text{ the net flow in favour of } a\,. \qquad (4)$$

Usually, by ranking the net flows in a decreasing order, we obtain the preference ranking of the items, the positive flows being associated to the dominating items and the negative ones to the dominated ones. An important point is that we did not divide the flows by $(n-1)$, n being the number of items in Eq. (3), like in the classical method, in order to point out the maximum number of possible dominances.

3.3. Methodology of the multi-criterion analysis at three levels

In order of analyzing by a multi-criterion method the performances of the telecommunication sectors in the four

[2]In a decision aid context, there is a subtle aid to supply to the deciders for the choice of weights attributed to the criteria (for instance, the software visual decision and, in some respect, ARGOS present a special aid for this allocation of weights). In a context of multiple criteria analysis where no stakeholders are considered, the choice of weighting must be neutral if no socio-economic consideration indicates a special weighting. This is the reason why we have adopted everywhere the same weights of criteria, families and dimensions of performance.

Table 4

Multi-criterion data representing the networks of 4 countries of Maghreb

Networks of the country	Years	Outgoing total traffic [minutes] (1)	Personnel (full time) (2)	Principal lines (3)	Teledensity* (4)	Income [USD] (5)	Investment [USD] (6)	Population (*1000) (7)
Algeria	1992	118 014 368	19 208	962 247	36.627	245 192 307	173 397 435	26 271
	1993	78 289 000	22 712	1 068 094	39.715	287 066 381	148 479 657	26 894
	1994	79 000 000	18 492	1 122 409	40.820	250 114 090	108 528 237	27 496
	1995	84 332 632	18 423	1 176 316	4.921	224 464 960	77 465 379	28 060
	1996	93 040 368	18 554	1 278 142	44.743	227 506 849	128 493 150	28 566
	1997	157 712 352	18 817	1 400 343	48.213	224 345 867	98 631 086	29 045
	1998	121 282 248	18 230	1 477 000	50.056	287 248 893	146 305 754	29 507
	1999	143 415 168	17 809	1 600 000	53.422	290 821 691	114 465 975	29 950
	2000	151 837 328	17 900	1 761 327	57.965	305 607 228	105 465 311	30 386
	2001	209 191 000	17 900	1 880 000	60.968	361 642 061	96 464 646	30 836
Morocco	1992	102 577 360	11 484	654 000	25.600	563 700 234	257 611 241	25 547
	1993	125 073 168	12 632	827 000	31.724	500 537 634	193 655 913	26 069
	1994	130 011 616	13 396	1 007 000	38.621	550 108 695	269 130 434	26 074
	1995	129 986 000	14 626	1 128 000	42.373	659 367 681	312 412 177	26 621
	1996	129 343 496	14 772	1 208 000	44.461	695 183 486	197 591 743	27 170
	1997	149 570 000	14 208	1 300 528	47.261	683 001 049	180 797 481	27 518
	1998	181 000 000	14 150	1 393 355	50.316	773 541 666	131 666 666	27 692
	1999	219 500 000	14 068	1 471 000	52.786	867 857 142	237 346 938	27 867
	2000	245 000 000	14 511	1 425 000	49.643	1 128 880 526	221 072 436	28 705
	2001	270 000 000	16 200	1 191 335	40.841	1 415 929 203	229 209 687	29 170
Mauritania	1992	4 357 334	400	6 750	3.262	25 830 173	1 401 815	2 069
	1993	4 277 511	410	7 499	3.531	20 428 772	3 294 429	2 124
	1994	4 503 822	456	8 426	3.865	24 081 566	9 443 275	2 180
	1995	4 127 943	451	9 249	4.135	24 936 425	12 391 153	2 237
	1996	4 889 159	443	10 204	4.444	27 444 978	17 431 861	2 296
	1997	5 475 163	456	13 045	5.535	29 720 118	12 683 569	2 357
	1998	6 300 266	454	15 030	6.213	28 278 862	5 565 577	2 419
	1999	8 078 267	480	16 525	6.655	2 880 974	4 095 269	2 483
	2000	9 029 041	720	18 969	7.445	25 230 202	4 830 423	2 548
	2001	9 800 000	600	25 199	9.640	26 905 588	4 462 846	2 614
Tunisia	1992	68 767 000	7 500	374 848	44.329	200 000 000	126 136 363	8 456
	1993	69 392 000	6 314	421 362	48.679	194 000 000	129 000 000	8 656
	1994	80 000 000	6 432	474 253	53.978	218 811 881	121 782 178	8 786
	1995	87 529 704	5 800	521 742	58.217	262 105 263	134 736 842	8 962
	1996	94 052 984	5 975	584 938	63.984	296 907 216	198 969 072	9 142
	1997	97 903 000	6 221	654 242	70.821	326 126 126	138 738 738	9 238
	1998	115 000 000	6 421	752 180	80.576	350 877 192	156 140 350	9 335
	1999	140 000 000	6 567	850 381	89.892	378 151 260	104 201 680	9 460
	2000	164 000 000	7 011	955 131	99.638	400 729 927	159 124 087	9 586
	2001	174 000 000	7 400	1 056 209	108.887	475 694 444	212 500 000	9 700

* This column is expressed in: –/1000 inhabitants.

Sources: Algeria – Ministry for the Post and Telecommunication (MPT); Morocco – National Office of the Post and Telecommunications (NOPT); Mauritania – Office of the Post and Telecommunications (OPT); Tunisia – Tunisia Telecom.

Table 5
Ventilation of the criteria according to the families and dimensions, the thresholds

Dimensions, families, criteria	Threshold q	Threshold p	Criterion direction
Technical-economic evaluation			
Economic family of criteria:			
– traffic part for 1000 inhabitants = (1) / (7)	500	5000	Max
– traffic(*1000 min) / sector income = 1000*(1) / (5)	10	100	Max
– investment part for 1000 inhabitants = (6) / (7)	500	5000	Max
Lines family of criteria:			
– teledensity = (4)	5	50	Max
– number of lines / number of personals = (3) / (2)	2	20	Max
– number of lines / sector investment = (3) / (6)	1	100	Max
Sector performance evaluation			
Traffic family of criteria:			
– traffic / number of lines = (1) / (3)	20	200	Max
– traffic / number of personals = (1) / (2)	100	1000	Max
– traffic / investment = (1) / (6)	0.25	1	Max
Income family of criteria:			
– sector income / number of lines = (5) / (3)	200	2000	Max
– sector income / number of personals = (5) / (2)	1000	5000	Max
– sector income / sector investment = (5) / (6)	2	8	Max

Maghreb countries, we took as a starting point the work [7]. Authors based their analysis on two dimensions of performance of the public companies: namely the effectiveness of the public service and the efficiency of those in terms of using resources. These authors constituted a hierarchy at 3 levels of the selected criteria. Here we have initially incorporated 3 basic criteria to constitute a coherent family and that for 4 families, which are then gathered into two dimensions of analysis. Table 5 presents this hierarchy and the preference and the indifference thresholds of the adopted twelve pseudo-criteria. According to the first dimension described as technical-economic, we aim at collecting the performance of the sector from the points of view of the user in technical terms and of the economic health of the sector: they will be the two families: economic and lines. This first dimension is a general performance function measuring the importance of the (public) service given to the user and to the country by the telecom sector. The second dimension evaluates the physical and financial enterprise performances of the set of companies of the telecom sectors; this is done by countries and they are entitled under the names of family: traffic and income. We adopted only one type: the linear pseudo-criterion (with two thresholds of decision), since this type fits well less reliable data than usual and avoids a strong preference for a small variance. The first threshold (q) is the limit between a decision of indifference between two actions and a decision of weak preference.

For the calculation of multi-criterion preference flows of all sectors, we used software ARGOS [5], which has the advantage of being able to treat directly two levels of hierarchy of criteria. Recall however that the multi-criterion flows are not reduced in an interval [0,1] in this software, as it was in the original Promethee method.

Table 5 synthesizes the criteria and the families with their thresholds. The second column indicates the thresholds q which mark the end of an indifference between two operators due to the weakness of the differences in evaluation between these two operators on a same criterion; a third column indicates the thresholds p and a last column shows the preferable direction (max or min) for each criterion.

3.4. Interpretation of the results of the multiperiod and multi-criterion rankings

According to Table 5, we got three levels of analysis of the performances of the telecom sectors in Maghreb for every year 1992 to 2001. At the upper level, we obtained Table 6 that is the aggregation of preference flows of Table 7, that are the four applications of Promethee II to the data of Table 4 for each family of criteria, taking into account the ventilation and the thresholds of Table 5. Each cell of Table 7 contains a multi-criterion net preference flow (multiplied by ten for more readability) indicating how much the corresponding country sector dominates the other ones in this family, if it is positive. A negative flow indicates how much the sector is dominated by the others in its family. For instance in 1992, the Mauritanian sector dominates the 3 other sectors in traffic and income (the flow is then +30/10 = +3). All other figures results of flows additions, vertically or horizontally. The horizontal total additions

Table 6
Promethee II preference flows of general performance dimensions by sub-periods for telecom in four Maghrebian countries

Years	Algeria		Morocco		Mauritania		Tunisia		Balances
	Enterprise performances	Service	Enterprise performances	Service	Enterprise performances	Service	Enterprise performances	Service	
1992	–32	12	–5	–1	60	–39	–22	27	0
1993	–27	–6	–2	6	30	–38	–2	39	0
Before change	–59	6	–7	5	90	–77	–24	66	0
1994	–27	–6	–3	7	30	–38	1	36	0
1995	–24	–3	–9	3	28	–37	5	37	0
1996	–30	–1	–10	–5	32	–32	7	39	0
1997	–20	4	–14	–1	28	–40	5	38	0
During change	–101	–6	–36	4	118	–147	18	150	0
1998	–30	2	–3	–3	32	–40	2	40	0
1999	–31	–1	–5	2	32	–41	5	39	0
2000	–19	1	7	–6	9	–36	3	41	0
2001	–22	7	9	–11	15	–35	–2	39	0
After change	–102	9	8	–18	88	–152	8	159	0
Total	–262	9	–35	–9	296	–376	2	375	0

Source: calculation of Karim Sabri from ARGOS results.

Table 7
Promethee II preference flows of performance by families of criteria by sub-periods for telecom in four Maghrebian countries

Years	Algeria					Morocco					Mauritania					Tunisia					Balances
	Traffic	Income	Economic	Lines	**Total**	Traffic	Income	Economic	Lines	**Total**	Traffic	Income	Economic	Lines	**Total**	Traffic	Income	Economic	Lines	**Total**	
1992	–14	–18	7	5	–20	–3	–2	–6	5	–6	30	30	–23	–16	21	–12	–10	21	6	5	0
1993	–12	–15	–6	0	–33	–1	–1	–1	7	4	6	24	–21	–17	–8	6	–8	28	11	37	0
Before change	–26	–33	1	5	–53	–4	–3	–7	12	–2	36	54	–44	–33	13	–6	–18	49	17	42	0
1994	–15	–12	–7	1	–33	–3	0	0	7	4	10	20	–20	–18	–8	8	–7	26	10	37	0
1995	–10	–14	–4	1	–27	–6	–3	–2	5	–6	8	20	–18	–19	–9	8	–3	24	13	42	0
1996	–15	–15	–1	0	–31	–6	–4	–9	4	–15	12	20	–13	–19	0	9	–2	24	15	46	0
1997	–5	–15	4	0	–16	–9	–5	–6	5	–15	9	19	–20	–20	–12	5	0	23	15	43	0
During change	–45	–56	–8	2	–107	–24	–12	–17	21	–32	39	79	–71	–76	–29	30	–12	97	53	168	0
1998	–15	–15	4	–2	–28	–3	0	–8	5	–6	14	18	–20	–20	–8	5	–3	23	17	42	0
1999	–17	–14	0	–1	–32	–10	5	–1	3	–3	20	12	–21	–20	–9	8	–3	21	18	44	0
2000	–6	–13	–1	2	–18	–4	11	–5	–1	1	8	1	–16	–20	–27	2	1	22	19	44	0
2001	–9	–13	1	6	–15	–5	14	–5	–6	–2	15	0	–16	–19	–20	–2	0	20	19	37	0
After change	–47	–55	4	5	–93	–22	30	–19	1	–10	57	31	–73	–79	–64	13	–5	86	73	167	0
Total	–118	–144	–3	12	–253	–50	15	–43	34	–44	132	164	–188	–188	–80	37	–35	232	143	377	0

Source: calculation of Karim Sabri from ARGOS results.

give zero, since the flows of dominating sectors are exactly compensated by those of the dominated ones.

Let us interpret some figures of Tables 6 and 7. For instance we shall compare the first line representing the year 1992 and the last line of the year 2001. In these lines, we observe first the traffic performance, remembering that the traffic family will synthesize 3 criteria that are the ratios where the importance of traffic is reported respectively to the number of lines, to the number of personals and to the investment. These 3 ratios can be considered as 3 measures of productivity in terms of traffic produced by the available resources of each sector in lines, manpower and capital variation. In 1992, we observe that Mauritania has a positive flow of 30 while Morocco, Tunisia and Algeria have the three negative flows –3, –12 and –14 respectively. For understanding well these figures, we must remember that we proceed to a multiple criteria comparison of *relative* performances of the 4 sectors and that the sum of these four figures is zero (at the rounding close) – the balance of flows by family and by line should be zero. By obtaining a high positive preference flow of 30, we see that the telecom sector of Mauritania in 1992 is dominating the other 3 sectors of Morocco, Tunisia and Algeria in terms of its capacity to produce good ratios traffic/resources. The Algerian and Tunisian sectors have relatively weak ratios.

Nine years later in 2001, the situation of this family is quasi similar except for 2 observations: if the dominance of the Mauritanian sector (a flow of 15) on the 3 others still exists, its importance has been divided by 2, while the Tunisian sector is now second in the ranking for this family. Let us consider for these two lines 1992 and 2001, the relative performances in terms of production of income with these same resources, i.e., the family of income.

In 1992, the situation of the four compared sectors is nearly the same for this ratio income/resources: Mauritania has the same dominance (30) and the ranking is identical; this is not a surprise since a higher traffic for given resources should generally produce a higher income. However, it must be observed that the Moroccan sector has reached the Mauritanian sector performances in 2000 with a flow of +11 in 2000 and +14 in 2001. This disparity of performance of the Moroccan sector in terms of traffic and income could be explained by an increasing of the price paid by minute in this sector since 2000. This tendency of a better relative financial performance of the Moroccan sector is perceptible during all the sub-period 1998–2001 that follows the phenomena of privatization of this sector.

If we aggregate the flows of the 2 families for obtaining the flows of the general enterprise performance showed in Table 6, we are not surprised that the Mauritania's sector is still considered more dominating in 1992 with a mark of 60. Since the traffic and the income ratios reported to the same resources are normally highly correlated, it seems that there is some kind of double counting in these aggregated flows in this dimension of enterprise performances. This potential double counting tends to disappear when a differential of prices marks the compared sectors like in the years 2000 and 2001.

Until now, we can summarize our analysis by observing that the Mauritanian sector has higher ratios of traffic and income than the 3 other sectors but that this relative better performance is no more true with respect to the Moroccan sector that becomes the best or equivalent in terms of income and not in terms of traffic at the end of the period 1992–2001. The last observation that the Moroccan sector has relatively progressed in terms of income but not in traffic ratios may leave us with the supposition that the privatization was not so favourable to the consumer who will pay a relatively higher unit price.

So it is useful to observe the other general objective of a telecom sector: **its capacity to supply some public service**, measured here by the production of lines by 1000 inhabitants and by used resources in terms of manpower and investment for the family lines, and measured for the family economic by the traffic and the investment of the sector reported to the number of 1000 inhabitants or to the sector income. As the theory announced it, there may exist some conflict between the two general objectives of the enterprise performance and of the public service, although the relative excellence of a sector would be to be very good in the two dimensions. Clearly this kind of relative excellence is not reached by any Maghrebian telecom sector. Indeed, we observe on Tables 6 and 7, that the Mauritanian sector, that is the best in terms of enterprise performances, is also the worst in terms of service to the consumers and to the economy, and that for all the period 1992–2001. This result is no more astonishing if we recall that the Mauritanian telecom sector is still little developed.

From a very general point of view, by looking at the total of flows for the whole period at the bottom of Table 6, we read the following figures for the service: Tunisia 375, Algeria 8, Morocco –10 and a very low score of –374 for Mauritania!

This clearly means that the Tunisian telecom sector produced the relatively best service in Maghreb and the Mauritanian one the relatively worst. For the other dimension of enterprise performances, it is also clear that the Mauritanian sector was the best with a score of 297 and the Algerian one was the worst with a mark of –261.

The general rankings according to each of these 2 dimensions and together are thus:

- **Enterprise performances**:
  1. Mauritania (296),
  2. Tunisia (2),
  3. Morocco (–35),
  4. Algeria (–262);
- **Service technical-economic**:
  1. Tunisia (375),
  2. Algeria (9),
  3. Morocco (–9),
  4. Mauritania (–376);
- **Together**:
  1. Tunisia (377),
  2. Morocco (–44),
  3. Mauritania (–80),
  4. Algeria (–253).

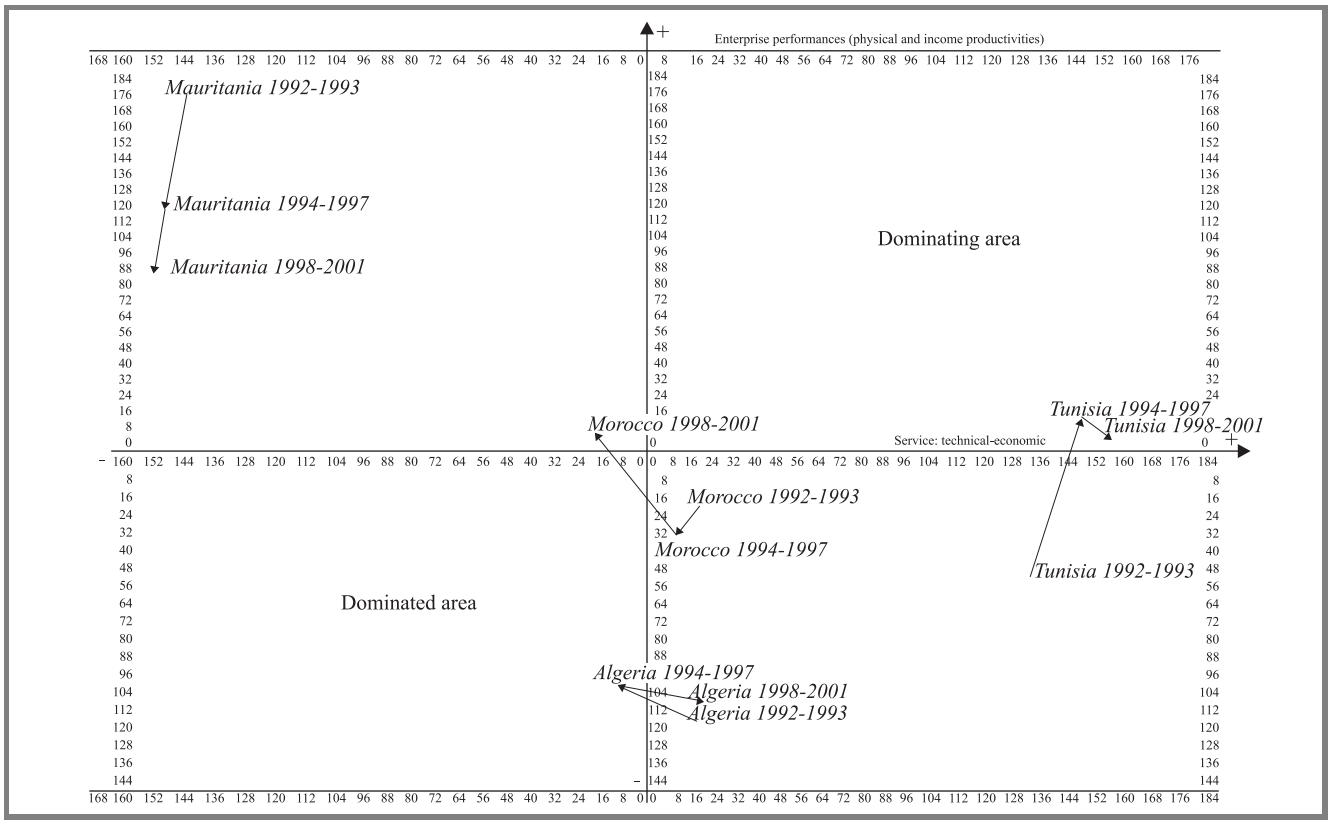

Fig. 3. Relative positions and moves of the telecom sectors in terms of performances versus service (1992–2001).

Finally, let us observe that the Tunisian telecom sector is the sole sector having a positive preference flow for the two dimensions for all the period (3;375): it is not so far from a relative excellence[3] in Maghreb.

3.5. Interpretation of the results by sub-periods

The main changes of laws and regulation for the Maghrebian telecom sectors took place during the sub-period 1994–1997. Therefore, we want to compare the changes of relative positions of the telecom in the four countries between the three sub-periods: before the change (1992–1993), during the change (1994–1997) and after the change (1998–2001). Thus, we shall use the corresponding lines in Tables 6 and 7 that have been visualized under the form of moves in the three figures (Figs. 3–5). First, let us remark that in Tables 6 and 7 and in Figs. 3–5, the figures of the sub-period totals before change have been doubled for a possible comparison with the other sub-periods totals that aggregate the flows of 4 years rather than 2 years. Also, in Fig. 3, the relative flows per period for the four telecom sectors have been mapped into the two dimensions (enterprise and service performances), in Fig. 4, the mapping is done for the two components of service: economics and lines and the mapping of the two other components of enterprise performance: traffic and income productivities is sketched in Fig. 5.

By looking at Fig. 3, the following general moves between periods can be observed:

1. Tunisia's sector, that had the best position in terms of service but a dominated position in terms of enterprise performance before the change, seems to benefit of the change in improving a bit its enterprise performances by passing in a dominating position (positive flows) during and after the change periods.

2. Mauritania's sector, that had the best position in terms of enterprise performance but a dominated and the lowest position in terms of service before the change, seems to deteriorate relatively its enterprise performances, however keeping its leader's position (positive flows) during and after the change periods.

3. Morocco's and Algeria's sectors are and remain in median positions with respect to the service, while Algeria's sector remains in the lowest position in terms of enterprise performances all through the changes.

4. We can summarize the ranks evolutions in Table 8.

[3]It must be underlined again that all performances are only expressed in **relative terms** issued from the comparison and that no assessment of absolute value is attempted in this method.

Fig. 4. Relative positions and moves in service performances in terms of economy and lines (1992–2001).

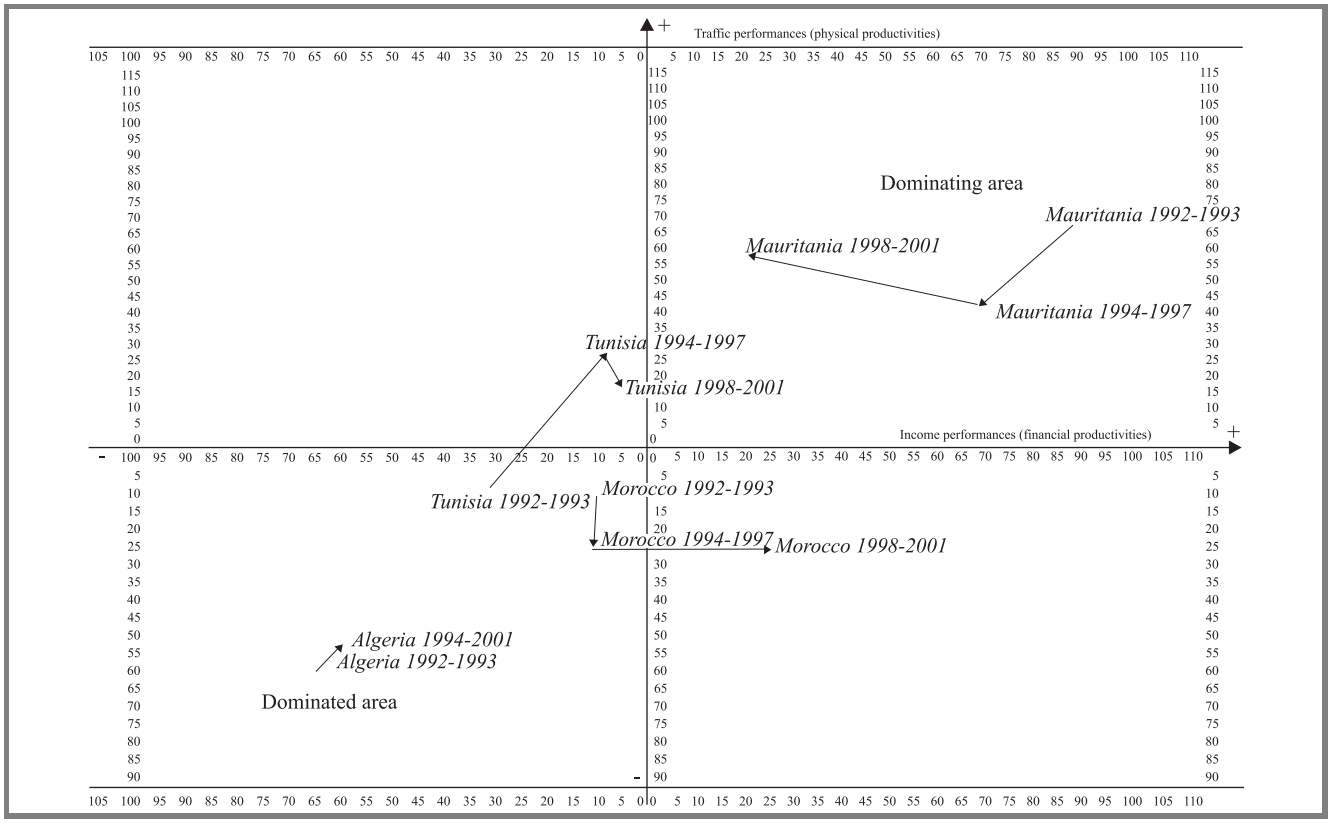

Fig. 5. Relative positions and moves in enterprise performances in terms of traffic and income (1992–2001).

Table 8
The telecom sectors' ranks for each sub-period and by dimension; the general ranking of the countries sectors for 1992–2001

Sub-periods	Sector of the country							
	Algeria		Morocco		Mauritania		Tunisia	
	Performance	Service	Performance	Service	Performance	Service	Performance	Service
1st sub-period 1992–1993	4	2	2	2	1	4	3	1
2nd sub-period 1994–1997	4	**3**	**3**	2	1	4	**2**	1
3rd sub-period 1998–2001	4	**2**	3	**3**	1	4	2	1
1992–2001 ranks	4	2	3	3	1	4	2	1
General ranking	4		2		3		1	
Note: the changes of ranks from one sub-period to another are in bold characters.								

The consideration of the small and few changes of ranks from one sub-period to another reveals the weak influence of the technological and lawful changes on the relative positions of the telecom operators in Maghreb. Although its relative backward in terms of service, the Mauritanian telecom sector remains in the same positions: 1st on the enterprise performance and last (4th) on the service, all through the sub-periods. Except for Tunisia, the changes of ranks are not significant. *We could say that the Tunisian telecom sector benefited more than the others of the changes in laws and techniques, during the sub-period 1994–1997*.

Looking now at the level of families, we consider Figs. 4 and 5.

In Fig. 4, the service supplied by the sectors is decomposed into the physical service of production of lines and the influence to economy.

The issues are as follows:

1. The Tunisian sector maintains its high positive flows all through the period with an improvement of the productivity on lines and a small decrease of the economic flows.
2. The Mauritanian sector is nearly stationary with a small increase of economic flows counterbalanced by the small decrease of the productivity on lines.
3. In the middle remain positioned Algeria's and Morocco's sectors that change a little.
4. There is a significant relative decrease of the productivity on Moroccan lines and a less important decrease in both components of service for the Algerian sector.

On Fig. 5, the enterprise performances achieved by the sectors are decomposed into the physical traffic and financial income reported on the used resources. The issues are as follows:

1. Here, the relative moves are greater, except for the Algerian sector, that improves a little both performances.
2. Like for the falling move in economic service, the Mauritanian sector movement is characterized by a considerable fall of flows in the income ratio and a small decrease of the traffic ratio followed by an increase. This differential of behaviours is explained by a fall of the unit prices of Mauritanian telecommunications with the respect to other Maghrebian prices, and by the increase of Moroccan unit prices.
3. Both Tunisian performances are marked by a considerable relative improvement during the change period, followed by a small decrease of the sole productivity on lines.
4. After a short fall in both components of performance enterprise, the Moroccan sector makes a bound of income performance in 1998–2001 while the physical performance remains stationary. Combining this observation with the point 2, it appears clear that a differential of unit prices between Morocco and Mauritania is the probable cause of such compensatory moves of relative financial performance of both sectors.
5. The Algerian sector sees its physical more than its financial enterprise performance progresses slightly.

Summarizing the issues about sub-periods moves, observed in Figs. 4 and 5, and Tables 6, 7 and 8, we could dare the following conclusions:

1. In terms of enterprise performances, there is some centripetal tendency to a convergence of performances of the telecom sectors in Maghreb.
2. But in terms of service to the community, the divergence is rather observed for the two extremes: Tunisian and Mauritanian sectors, the Tunisian telecom increasing its advance in lines production and the Mauritanian telecom being distanced.

4. Synthesis and conclusion

According to the productivities analysis, the Malmquist index revealed a general growth of the Maghrebian telecom productivities, reaching only 3.7% on a decade 1992–2001, decomposed in 2.5% due to the technological change and 1.2% in a better management. These growths are very unequal for the four compared countries, attaining 9.1% for Tunisia, 7.6% for Morocco, 4.9% for Algeria while Mauritania had a negative growth of 6%. Now in all these countries, we observed very small changes due to the management change and this result may indicate that the regulatory reforms and privatisation has not yet produced their expected effects of improvement of the management.

We enlarged the scope of analysis in the Promethee II multiple criteria comparisons of the four countries, taking into account the two general objectives of a utility enterprise: the performances of the enterprises in the sector, on one hand, and the service given to the community, on the other hand. According to these two classical dimensions, we observed, without a great surprise, that the leader in enterprise performances: Mauritania's sector becomes the last ranked in service. This is some confirmation of the theoretical hypothesis that it is difficult to be the best or even good in both dimensions, which are often in conflict. However, the Tunisian sector is close to the relative excellence, by being largely the best in service and the second in enterprise performances. More details were obtained in decomposing these two general objectives, each one into two families of criteria.

For the first dimension: the enterprise performances, the productivities of the resources were declined in terms of traffic and of income of the telecom sectors. Of course, these two kinds of ratios are mainly different by the unit price of the traffic: when the countries tariffs are constant or have the same moves, both ratios produce the same moves. So, the Mauritanian sector practiced a strong unit prices decrease from 1995 to 2001 while a small unit prices rise was observed in Morocco since 1998. These corresponding "bad" income performance move for Mauritania and "good" income performance move for Morocco were indeed a benefit for the Mauritanian consumers and a loss for the Moroccan ones, since both traffics increased in the same periods. For the second dimension: the relative service progress in lines productivity was observed for the service leader, i.e., the Tunisian sector of telecom.

In terms of evolution of the relative positions of the different sectors, we tried to observe the impact of regulatory and competition changes on these countries relative positions: the main conclusion for these moves through the three subperiods of analysis is that only small changes of ranks were observed revealing the weak influence of the technological and lawful changes on the relative positions of the telecom operators in Maghreb. This latter conclusion, in turn, can be explained by two hypotheses: either the differences in technical and legal evolutions are too weak or these differences have not yet produced all their effects.

A last comparison was potentially possible between the results issued from the Malmquist DEA TFP index analysis and the Promethee method. From a general point of view, the rankings of the countries telecom sectors were similar. According to the Malmquist index, the best progress was observed in Tunisia, then for Morocco, Algeria being the third and Mauritania being the last with a regression while in terms of management progress, Morocco is a bit before Tunisia, both before Algeria and Mauritania. By adding the preference flows of the two dimensions service and enterprise performance, we obtained the ranking: the Tunisian sector remains the leader, then comes the Moroccan sector followed by the Mauritanian sector, far before the Algerian one. The only difference of general ranking between both methods bears on the last position of Mauritania's or Algeria's sectors. Now, on the field, we can consider that these two countries as nearly incomparable in terms of populations, political regimes, and telecom sectors. Anyway the two methods of analysis are indeed different in scope and used data and they give complementary information. While the Malmquist index analysis can separate the effects of technological and management changes, the 3-levels multiple criteria method can score the preference flows via two levels of aggregation and highlights the two dimensions of service and enterprise performance, useful to fully evaluate a utility sector like the telecom.

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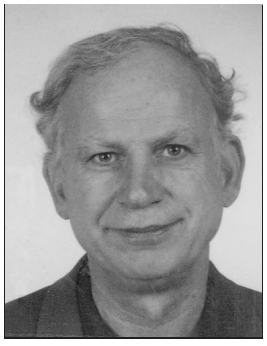

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Paper

Modeling preparation for data mining processes

Timm Euler

Abstract— Today many different software tools for decision support exist; the same is true for data mining which can be seen as a particularly challenging sub-area of decision support. Choosing the most suitable tool for a particular industrial data mining application is becoming difficult, especially for industrial decision makers whose expertise is in a different field. This paper provides a conceptual analysis of crucial features of current data mining software tools, by establishing an abstract view on typical processes in data mining. Thus a common terminology is given which simplifies the comparison of tools. Based on this analysis, objective decisions for the application of decision supporting software tools in industrial practice can be made.

Keywords— data mining, data preparation, KDD process.

1. Introduction

Knowledge discovery in databases (KDD) and data mining are, in practice, complex processes whose development requires advanced skills and precise data understanding. Increasingly, however, software systems that support many aspects of data mining on a high level are becoming available, which makes the development of industrial mining applications easier even for less experienced users. Examples for such systems are SPSS Clementine, SAS Enterprise Miner or MiningMart. For industrial decision makers, the choice of the most suitable decision support software is becoming an important challenge, as many software tools are available, but each has its own strengths and weaknesses. This paper addresses the question of how the different technologies can be compared with respect to their work-saving potentials. The main thesis is that there are critical aspects of a mining process which must be explicitly supported on a conceptual level by a high-quality data mining software. The paper contributes a detailed, conceptual analysis of these critical aspects. This allows to establish a common terminology for a broad range of functionalities, and thus to easily compare different software solutions based on detailed, objective and quantitative criteria.

The general focus of the paper is data processing during data preparation and mining. For the preparation of data, the conceptual analysis presented in Section 3 yields a list of essential operators that must be available in order to be able to compute arbitrary data representations. Arbitrary data representations may be needed for a successful mining phase. During mining itself there are also central data processing tasks that must be supported, such as cross validation and parameter tuning, which is explained in Subsection 3.5. Based on this conceptual view on critical aspects of the process, criteria for existing software solutions in data mining and knowledge discovery are derived in Section 4. The criteria reveal the strengths and weaknesses of such software solutions, allowing clear and objective decisions for the application of data mining software tools in industrial practice.

2. Related work

Conceptual models of knowledge discovery processes (of which data mining is the crucial part) have mostly been developed in the context of data mining supporting tools. In particular, [4, 20] attempt to assist users during the development of discovery processes by automatically exploring various options for the process. To this end, the basic steps in a KDD process are realized by agents in [20]; meta-agents (planners) organise them to a valid process using their input and output specifications. The authors provide an ontology of KDD agents that distinguishes between three phases of the process, namely preprocessing, knowledge elicitation (modeling) and knowledge refinement. The particular choice of agents is not explicitly justified in the published articles; compare the minimal and complete list of operators in Subsection 3.3. Further, the present work includes a conceptual view on the data which is missing in [20]. The same is true for [4], where a system to systematically enumerate and rank possible KDD processes is presented, given some input data and a mining goal. These authors have developed a metamodel for KDD processes. In this respect, there system is somewhat similar to MiningMart [16], which is the system that inspired much of the present work.

A well-known standard to model the KDD process is CRISP-DM [7]. While it gives an overview of different, interdependent phases in a KDD process and defines some terminology, it is not detailed enough to model concrete instances of data preparation and modeling operations, and does not include a data model. An early sketch of a formal model of the KDD process was presented in [19]. The new predictive modeling markup language (PMML) Version 3.0[1], a standard to describe machine-learned models in extended markup language (XML) [18], includes facilities to model the data set and data transformations executed on it before modeling. However, it is not process-oriented,

[1] See http://www.dmg.org/pmml-v3-0.html

thus it does not allow to model a data flow through a complex KDD process, and the data model is restricted to one table. Other standards around data mining are Java data mining (JDM) [13], which includes web service definitions, and structured query language/multimedia extension (SQL/MM) data mining. Though extensible, they currently provide interfaces to modeling algorithms rather than to complete KDD processes.

Recently, some new research attempts to employ grid infrastructures for knowledge discovery; a good overview is given in [5]. To enable the execution of KDD processes on a grid, these processes have to be modeled independently from the machines that execute them, and heterogeneous data schemas and sources have to be modeled. In [2], a discovery process markup language (DPML) is used, based on XML, to model the complete KDD process. This language is used to formalize a conceptual view on the data mining process. Unfortunately, from the available publications it is not clear how comprehensive and detailed DPML is.

Criteria for the comparison of KDD and data mining tools have been listed in several papers [1, 8, 10, 11], but have not been linked with the conceptual works above. The criteria are therefore not consistent across publications and their selection is not justified. The present paper attempts to support the choice of criteria by a conceptual analysis of the essential data processing tasks in data mining for the first time.

3. Data preparation

This section introduces the conceptual notions that are needed to describe a data mining process. Subsection 3.1 introduces two description levels which are used to describe the data (Subsection 3.2) and the data processing (Subsections 3.3 and 3.5).

3.1. Two levels of KDD descriptions

It is generally possible to describe both the data and the preparation tasks on two different levels: a more technical one and a more KDD-related one. The technical level describes the data and any operations on the data independently of any application purpose. The higher level deals with KDD concepts: the role that the data plays, and the purpose of applying a preparation method, are seen in the context of the knowledge discovery application. This level will therefore be called *conceptual*. The differentiation of the two levels will be detailed below. One may relate the different levels to different types of users of data collections: while for example database administrators are concerned with the technical level, KDD experts and statisticians (data analysts) tend to think and work on the conceptual level, as they cannot take the application out of their focus.

One of the purposes of this work is to argue that the two levels should be explicitly supported by KDD software. This has the following advantages:

- If the higher level is made explicit, the lower one can be hidden. A software that hides the technical level can present the entire KDD process to a user in terms of familiar concepts. This eases the development of and daily work on KDD applications.
- By making the conceptual level explicit, it is automatically documented and can be stored and retrieved for later reference [16].
- Independence of the conceptual level allows to reuse parts or all of a conceptual process model on new data by simply changing the mapping to the technical level. Though this may require conceptual adaptations, it saves much effort compared to a development from scratch.
- The use of the conceptual level allows the comparison of different software tools by abstracting from technical details. Criteria for comparison can be formulated on the conceptual level, which makes their communication and application much easier (see Section 4).

3.2. Data description

Throughout the paper, the data is assumed to be in attribute-value format. On the technical level, it is common to think of *tables* which are organized in columns and rows. Conceptually, data is seen as representing objects from the real world; the objects are described in terms of their *attributes*; and each attribute has a *domain* whose *values* it can take. There can be different *sets* of data, with different attribute sets; it is common to refer to the different sets as tables even on the conceptual level, though the term *concept* will be used below. Thus there is a direct and simple mapping from attributes to columns and representation of objects to rows. Whether the columns and rows are gained from a flat file or a database system is unimportant on the conceptual level.

While attributes and concepts are used to describe the data *schema* – the organization of the data – on the conceptual level, a description of the data *contents* is also very useful on this level, since the data processing operations in a KDD process depend on both. Schema- and content-related information are usually referred to as *metadata*. During processing, both the data schema and the data contents, the data itself, change. To have the data characteristics listed below available on the conceptual level requires a data scan, which typically consumes a substantial amount of time because the data sets are large. Therefore, this analysis should be performed as rarely as possible, preferably only once, on the input data (even then, it may have to be performed on a sample of the data). Based on the characteristics of the input data, many characteristics of later, intermediate

data sets in the process can be inferred from the types of operations that were applied, rendering new data scans superfluous.

The useful metadata for modeling a KDD process includes: the number of rows in every table/concept; the minimum and maximum values of each attribute with ordered values; the list of values each discrete attribute takes; and the number of occurrences of each value of each discrete attribute.

One important task that can be solved based on this metadata is the estimation of the storage size needed for storing the data set. This is important during the declarative set-up of the KDD process, as it allows to decide *before* executing the process whether and how to store intermediate data sets (due to limited main memory), because the necessary metadata can be inferred in many cases (though not always). An intelligent administration of intermediate data processing results is important for a smooth execution of the process.

The administration of the above metadata allows not only size estimation, but also an easier declarative development of the KDD process model, as many operations depend on the values that certain attributes take. For example, *Value mapping* is an operator used to change these values, and a specification of an instance of this operator is easy if the available values can be chosen in a graphical interface, rather than be looked up elsewhere and typed in by hand.

Another useful kind of metadata is given by data *types*. On the technical level, the common data types are numbers (integer or real), strings, and calendar dates/clock times. Conceptually, however, one would distinguish types according to the way they represent real-world objects. In this work, four conceptual data types are proposed as essential because their distinction is needed during the development of a KDD process using the processing operators. These types are *date/time*, *key*, *discrete* (further divided into *binary* and *set*), and *continuous*.

In principle, every conceptual domain type can be realized by any technical data type. For example, keys can be realized by strings or by numbers; dates can be represented by strings; and so on. To hide the technical level, a flexible mapping is needed. When new data is introduced, the mapping from the technical to the conceptual level can be done automatically (by inspection of occurring values), but must also be manually manipulable to allow uncanonical mappings, like strings representing dates. The need for flexibility arises from the unpredictable ways in which data preparation operations may be combined. For example, one certain operator produces a binary output consisting of the numbers 0 and 1, where from the view of this operator the output is discrete (no ordering implied). Yet the next operator in a given application chain may compute the mean of that output, interpreting the 0s and 1s as real numbers, which is a neat way of computing the ratio of 1s. Even when such interpretation changes occur, it is still possible to hide the technical level by adjusting it automatically, determined by the kinds of manipulations that an operator defines. This is one example of how a conceptual analysis leads to objective criteria for software.

Further information about attributes (beyond conceptual data type and data characteristics) is given by the *role* it plays in the KDD process. Some attributes are used as input for learning; one or more may contain the target (the label) for learning; still others relate several tables to each other. Thus, four roles are distinguished on the conceptual level (without a correspondence on the technical level): *predictor*, *label*, *key* and *no role*.

Changing the perspective now from (domains of) attributes to whole tables, their contents, and how they relate to each other, it is easy to see that the two levels of description can be applied in a similar fashion. Data represents objects from the real world and describes them along several dimensions. Some objects share similarities, which allows to subsume them in a *class*; the science of what classes exist and how they should be described is called *ontology*. Leaving philosophical approaches aside, the word *ontology* is used in computer science as a countable noun, where an ontology is the description of a shared conceptualization of an application domain [12]. Obviously, a conceptual description of data sets could make use of ontologies. If an ontology exists for the application domain from which the data is collected, it would be very helpful to describe a KDD application on that data in terms of that ontology [6, 9]. However, realizing this idea is fraught with the difficulty that not all ontology formalisms are suitable for supporting KDD-oriented data processing. Data for KDD comes in tables, and the tables are the objects of the extensive modifications which are usual during data preparation. A useful conceptualization, from an operational point of view, should therefore introduce a concept for each table, even though some concepts from the application domain may, in a given data set, be described using several tables, or only a part of one table. The latter problem can however be remedied by the availability of data transformations in KDD to bring the tables into a suitable shape [9].

A mining process consists of a sequence of transformation operations, as explained in Section 3, and each operation introduces a new data set, or in the conceptual view, a new concept. Thus a large number of intermediate concepts is created in a large process, and the intermediate concepts are related by the data flow or process view. However, they are also related in a different way, namely by the nature of their creation: some processing operations create *subsets* of the input data, while others create *specializations*. It will be seen in Subsection 3.3 that several essential processing operations produce such relations. Further, the intermediate concepts may be related by *foreign key links*. The web of these relations allows an alternative view on the data mining process which can help the user to keep an overview of it.

3.3. Data preparation operators

Usually, data preparation is seen as the execution of basic steps, each of which applies some predefined data transformation to the output of the previous step(s), resulting

in *dependency graphs* of data preparation. The predefined data transformations are defined through *operators*, which are specified by their input, their transformation task and their output. It is important to note that input and output can be specified on the higher, conceptual description level. In [14], a list of atomic operations for data preparation was given for the first time. One main contribution of the present work is the classification of these and other operators into *primitive* and *convenience* operators. The former correspond to the technical description level. They provide basic operations without which no complex data preparation can be performed. Their computational power is examined in Subsection 3.4.

The operator *Model learning* has a special status; it is not a primitive operator for data preparation, and does not produce output in terms of the data ontology, but is indispensable for a complete KDD process.

The convenience operators describe data transformations in conceptual, KDD task-related terms; they are mere combinations or special cases of the primitives. As an example, the convenience operator *Dichotomization* takes a discrete attribute and outputs several attributes, one for each value occurring in the given attribute, where the output attributes contain a Boolean flag indicating whether the value they correspond to occurs in that row in the input. This convenience operator can be realized by a repeated application of the primitive operator *Attribute derivation*. However, for a KDD expert user, using the convenience operators where possible is more intuitive than using many primitive operators, and provides an aggregated, high-level view of the preparation graph. Thus again the claim that KDD can be extensively supported on the conceptual level is justified.

In the following, brief descriptions of all primitive and some convenience operators are given.

- *Attribute derivation (primitive)* – a very general operator to create a new attribute, usually based on values of existing attributes. To allow this, extensive date, string and numeric arithmetics must be offered by this operator. In fact, to make the list of primitive operators complete in the above sense, arbitrary computations must be allowed to derive a new attribute. This requires a computationally complete formalism such as a programming language. The input for this operator is any concept; the output is a concept that is a specialization of the input concept.

- *Attribute selection (primitive)* – this operator removes attributes from the input concept. The selection of attributes to be removed is either done by the user or, for advanced applications, automatically, using redundancy criteria or the performance of a modeling algorithm on different attribute sets. The input is any concept with at least two attributes. The output is a concept of which the input concept is a specialization.

- *Join (primitive)* – this operator joins two or more input concepts according to the values of a key attribute specified for each concept. All attributes from the input concepts occur in the output concept without duplicating keys. The input are two or more concepts, each of which has a key that relates it to one of the other input concepts. The output is a concept that is a specialization of all input concepts.

- *Model learning (special)* – this operator is a general place holder for model learning algorithms. In predictive settings, the model gives a prediction function that can be applied to other concepts in the *Attribute derivation* operator. In descriptive settings, only the model itself is produced.

- *Row selection (convenience)* – this operator copies certain rows from the input concept to the output concept, according to some criteria. The input is any concept. The output is a concept that is a subconcept of the input concept.

- *Union (convenience)* – this operator unifies two or more concepts that have the same attributes. The extension of the output concept is the union of the extensions of the input concepts. The input are two or more concepts, each with the same set of attributes. The output is a concept with again the same attributes, of which every input concept is a subconcept.

- *Aggregation (convenience)* – this operator aggregates rows of the input concept according to the values of given *Group by*-attributes. Aggregation attributes are chosen in the input concept; in the output concept, values that are aggregated over an aggregation attribute appear for each combination of values of the *Group by*-attributes. The input is any concept with at least two attributes. The output is a new concept not related to the input concept.

- *Discretization (convenience)* – this operator discretises a continuous attribute. That is, the range of values of the continuous attribute is divided into intervals, and a discrete value is given to every row according to the interval into which the continuous value falls.

- *Value mapping (convenience)* – this operator maps values of a discrete attribute to new values. In this way, different values can be mapped to a single value, thus be grouped together, if they should not be distinguished later in the process.

- *Dichotomization (convenience)* – this operator takes a discrete attribute and produces one new attribute for each of its values. Each new attribute indicates the presence or absence of the value associated with it by a binary flag.

- *Missing value replacement (convenience)* – this operator fills gaps left in an input attribute (the target attribute) by missing or empty values.

3.4. Computational power of the primitive operators

This section considers the range of convenience operators that are definable by the primitives from Subsection 3.3. Though *Attribute derivation* is assumed to use computationally complete mechanisms, it does not add tuples to its input, and it derives only one attribute. Thus it is a natural question to ask whether the three primitives can produce any output concept that is computable from given input concepts. This is a notion of computational completeness based on the given data model. The answer is that the three primitives alone cannot provide this computational completeness without iteration or recursion constructs and counting devices. However, even using the three primitives in schematic algorithms (without looping, recursion or counting) allows to realize a number of important convenience operators as demonstrated above. Below some more precise observations on the expressiveness of the primitives are given, without proof because they are easy to verify.

First, it is easy to see that the six basic operators of the relational algebra, which is used in relational databases, can be reduced to the three primitives above. For projection (*Attribute selection*) and natural join this is trivial; for the other relational operators the reduction to primitives is not difficult either, but is omitted here.

Second, the primitives are in fact more expressive than the relational algebra: it is well-known that the transitive closure of an arbitrary directed graph cannot be computed using the relational algebra [3], but this is possible using the three primitives defined here. Indeed, any function on a given set of input concepts that produces a fixed number of output concepts with known arities, and where the sizes of these output concepts are bounded in terms of the input sizes, can be computed by the three primitives, essentially by computing the cross-product of all values occurring in inputs as many times as needed to create enough entities, and using the power of *Attribute derivation* to compute the function results[2]. The number of edges in the transitive closure of a graph is of course bounded by $O(n^2)$ for n nodes and the arity of the result concept is 2. Thus the transitive closure of any graph given as a 2-ary concept can be computed by forming the cross-product of its nodes (joining the input concept with itself) and then using *Attribute derivation* to mark the relevant edges. The latter are then selected using *Row selection*.

Third, it is easy to see that given an arbitrary function on sets of input concepts, there may be no fixed schema of applying the three primitives to compute it. For example, the function that creates n copies of an input concept, where n is the number of entities in the instance of the input concept, is dependent on the input size and thus cannot be expressed on arbitrary inputs using a constant number of primitives.

From these observations it is clear that the three primitives do not provide full computational power, but they do allow to construct powerful convenience operators for practical purposes, since for many computations which are needed in practical applications, bounds on the output size can easily be given and thus no iteration or recursion constructs are needed in addition to the three primitives.

Obviously the computational power of the primitive operators stems from *Attribute derivation*, but it is a useful insight that these three conceptually simple operators, applied to instances of the intuitive modified entity-relationship (ER) data model, provide very powerful operations. So it suffices to think of data processing on instances of the modified ER metamodel in terms of attribute addition, attribute removal, and joining by keys. For many KDD applications, the full computational power of *Attribute derivation* is not even needed. It will often suffice to employ some of the simpler functions it offers. But KDD is a complex field, and users will need flexible devices to cope with very different situations in different KDD projects. Therefore a KDD workbench should offer the full computational power of *Attribute derivation* as a fall back mechanism for unforeseen situations.

3.5. Data mining

During modeling, conceptual support is mainly needed for training, testing (evaluation of models), and parameter tuning, as well as the visualization of models. Conceptual support here means again to present these tasks in suitable terms; for example, standard operations should be offered to split a data set into training set and test set, to learn, evaluate and apply a model, to automatically find optimal parameter settings, and so on. Since modeling is in itself a complex process, in fact this often leads to a separate graph of processing tasks. Following [15], trees of nestable operators are a suitable, conceptual representation for these tasks. The leaves of the trees represent operations such as the learning or application of a model, while the inner nodes correspond to more abstract, control-oriented tasks such as cross validation or meta learning. This representation provides great flexibility for the design of complex mining experiments, which are independent of the data preparation in that they take a single, fixed data table as input.

4. Criteria for data mining tools

How can the conceptual analysis from Section 3 be applied in practice? The analysis focused on data processing during preparation and mining. According to [17] and a 2003 KDnuggets poll[3], most of the efforts spent on a KDD project are consumed by data preparation. Therefore the analysis above directly concerns work-intensive areas of KDD. It provides the details for a *declarative* development of KDD processes on the conceptual level, given a system that realizes a translation to the technical level.

[2]This argument presumes that at least one input instance has more than one entity.

[3]See http://www.kdnuggets.com/polls/2003/data_preparation.htm

Various data mining systems, like Clementine, IBM Intelligent Miner, SAS Enterprise Miner or MiningMart already realize certain parts of the notions from Section 3, but no "ideal" system exists (yet) that includes all of these notions. The concepts can be directly translated to functional criteria for data mining systems that include data preparation facilities. As a simple example, *all* primitive operators from Subsection 3.3 *must* be available in such systems, otherwise the data preparation facilities are incomplete (the operator *Row derivation* is an exception). The more convenience operators are available, the better. Attribute *roles* must be supported as well as the three types of relations between intermediate concepts (see Subsection 3.2); for this, concepts (representing tables) must be explicitly represented; and so on. These criteria can be objectively and simply checked in any data mining tool. They can also be easily *quantified*, as explained in the following.

Every notion from Section 3 can be broken down into a number of *Boolean* criteria. For example, each operator from Subsection 3.3 corresponds to one Boolean flag indicating its presence or absence in a given tool. The same is true for the four attribute roles. Other ideas from Section 3 can be set up as Boolean lists as well: for example, the explicit support for conceptual data types can be present or absent; the mapping from conceptual data types to technical types may be adjusted automatically in a given tool or not; and so on. This results in a set of detailed, Boolean criteria for data mining tools.

However, while a long list of Boolean criteria is very detailed, it does not serve well to gain a quick overview of the strengths and weaknesses of a tool. To make the evaluation clearer, related criteria can be *grouped*. Assuming a group of $m > 0$ criteria, any given data mining tool will fulfill $0 \leq n \leq m$ of them. This leads to the *n-of-m* metric for evaluating KDD tools, or indeed any type of systems given functional criteria. The size of the groups is variable; each group can have an own value of m. Further, the grouping itself can be adjusted to different purposes. To gain a quick, broad overview, larger groups (larger values of m) can be used, while for detailed inspections smaller groups are recommended. So the *n-of-m* metric is adaptable to different evaluation purposes and different audiences for the presentation of evaluation results. Based on a single, detailed list of Boolean criteria, humanly comprehensible quantitative scores can be formed to compare and evaluate KDD tools.

5. Conclusions

This paper has addressed the important, time-consuming data processing phases of the KDD process, namely data preparation and data mining. It was shown how these tasks and the methods to solve them can be described on two levels, a higher, conceptual one which is independent of the realization of the KDD process, and a lower one that realizes the process. Critical aspects for declarative models of KDD processes were identified, in the area of data descriptions (data models), preparation operators (with a minimal and complete list of essential operators), and data processes around the actual mining algorithm (such as cross validation or parameter tuning). Based on these critical aspects, a methodology to set up objective and quantifiable criteria for the comparison and evaluation of KDD tools was presented. The methodology is adaptable to different evaluation purposes and audiences for the presentation of evaluation results.

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Paper

A framework for event based modeling and analysis

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Abstract— In this paper we will present a framework for modeling and management of complex systems. There are various approaches for modeling of these systems. One of the approaches is events driven modeling and management of complex system. Such approach is needed in information systems that provide information in real-time. Most of the existing modeling approaches use only information about type of event and the time when an event occurs. However, in the databases we can store and then we can use much richer information about events. This information might be structured as well as unstructured. There are new challenges in algorithms development in case of description of event by various attributes.

Keywords— event mining, temporal data mining, telecommunications.

1. Introduction

Recently, the focus is on real-time decision support [1] what requires a new class of the data processing, the analytical algorithms as well as modeling approaches. The actions have to be taken immediately after the event occurred. The delay may cause the fault of the system or significant loses. It should be stressed that we can distinguish a broad spectrum of various types of events. It will often require dedicated algorithms and approaches. However, the framework will help in generalization of the specified methods and algorithms. Moreover, this framework may help in integration of achievements in event based modeling in different scientific disciplines. At this time there are separate developments in temporal data mining, stochastic systems, event based control, etc. The combination of these approaches might significantly improve the results of new algorithms.

2. The modeling framework

Figure 1 shows the basic components of the event driven modeling framework: the system that is influenced by external as well as internal events, data and textual information about the system as well as about the events, models, algorithms, event detection algorithms, knowledge representation, description of decision maker behavior and actions.

In order to build models or algorithms we have to store the data about the system and the events. The existence and the proper quality of data are crucial to any further steps. We can distinguish primary data that are stored in relational databases and data that are prepared for specific modeling tasks. The data can be stored in one central database or can be stored in distributed databases. Moreover, the designers apply event based system design approach which leads to well structured databases that contain information about events. There is also increased importance of using textual information about events. Recently, the video sequences are becoming important source of data for event discovery.

The models use mathematical formulas to describe behavior of the system. In case of the presented framework the models describe dependencies between events and observable variables. Various models can be considered like stochastic models, temporal relationships, temporal sequence associations, etc. The algorithms on Fig. 1 are understood as algorithms that work with analytical models as well as algorithms for event mining or event processing. A key to understanding events is knowledge of what might have caused them and having that knowledge at the time the events happen. Event mining is one of key approaches. Event mining can be defined as a process of finding: the frequent events, the rare events, unknown event (it occurrence can be deduced from observation of the system), the correlation between events, the consequences of event and what caused the event. There is a special class of algorithms for event detection. We distinguish two classes of algorithms. Events detection based on numerical and categorical data analysis and event detection by analysis the textual information. The results of algorithms, data and textual information and results of algorithms go to the block called knowledge representation. In this block there is unifying representation of the results. However, the results are very simple form of the knowledge. Here, there is a place for introducing contextual knowledge and more advanced algorithms that support knowledge creation and management. There will be also represented the knowledge about the consequences of events. The ability to track event causality and consequences is an essential step toward online decision support and important challenge for new algorithms for event mining. The models and algorithms as well as data provides the decision maker important knowledge about the system. Then decision maker can specify various actions that will be applied in the system and reduce the influence of events on the system. The information about actions should be stored in computerized form. That will help later the evaluation of consequences of the chosen actions. In some cases the results of the algorithms can be directly applied to the system (for example the event based control algorithms).

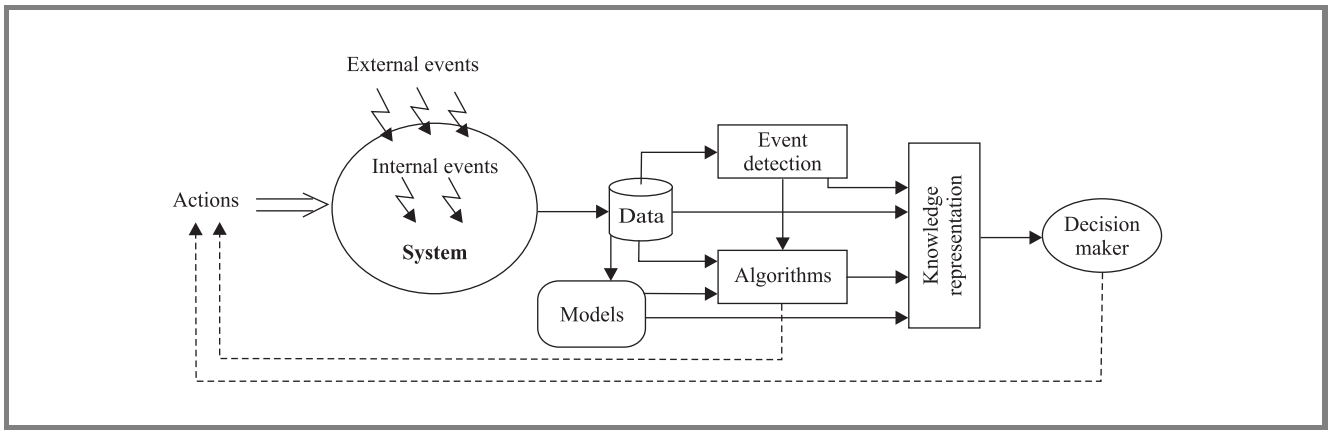

Fig. 1. Basic components of the modeling framework.

3. The applications

The presented approach has various applications in business monitoring, network management, intrusion detection, fault detection, etc. In this section we will present selected examples of event driven modeling: events monitoring, event processing networks, events in environmental scanning, event based control, temporal sequence associations for rare events, event mining and events in alerting systems. There is research on events monitoring in given environment. The sensor networks are applied for events monitoring. Sensor networks are systems of many sensing elements endowed with computation, communication and motion that can work together to provide information about events in an environment. In this case we have information about the type of event, the time and location of events. The control algorithms are used for positioning mobile sensors in response to a series of events. Many monitoring problems can be also stated as the problem of detecting a change in the parameters of a system called event detection. Another important concept are event processing networks (EPN) such networks consist of event processing agents called event sources, event processors and event viewers. EPN have been applied for computer network monitoring. The events sources were middleware sniffers. The aggregated information about events has been displayed by viewers and additionally has been used for event mining. This concept has been also applied for solving business problems. The organizations are working on improvement of the analysis of the external environment and influence of this environment on the performance of the organization. Environmental scanning is a new term and it means the acquisition and use of the information about events, trends, and relationships in an external environment. In this case the methods of dealing with unstructured information about events are especially important. In event based control the sampling is event-triggered instead time-triggered. The event-based PID controller can be build. Such approach reduces CPU utilization. The event-triggered PID controller is nonlinear system of hybrid nature. In many cases we have to monitor and analyze rare events like credit card frauds, network faults, etc. However, if we store the data about the system in the database it is very difficult to identify rare events. In this case the events are characterized by type of event and the time of occurrence of the event. Temporal sequence associations for rare events can be applied to solve this problem. Sometimes, it is impossible to observe the events directly. In such cases the data are stored in databases in form of time series. This data represents observations of the system in selected points. The observations are analyzed by the system and alarms are generated in case of abrupt changes in the values of observations. In the next step another algorithms finds the events that caused changes in the system.

The following algorithms can be considered: for significant change of observation find events that are the reasons of this change, prediction of future events by analyzing the changes of observations, prediction of changes of observations after the event occurs.

4. Conclusion

Presented modeling framework might help in developing future event driven approach for management and modeling of complex. This type of environment might help in real-time decision support. There is a new and challenging area for algorithms development.

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Paper

Decision support for extracting and dissolving consumers' uneasiness over foods using stochastic DEMATEL

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Abstract— In this paper we try to extract consumers' uneasy factors on foods such as carcinogenic substance, bovine spongiform encephalopathy (BSE) problem, genetic recombination, etc., and try to construct structural models among these uneasy factors using stochastic DEMATEL. Stochastic DEMATEL is developed as a revised DEMATEL (Decision Making Trial and Evaluation Laboratory) to extract structural models of a complex problematique composed of many factors under uncertainty. For structural modeling of uneasy factors on foods we look at the binary relation such that "How much would it help to dissolve uneasy factor *j* by dissolving uneasy factor *i*?" Finally, we try to find the priority of dissolving each factor among all the uneasy factors based on the information of stochastic composite importance. This would contribute for decision support to dissolve uneasy feeling and to get sense of security on foods.

Keywords— safe, secure and reliable society, structural modeling, decision support, stochastic DEMATEL, stochastic composite importance.

1. Introduction

Food safety has been the focus of concern of the general consumers, policymakers and risk assessors. Fisher *et al.* [1] presented the potential of a transdisciplinary approach to food risk analysis in terms of delivering additional improvements to public health. As one of such approaches we will try to extract a structural model among consumers' uneasy factors on foods such as carcinogenic substance, bovine spongiform encephalopathy (BSE) problem, genetic recombination, etc., and try to construct structural models among these uneasy factors using stochastic DEMATEL.

DEMATEL (Decision Making Traial and Evaluation Laboratory) [2–4] has been widely used to extract a problem structure of a complex problematique. By using DEMATEL we could quantitatively extract interrelationship among multiple factors contained in the problematique.

It is important and useful to get the structural model of a problematique from which we could find the priority among multiple strategies to improve the structure. This is the main aim of DEMATEL. However, the conventional DEMATEL is insufficient to obtain significant implication of the priority of the strategies for decision making as follows:

1. Shortage of information on the importance of each factor

   In the conventional DEMATEL it is hard to find the superiority of factors, since we could get only interrelationship of factors contained in the problematique. To overcome this difficulty we developed a new criterion "composite importance (CI)" [5] combining the interrelationship of factors and the importance of each factor.

2. Shortage of flexibility to describe structural uncertainty

   Conventional DEMATEL describes the deterministic interrelationship among factors contained in the problematique. However, the strength of the interrelation among factors may be dependent on the various situations, and the fluctuation may depend on the factors taken into account. To overcome this difficulty we developed a stochastic DEMATEL [6] to deal with flexible interrelationship among factors in the problematique.

In this paper in the context of finding priority among multiple strategies to improve the structure of the problematique, we aim at three objectives as follows:

- We describe the method of stochastic DEMATEL briefly.
- We show usefulness and future problem of stochastic DEMATEL through an empirical analysis of ordinary consumers' and food specialists' uneasiness over foods where we deal with structural modeling of uneasy factors on foods.
- Using the information of stochastic composite importance (SCI) we try to find the priority of dissolving uneasy factors on foods.

2. Outline of stochastic DEMATEL

2.1. DEMATEL

Suppose, in a complex problematique composed of n factors, binary relations and the strength of each relation are investigated. An example of binary relation is such that "How much would it contribute to resolve uneasy factor j

by resolving uneasy factor i?" We would get $n \times n$ adjacent matrix X that is called the direct matrix. The (i, j) element x_{ij} of this matrix denotes the amount of direct influence from factor i to factor j. If the direct matrix X is normalized as $X_r = \lambda_1 X$, by using $\lambda_1 = 1/$(the largest row sum of X), we would obtain

$$X^f = X_r + X_r^2 + \Lambda = X_r(I - X_r)^{-1}. \quad (1)$$

Matrix X^f is called the direct/indirect matrix. The (i, j) element x_{ij}^f of the direct/indirect matrix denotes the amount of direct and indirect influence from factor i to factor j.

Suppose D_i denotes the row sum of ith row of matrix X^f. Then, D_i shows the sum of influence dispatching from factor i to the other factors both directly and indirectly. Suppose R_i denotes the column sum of ith column of matrix X^f. Then, R_i shows the sum of influence that factor i is receiving from the other factors. Furthermore, the sum of row sum and column sum $(D_i + R_i)$ shows the index representing the strength of influence both dispatching and receiving, that is, $(D_i + R_i)$ shows the degree of central role that the factor i plays in the problematique. If $(D_i - R_i)$ is positive, then the factor i is rather dispatching the influence to the other factors, and if negative, then the factor i is rather receiving the influence from the other factors. We call D_i the degree of dispatching influences, R_i the degree of receiving influences, $(D_i + R_i)$ the degree of central role and $(D_i - R_i)$ the degree of cause.

2.2. Composite importance

Suppose based on the degree of dispatching influences we found a factor that may contribute to improve the overall structure. In this case to resolve this factor is not necessarily the best choice, since the factor that could contribute to resolve some important factors may be more efficient to resolve even if it may not contribute to improve overall structure. Since the original DEMATEL is not taking into account the importance of each factor itself, it is not possible to evaluate the priority among the factors. Similarly, it is not possible to evaluate the priority of each factor by just looking at the importance of each factor. We need to take into account both the strength of relationships among factors and the importance of each factor. To reflect both viewpoint we proposed the composite importance z as [4]

$$z = y_r + X^f y_r = (I + X^f) y_r, \quad (2)$$

where y_r denotes the normalized n-dimensional vector of y that denotes n-dimensional vector composed of the importance of each factor, where normalized means to divide each element of y by the largest element in y.

2.3. Stochastic direct matrix

In the ordinary DEMATEL the direct influence from factor i to factor j is written in the (i, j) element x_{ij} of the direct matrix X. Suppose the structure of the problematique is uncertain and x_{ij} is a random variable. Furthermore, suppose the stochastic parameter values of x_{ij} are different for different pair of i and j. When each element of the direct matrix is a random variable, each element of the direct/indirect matrix X^f is also a random variable. Furthermore, the composite importance z is also a random variable. Therefore, it is necessary to extend the ordinary DEMATEL to deal with uncertainty in the problem structure. We developed a stochastic DEMATEL [6] in which we could take care of various uncertainties in the problem structure.

In the stochastic DEMATEL it is postulated that we describe the amount of direct influence by expectation and the amount of uncertainty by variance and the shape of distribution. Suppose we got $n \times n$ direct matrix X and the matrix E of probability density function as

$$E = \begin{bmatrix} g_{11}(x|\theta_{11})\Lambda\ g_{1n}(x|\theta_{1n}) \\ \mathrm{M} \quad \mathrm{O} \quad \mathrm{M} \\ g_{n1}(x|\theta_{n1})\Lambda\ g_{nn}(x|\theta_{nn}) \end{bmatrix}, \quad (3)$$

where $g_{ij}(x|\theta_{ij})$ denotes the probability density function of direct influence from factor i to factor j, and θ_{ij} denotes the parameters of this probability distribution including expectation and variance of the random variable x_{ij}.

Let G_X be a set of stochastic direct matrices X^s generated by a Monte Carlo method from the direct matrix X with probabilistic information. Then, we obtain

$$G_X = \left\{ X_1^s, X_2^s, \Lambda, X_t^s \right\}. \quad (4)$$

In the stochastic DEMATEL we need to collect the information on the variance as well as on the expectation of influence.

2.4. Manipulation in stochastic DEMATEL and stochastic composite importance

We normalize the stochastic direct matrix as

$$X_r^s = \lambda_2 X^s, \quad (5)$$

where $\lambda_2 = 1/$(the largest row sum of X^s). Then we obtain

$$X^{sf} = X_r^s + (X_r^s)^2 + \Lambda = X_r^s(I - X_r^s)^{-1}, \quad (6)$$

where X^{sf} denotes a stochastic direct/indirect matrix that has the same property as the ordinary direct/indirect matrix. If we obtain X^{sf} for all X^s contained in G_X, we obtain a set G_{XF} of stochastic direct/indirect matrix as

$$G_{XF} = \left\{ X_1^{sf}, X_2^{sf}, \Lambda, X_t^{sf} \right\}. \quad (7)$$

Stochastic composite importance is obtained as

$$z^s = y_r + X^{sf} y_r = (I + X^{sf}) y_r. \quad (8)$$

The set G_Z of SCI is obtained as

$$G_z = \left\{ z_1^s, z_2^s, \Lambda, z_t^s \right\}. \quad (9)$$

Furthermore, we could obtain the set of the degree of dispatching, the set of the degree of receiving, the set of the degree of central role and the set of the degree of cause, respectively.
In the ordinary DEMATEL we could decide the priority of each factor based on the value of composite importance itself. In the stochastic DEMATEL we use three stochastic decision principles as follows:

- **Expectation principle**. We decide the priority based on the expected value or median of SCI.
- **Max-min principle**. We decide the priority of each factor by maximizing the worst value (either 2.5% or 25%) of SCI. This principle reflects a pessimistic decision.
- **Max-max principle**. We decide the priority of each factor by maximizing the best value (either 75% or 97.5%) of SCI. This principle reflects an optimistic decision.

The stochastic DEMATEL could describe the uncertainty of the structure of complex problematique, could describe the uncertainty of priority by SCI and could decide the priority of each factor reflecting the decision makers attitude whether he/she is pessimistic, neutral or optimistic.

3. Structural modeling of uneasy factors over foods by stochastic DEMATEL

We use the data obtained from ordinary consumers and food specialists. For both groups 10 uneasy factors are chosen as follows:

1) food additive (FAD),
2) genetic recombined food (GRF),
3) food forged display (FFD),
4) agricultural chemical problem (AGC),
5) imported food (IPF),
6) BSE problem (BSE),
7) environmental hormones (EVH),
8) carcinogenic (CAR),
9) allergic (ALL),
10) food poisoning (FPO).

Respondents to the questionnaire are 10 ordinary consumers and 10 food specialists. The importance of each factor is asked to the respondents by 5-grade evaluation where the importance of each factor means the degree of feeling uneasy for each factor. Then, the strength of binary relation for each pair of factors is asked by 3-grade evaluation. We look at the binary relation such that "How much would it contribute to resolve uneasy factor j by resolving uneasy factor i?"
The direct matrix is obtained by averaging the data of 10 people on the strength of binary relations. The data for the importance of each factor are first normalized between 0 and 1 and then averaged for 10 people.
The dispersion of the data of the strength of binary relations obtained from the respondents are used as the variance of the strength of binary relations. This implies that the variations among people would induce structural uncertainty. The shape of the stochastic distribution is assumed to be cutting normal distribution defined on [0–1,000,000], since the data obtained from respondents are all positive numbers. The number of stochastic direct matrices generated by random numbers are 1,000.
Structural model for uneasy factors of ordinary consumers is described as follows: the degree of central role for FFD (1.45) is high and FFD has the property of both cause factor and effect factor, but since the degree of cause for FFD (0.55) is positive, FFD is rather a cause factor. Actually, FAD, GRF, IPF, BSE and ALL are greatly affected by FFD.

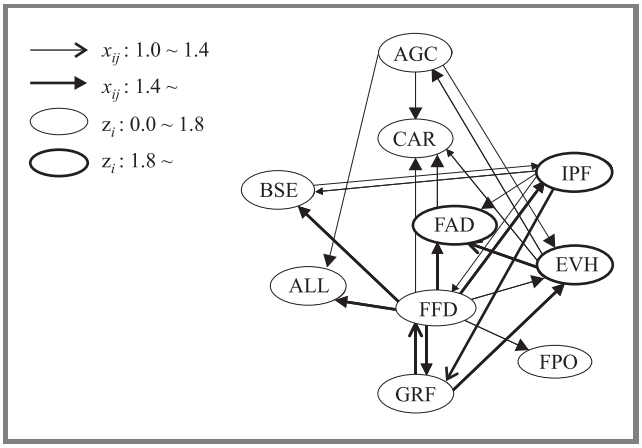

Fig. 1. Structural model of uneasy factors for ordinary consumers.

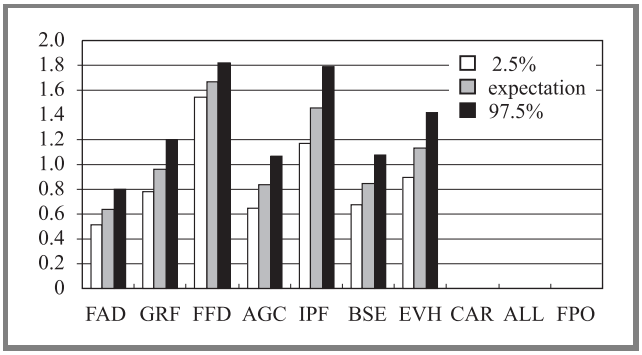

Fig. 2. Degree of dispatching influences for ordinary consumers.

The degree of central role for FFD (1.45) is the highest, and then IPF (1.33), EVH (1.24), GRF (0.941) and AGC (0.850). These factors are all cause factors. On the other hand CAR, ALL and FPO are completely effect fac-

tors. Figure 1 shows a structural model of uneasy factors for ordinary consumers.

Figure 2 presents the degree of dispatching influence, and Fig. 3 shows the stochastic composite importance, for ordinary consumer. In these figures, besides expected values, 2.5% and 97.5% data are also shown.

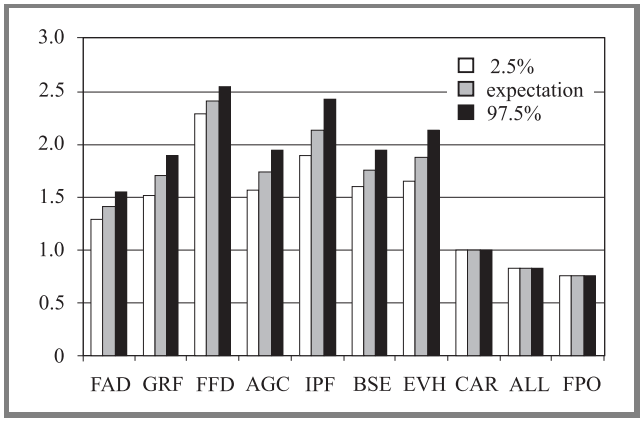

Fig. 3. Stochastic composite importance for ordinary consumers.

Figure 4 shows a structural model of uneasy factors, Fig. 5 presents the degree of dispatching influences, and Fig. 6 shows the stochastic composite importance, for food specialists.

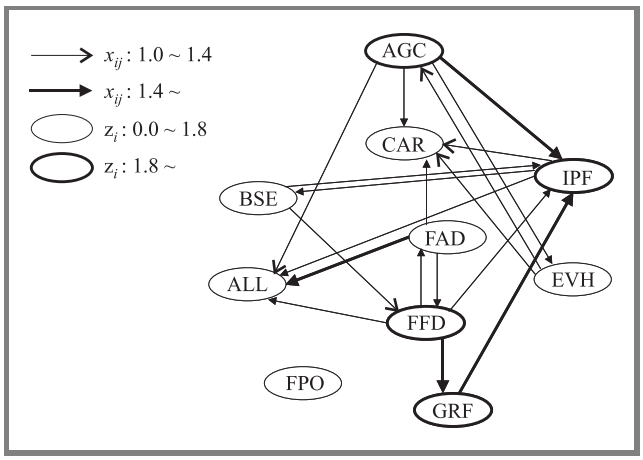

Fig. 4. Structural model of uneasy factors for food specialists.

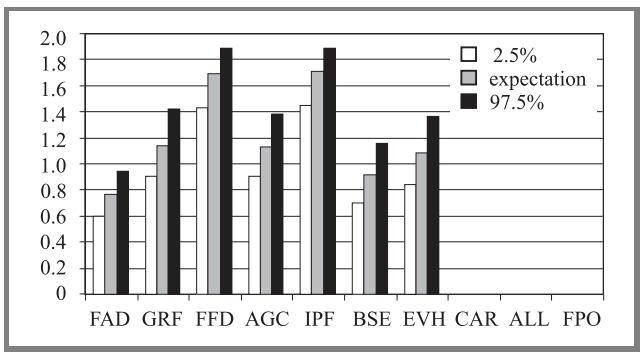

Fig. 5. Degree of dispatching influences for food specialists.

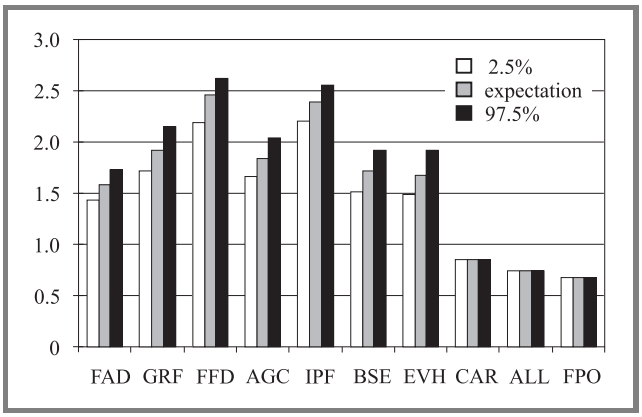

Fig. 6. Stochastic composite importance for food specialists.

For ordinary consumers is as follows: FFD, IPF, EVH, BSE, AGC, GRF, FAD, CAR, ALL, FPO. On the other hand, the priority for food specialists is as follows: FFD, IPF, GRF, AGC, BSE, EVH, FAD, CAR, ALL, FPO. These results are obtained from SCI.

4. Concluding remarks

In this paper a stochastic DEMATEL is applied to structural modeling of ordinary consumers' uneasy factors on foods taking into account the uncertainty of structure. It is demonstrated that the stochastic DEMATEL and the information obtained by the SCI are quite useful for structural modeling of complex problematique under uncertainty. A new knowledge obtained in this study is that in order to resolve uneasy factors over foods it is effective to solve the problems of food forged display (FFD) and imported food (IPF).

For further study we need to develop a method of identifying appropriate probability distribution function or we need to develop a non-parametric approach. We also need to develop a method of collecting information on variance. For these purposes we need to experience more empirical analysis of various case studies.

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Regular paper

Band energy approach and nonlinear optical phenomena in large-sized nanoparticles

Ivan V. Kityk

Abstract— This report presents principal optical properties of semiconducting large-sized nanocrystalline (LSNC) material with thickness within 10–50 nm range. A quantitative parameter for description of the LSNC state is introduced. Influence of the surrounding polymer matrices on the properties of the LSNC is shown. Role of the surrounding polymer matrix background in the manifestation of material optoelectronic properties is studied. Superposition of long-range ordering with the localized nano-quantized effects is analyzed within a framework of different one-electron band energy approaches. Manifestation of the LSNC nonlinear optical (NLO) properties will be presented separately. It is clearly demonstrated that appropriate choice of LSNC geometry parameters together with proper modification of chemical composition could enhance the key NLO parameters.

Keywords— nanocrystallites, nonlinear optics.

1. Introduction

A new direction in the nanotechnology is intensively developed today, based on possibility of using semiconducting nanocrystallites as materials for optics and electronics, particularly quantum electronics, due to nano-confined size-dependent electronic and optical properties of these nanoparticles [1, 2]. The material attracts interest due to both immediate applications in electronics and communications [3–6] and fundamental science, due to specific properties of such materials [7, 8]. Their applications are based on a quantum confinement influence on material properties, which implies their grain size and shape topology dependence to the corresponding susceptibilities. Traditional nano-confined effects are usually observed for particle size below 8–10 nm [9], where effects of nano-quantization are observed. In such cases, ***k***-space bulk-like dispersion disappears and discrete excitonic-like nano-levels occur inside the energy gap [10].

There exist also an intermediate semiconducting crystallites possessing averaged sizes equal to about 10–50 nm. In this case a coexistence of typical bulk-like properties (first of all possessing energy ***k***-dispersion due to translation symmetry) disturbed by thin near-surface boundary interfaces with thickness about 1.5–4.5 nm is observed. Such properties can be useful for different applications, particularly for biological ones [11], light emitting devices [12], solar cells, laser modulators and deflectors [13], etc.

2. Principal parameters of large-sized nanocrystallites

The quantum-confined boundary interfaces possessing bulk-like as well as dot-like quantized excitonic properties can also be of importance for different experimental utilization of optical phenomena, particularly in nonlinear optics (NLO) due to large charge density gradients determining large values of NLO susceptibilities [14]. Reconstructed interface boundary sheets with thickness of 1.5–4 nm, separating the bulk-like crystallites and surrounding amorphous-like or disordered background, determine these features. Very often this fact is neglected, which precludes appropriate use of their properties.

Principal parameter for the large-sized nanocrystalline (LSNC) is a ratio between thickness of the reconstructed boundary layer (sheet) and total effective diameter of the nanoparticle. Coexistence of bulk-like long-range ordering possessing ***k***-space dispersion, high effective mass, low carrier mobility gradients and quantum size-confined layers possessing a blue spectral shift, gives a rare possibility to operate by principal electron parameters within the same material.

The ZnO and ZnS-based LSNC appear to be particularly appropriate for modelling of luminescent behaviour, exhibiting green and yellow luminescence. Moreover, ZnO LSNC is technologically complementary to GaN, which opens a way for optoelectronics applications. The investigated ZnO-based LSNC were synthesized using different techniques (mechanical crushing, RF sputtering, spray pyrolysis, etc.). This LSNC had grain size of 15–60 nm. The topological profile of the LSNC was prevailingly ovalloid-like. The ZnO-based LSNC were incorporated into polymethyl methacrylate (PMMA) matrices, with LSNC content of 2–6% by weight. Photoluminescent measurements were performed using nitrogen laser excitation ($\lambda = 337$ nm) and grating monochromator with spectral resolution of about 7 nm/mm.

In the present work we introduce a new approach to search for and design of new optoelectronic and non-electronic materials based on LSNC. This approach takes into account both bulk-like and interface contribution; influence of the long-range ordering and influence of the surrounding matrices.

For the luminescent measurements ZnO-based LSNC having sizes of 15–30 nm and reconstructed sheets with thick-

ness of 1–4 nm were used. One can operate by principal nanoparameters by varying the ratio between the LSNC and sheet sizes. Of particular importance is the behaviour of trapping levels caused by the LSNC. Because the LSNC possess prevailingly ovalloid form, one can assume that the LSNC, like spherolites, possess effective sizes corresponding to the sphere with effective averaged renormalized radius. One can introduce an effective coefficient of nanocrystallinity by labelling the effective radius of the LSNC by R and the thickness of the reconstructed sheet by d [3]:

$$Eff_{NC} = \left(R^2/(2Rd - d^2) - 1\right)^{-1}. \quad (1)$$

3. Luminescent features

Typical measured photoluminescence spectra of ZnO-based LSNC with different structural parameters are shown in Fig. 1; existence of two principal spectral bands of luminescence can be deduced. The first one – originating from bulk-like states (at energy about 3.2 eV), is spectrally stable with respect to the LSNC sizes. For the nano-confined effects the second band covering low-energy wide spectral range at 2.2–2.4 eV is more important. It demonstrates a clear blue shift with increasing thickness of the reconstructed surface region. At the same time, intensities of the main spectral maxima decrease with decreasing sheet thickness t. Generally, this effect may be explained by transfer of excitation between the localized nano-confined reconstructed layers and quasi-continuous bulk-like crystalline states.

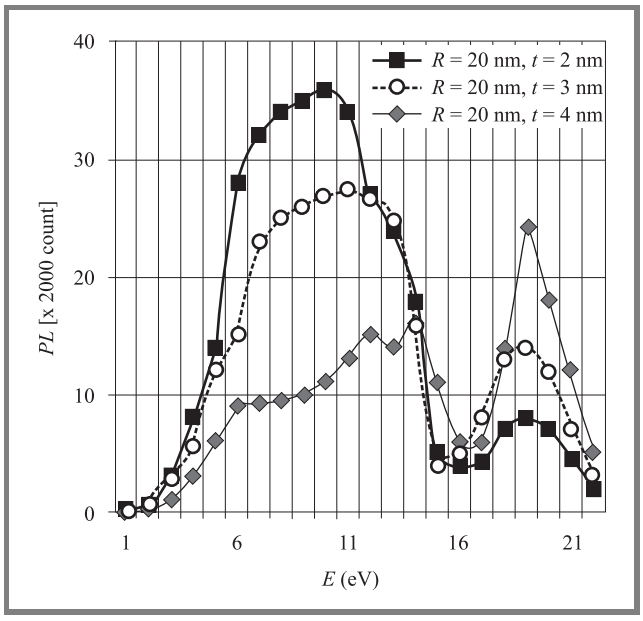

Fig. 1. Photoluminescence spectra of ZnO-based NC incorporated into amorphous-like ZnO matrix, with the different size parameters.

To clarify role played by parameter Eff_{NC}, Fig. 2 shows dependences of the low-energy maxima's spectral positions together with relative intensities of luminescent spectra versus the parameter Eff_{NC} calculated according to Eq. (1). For convenience, the wide spectral band may be divided into separate spectral regions related to different groups of nanoparticles. Another interesting fact is a presence of the near-surface recombination in LSNC determining the distance at which carriers can diffuse to the interfaces before their recombination. With decreasing sizes of the LSNC contribution of the defect states will decrease.

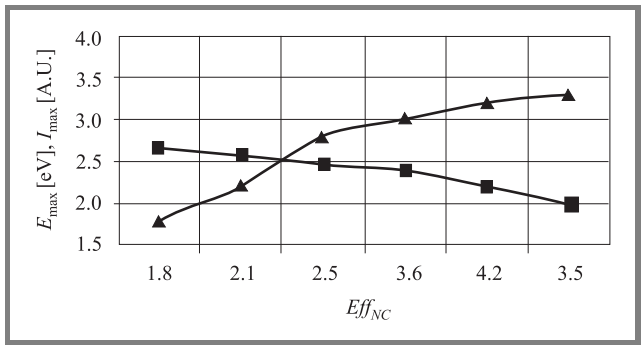

Fig. 2. Dependence of the low-energy photoluminescence spectral maxima (squares) and its relative intensities (triangles) on Eff_{NC}.

This report presents several examples of striking LSNC features and demonstrating key differences between LSNC nanochromophores and other types of nanomaterials – nanowires, dots or crystallites. However, main physical insights clearly indicate substantially different states, compared to nanomaterials studied earlier. This allows to evaluate suitability of LSNC for applications in optoelectronics.

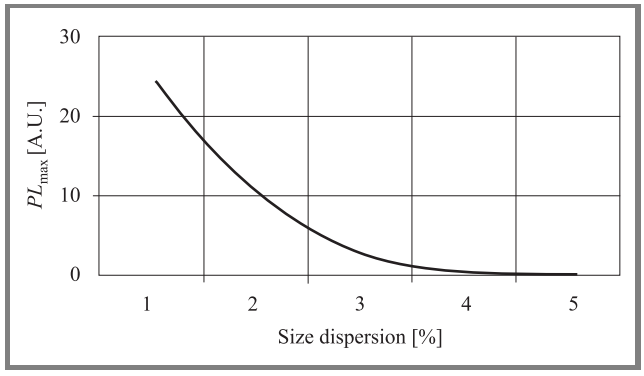

Fig. 3. Dependence of the photoluminescent (PL) maxima for the low-energy luminescence of the ZnO-based NC on size dispersion, defined as a ratio of average deviation of NC sizes to total thickness of reconstructed sheet.

Size dispersion also may be crucial for such effects – see Fig. 3. Usually this factor is neglected, which leads to appearance of substantially different results. Recent work performed on high quality ITO nanocrystallites has confirmed this assumption [14].

4. Manifestation of the LSNC in the FT-Raman spectra

The role of the phonon sub-system in nano-confined effects is not clear. This is a consequence of long range manifestation for such kind of states contrary to localized nano-confined effects. For this reason, the Fourier-transform (FT)-Raman spectra were investigated separately for semiconducting nanocrystallites, particularly for ZnS (Fig. 4) and LSNC incorporated into polymer matrices (Fig. 5).

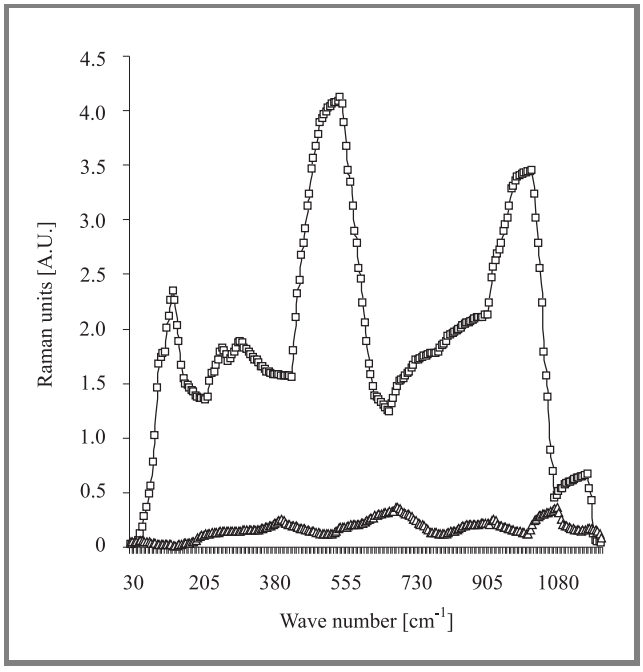

Fig. 4. FT-Raman scattering spectra for the ZnS-based large-sized NC (about 20 nm in diameter). A spectrum for the PMMA is given for comparison (lover curve). Estimated interface sheet thickness is about 2 nm.

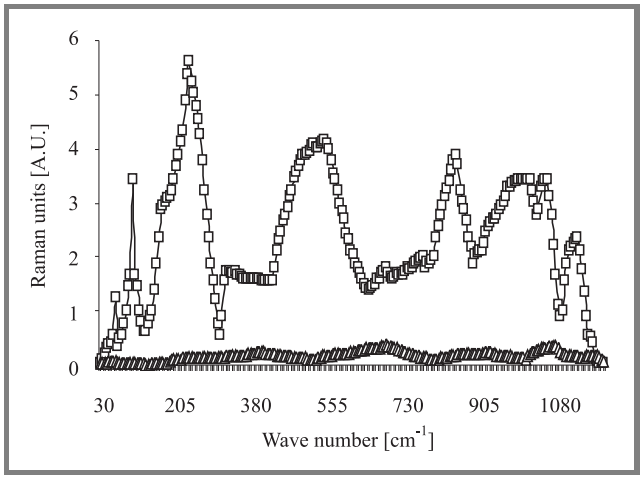

Fig. 5. FT-Raman scattering spectra for the ZnS incorporated into PMMA matrix. The parameters of the LSNC are the same as in the previous case. The PMMA spectrum is presented below for comparison.

Comparing these results with data obtained for the polymer matrices one should emphasize role of the reconstructed interfaces on the effects observed. This observation for different semiconducting nanocrystallites may be considered an additional confirmation that contrary to the traditional nanocrystallites, in case of the LSNC we deal with substantial influence of delocalized bulk phonon sub-system.

5. Band energy approach for research and design of LSNC with desired optoelectronic and nonlinear optical properties

The reconstructed surfaces in LSNC may substantially affect its optical properties, in particular optical nonlinearity, with LSNC having substantial advantages over traditional bulk-like or classical materials in this respect. This is caused by a coexistence of more delocalised band energy-like structural fragments and flat-like localized levels, originating from nano-confined states. The latter are formed exclusively in thin nanosheets separating the LSNC and surrounding material. Most of the published investigations are empirical and there is no systematic study of interfaces using the first principle theoretical methods. However, to use LSNC as material for optoelectronics, data on the state and transitions dipole moments, inter-level energies and related hyperpolarizabilities are needed. A novel situation compared to traditional case is the coexistence of delocalized and localized states, which restrains a possibility of application of the same basis, e.g., plane wave basis set.

The issue of reconstructed surfaces in the LSNC materials results in complicated application of principles of equilibrium thermodynamics, because such structures are very sensitive to nonequilibrium technological conditions. Unfortunately, several crucial parameters are difficult to be monitored; therefore building a reliable theoretical band energy picture is one of the main goals of this work.

Generally one can expect appearance of reconstructed near-surface structure possessing structural conformations corresponding to thermodynamically metastable (or even unstable) phases. Usually, the classical mechanics molecular dynamics method includes variation of principal structural parameters (bond lengths, angles, torsion angles, etc.) to find a structural configuration corresponding to a minimum of total energy. For the LSNC, we deal with the bulk-like perfect crystallites to which one can apply principles of long-range ordered symmetry and different one-electron band energy methods. However, the long-range ordering usually is broken in the interface region, contrary to perfect bulk-like crystals. In the present work we develop an approach introduced in [10], where an effective complex approach was proposed, combining principles of equilibrium thermodynamics with nonequilibrium perturbation created by surrounding disordered background. The method may be applicable to semiconductors and dielectrics with en-

ergy gaps larger than 1 eV and includes coexistence of many structural fragments (up to 120), renormalized by appropriate weighting factors. The main principle of such approach consists of formation of several coordinated layers (of thickness not more than 6 nm) possessing perfect long-range ordered translation symmetry. Afterwards, we add step-by-step 2–3 structurally disordered layers on the borders of crystalline layers. Geometry optimization is performed between the long-range ordered crystalline layer and disordered layers, assuming fixed atomic positions for crystalline layers.

Such coexistence of localized and delocalised states favours enhanced space gradients of the dipole moments, which determine large optical susceptibilities, being least an order of magnitude larger than in traditional bulk-like or classical nanomaterials. Despite a relatively low volume of such states we are able to achieve the large nonlinear optical susceptibilities. So a main direction of research is finding optimal parameters, like thickness of interfaces, degree of crystallinity of the interfaces, concentration of the LSNC, difference between the energy gaps and dipole moments of the LSNC chromophore and corresponding matrices. To solve this problem only the band energy approach may be useful.

The evaluation of structural form-factor is done following a procedure of superposition of the different structural phases with appropriate weighting factors [10]. As an example one can present ZnS-based LSNC interface phases at distances 1–3 nm from the border separating nanocrystalline and disordered phases (Fig. 6). One can see substantial flattering of the principal bands originating from the layers situated 1–3 nm away from the interfaces. This band dispersion flattering leads to occurrence of electron states possessing substantially different effective masses. Gradients of such effective masses are very large; his favours enhanced hyperpolarizabilities and corresponding nonlinear optical susceptibilities.

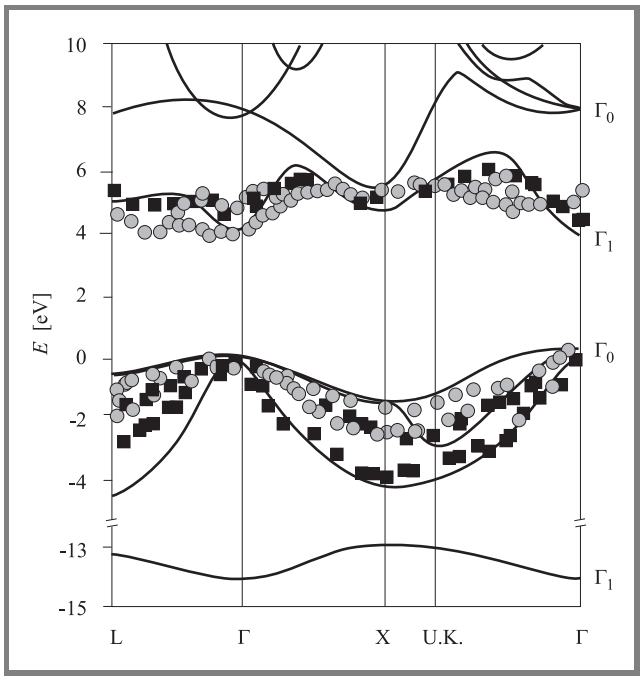

Fig. 6. Band energy structure of the ZnS-based LSNC with effective average sizes about 22 nm and film thickness sheet 2.3 nm. The calculations are done for distances 1.2 nm (circles) and 2 nm (squares) from the boundary interfaces.

Observed enhancement of susceptibilities is due to reconstructed surfaces, which are similar to sheets to which an approach of quantum well effects may be applicable. For narrow wells where with the well widths is less than 3D exciton Bohr radius, excitonic (hole-electron Coulombic interactions) effects dominate electro-optic properties, manifesting through the dipole momentums.

For example, Fig. 7 is a sketch of LSNC interfaces demonstrating the role of the nano-confined and trapping levels. When the thickness of layers is larger, quantum size effects become less important and long-range ordered phonon states seem to be more important. One can clearly see that in the same effective point of the space we have coexistence of several structural fragments. Technologically changes of the corresponding band energy parameters may be performed by monodispersion of LSNC sizes and their concentration. Another way is to find appropriate matrices to enhance the desired dipole moments and related susceptibilities.

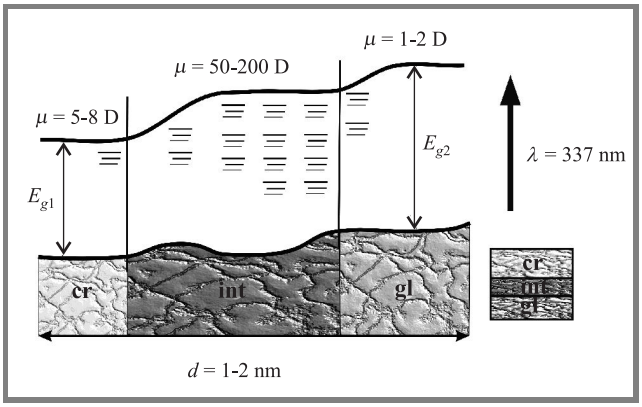

Fig. 7. Principal scheme of interfaces between nano-confined and trapping levels separating polymer matrix and crystalline semiconducting films.

The origin of the observed low-energy luminescence is closely associated with the trapping levels originating from near-surface states in the LSNC (see Fig. 6). With decreasing reconstructed surface-related volume we can suppress intensity of the output luminescent quantum efficiency. The latter play both diagnostic as well as the application role during changes of the effective charge transfer.

The behaviour presented in the previous pictures indicates a substantial difference between LSNC and traditional nanoparticles. Another important aspect substantially attributed to LSNC is an occurrence of oriented spin-polarized vacancy states in the near-surface surrounding polymer sheets. The latter create additional effective layers involved in the process.

The quantum efficiency of the *PL* is substantially sensitive to the type of the matrix or more precisely to the dipole momentums of the surrounding polymer matrices. This is substantially different compared to traditional nanocrystallites, where influence of the matrix on the intensity of luminescence is very low due to screening of local Lorenz field effects.

Using the data obtained, one can conclude that the deep-level core luminescence is substantially suppressed by near-surface trapping level luminescence. This fact may serve as an additional confirmation of important role played by near-surface states in the effect observed, contrary to pure nano-confined states in case of traditional nanoparticles. We probably deal with changes originating from both parts of the boundaries – from the polymer boundary states and from the reconstructed crystalline states (Figs. 4 and 5).

It was also revealed, that less polarized matrices (with lower dipole momentums) exhibit substantially less quenching

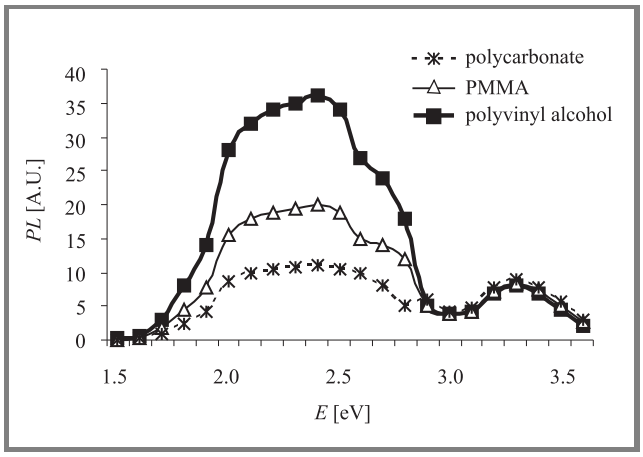

Fig. 8. Dependence of the ZnO-based LSCN *PL* in different host polymer matrix.

of the emission. One can use external cladding for operation by emission in different LSNC (Fig. 8).

6. Origin of nonlinear optical effects formed by the LSNC

Coexistence of localised and delocalised states is a main source of the observed giant optical susceptibilities. However, to utilize these large susceptibilities it is necessary to have an appropriate architecture of particular LSNC chromophore. The nonlinear optical effects used in the optoelectronic devices are of two types. The first of them is a second-order one described by third rank polar tensors and corresponding to linear Pockels effect, optical second harmonic generation. The other is a third-order optical device determining the two-photon absorption and third harmonic generation. Principal difference between the two effects consists in a requirement of the macroscopic non-centrosymmetry for the first class of the devices.

Out of a large number of nonlinear optical effects for such kinds of LSNC, here is presented only a dependence of the two-photon absorption (TPA) for several oxide-based LSNC (see Fig. 9). Figure 9 shows typical dependences of the TPA for the LSNC of different crystalline materials. For comparison, we have shown data for β-BaB_2O_4 and $KTiOPO_4$ LSNC. One can clearly see substantially better parameters of YAB borates (even with different Cr^{3+} content) compared to other materials, which indicate an important role of the LSNC content, contributing to the NLO susceptibilities.

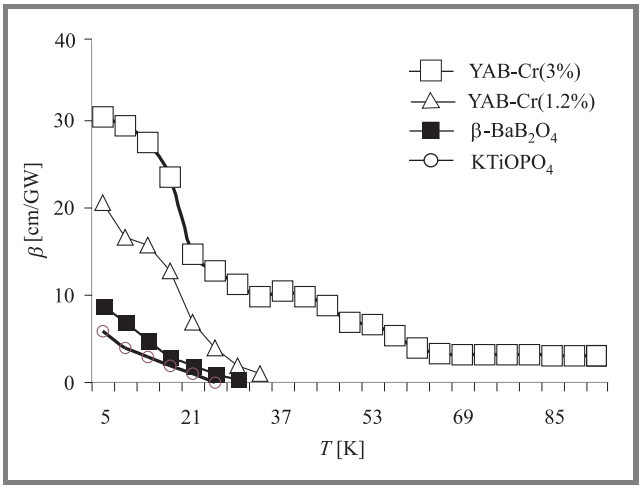

Fig. 9. Temperature dependence of the TPA for the 22–25 nm NC.

The LSNC effect is enhanced at low temperatures, which confirms the crucial influence of the phonon bulk-like subsystem. Moreover, the effects related to condensation of the so-called soft phonon modes responsible for the different kind of phase transformations role begin to play a substantial role at low temperatures. Simultaneously, the localized energy terms of the rare earths give additional contribution to the nano-confined states. Similar behaviours were observed for both second- as well as for the third-order nonlinear optical effects and may serve as an independent confirmation of the applicability of these materials simultaneously as light emitting materials and materials for optical limiters, modulators and for optical parametric transformations.

The effects observed allow to propose LSNC as promising materials for modulation of light, particularly for transmission of information over optical fibres. Their main advantage is large optical susceptibility in comparison to traditional materials. Another option is the possibility of changes of principal parameters like signal to noise ratio by varying their concentration and sizes. By appropriately selecting the host polymer, glass or amorphous-like matrices one can optimise parameters like optical windows [15, 16]. Simultaneous blue-shifted luminescence in the LSNC in combination with large nonlinear optical susceptibilities may be useful for making light emitting diodes, optically operated multi-functional optoelectronic devices, photode-

tectors, etc. Of particular interest is the possibility of photoinduced changes of the basic nonlinear optical parameters, which allows use of the same materials for multiple (erasable) optical recording of information.

7. Conclusions

A principal difference between large-sized semiconducting (dielectric) nanocrystallites and traditional (below 8–10 nm) material is demonstrated, and a parameter for description of the observed effects was introduced. Luminescent, FT-Raman and nonlinear optical spectra demonstrate a crucial role of LSNC interfaces in the observed effects. Following the band structure calculations and molecular dynamics simulations of the interfaces, it is shown that the interfaces are crucial for manifestation of the blue spectral shift and enhanced optical susceptibilities. Another important factor is related to the contribution of the phonon sub-systems, determining high temperature sensitivity of the corresponding effects. Optoelectronic devices having simultaneously efficient light emitting and nonlinear optical properties effectively controlled by LSNC sizes are proposed.

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